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THE

COMPLETE PRACTICAL ARITHMETICIAN:

CONTAINING

SEVERAL NEW AND USEFUL

IMPROVEMENTS

ADAPTED TO THE

USE OF SCHOOLS AND PRIVATE TUITION.

BY THOMAS KEITH.

THE EIGHTH EDITION.

Arithmetic is the easiest, and consequently the first sort of abstract reasoning which the mind commonly bears, or accustoms itself to, and is of such general use in all parts of life and business, that scarce any thing is to be done without it.—*Locke on Education.*

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PREFACE.

ARITHMETIC is justly considered as the basis of every part of mathematics; for, even in comparing magnitudes with each other, recourse is frequently had to numbers: several instances occur in the Vth Book of EUCLID, which, in many parts, would be almost unintelligible without a reference to numbers. Arithmetic must have originated as soon as mankind began to hold any commercial intercourse with each other; for when commerce began to be established, they would soon see the necessity of enquiring into the nature of Numbers, without which no Business could be carried on; but when, or by whom, it received its form as an Art, or Science, is very uncertain.

The Phœnicians, the Descendants of Noah, who settled on the Coasts of Palestine, were the first People in the World who made Navigation subservient to Commerce. Hence it is extremely probable that Arithmetic had its Rise among the Phœnicians, and that they introduced it into Egypt; and this opinion is supported by Proclus in his commentary on the First Book of Euclid. But

Josephus tells us, that, a Famine happening in Canaan, Abraham retired into Egypt, and was the first who taught the Egyptians the Sciences of Arithmetic and Astronomy, and these he brought with him from Chaldea. From Egypt they were transmitted to Greece, by Pythagoras and others, and thence to the Romans.

The first step necessary towards rendering the idea of Numbers intelligible and useful, would be to establish a method of Notation, upon which calculations were to be founded.

The Greeks, Hebrews, Romans, and several other nations, used a Notation by the letters of the alphabet. The best method made use of by the Greeks was that wherein the first nine letters of their alphabet represented the Numbers from One to Nine; the second nine any number of Tens, from Nine, as Ten, Twenty, &c. to Ninety. Any number of Hundreds they expressed by other letters, supplying what was wanting with some other marks; and, in this order, they proceeded, using the same letter again, with different marks, to represent Thousands, Tens of Thousands, &c. No particular treatise of their art of computation has been transmitted to us. There is a Commentary, by Eutocius, upon Archimedes' Treatise of the Dimensions of a Circle, in Dr. Wallis's Works, and some fragments of Pappus, which relate particularly to Multiplication, and sufficiently shew us the difficulty attending their practice, owing to their imperfect Notation.

The simple characters made use of by the Romans were taken out of their alphabet of capital letters, and were the seven following, viz. I. One; V. Five; X. Ten; L. Fifty; C. one Hundred; D. five Hundred; M. one Thousand. The intermediate numbers between these were expressed by a repetition of the same, and the sum of their values represented the number, the character of the greatest value being set to the left-hand; as II. Two; III. Three; VI. Six; VII. Seven; XII. Twelve; XV. Fifteen; XXI. Twenty-one; LX. Sixty; DX. five Hundred and Ten; DC. six Hundred; DCCCC. nine Hundred: DCCCCLXXXXVIIII. nine Hundred and Ninety-nine, &c. But to prevent too great a repetition of the same characters, they sometimes set the less character before the greater, and then the difference of their values represented the number; as, IV. Four; IX. Nine; XL. Forty; CD. four Hundred; CM. nine Hundred. When a number was expressed by more than two characters, they distinguished it from the character on the left-hand by a point; thus, C.XL. one Hundred and Forty; CD.XC.IX. four Hundred and Ninety-nine; and so on for numbers greater than a Thousand. Besides these, they had other expressions for numbers greater (and some less) than a Thousand; thus, for D. five Hundred, they wrote, IↃ; and then, by adding another Ↄ. it gradually increased tenfold; as IↃↃ, five thousand; IↃↃↃ, fifty thousand; &c. Again, for M. one Thousand, they wrote CIↃ; and, by joining Ↄ and C to the right and left, it expressed ten times the value; thus CCIↃↃ, ten Thousand, &c.; or, by drawing a line over any number less than one Thousand, it expressed as many thousands as the letter, or letters, contained units; thus $\overline{\text{V}}$. five

Thousand; $\overline{\text{VI}}$. six Thousand; $\overline{\text{LX}}$. sixty Thousand; $\overline{\text{C}}$. a hundred Thousand; $\overline{\text{M}}$. a Million, &c. Thus we see the difficulty the ancient Greeks and Romans laboured under for want of a more perfect method of Notation.

Archimedes invented a peculiar scale and Notation of his own, which he employed in his Arenarius to calculate the number of the sands.

In the second century of Christianity, to remedy the difficulty of the common method of Notation, particularly with regard to fractions, Claudius Ptolemy is said to have invented the sexagesimal division of numbers; which division is still used in astronomical calculations, and for the subdivision of circles. Every unit was supposed to be divided into sixty parts, and each of these parts into sixty, &c., hence any number of such parts were called sexagesimal fractions. And, to render the computation in integers more easy, he made the progression, in these likewise sexagesimal; thus from one to fifty-nine he marked in the common way, then sixty he called a sexagena prima, and expressed it thus, I′; two sixties, or 120, thus, II′; and so on to fifty-nine times sixty, or 3540, which he wrote thus, LIX′. For sixty times sixty, or 3600, he wrote I″, calling it a sexagena secunda; for twice 3600, or 7200, he wrote II″; for three times 3600, or 10800, he wrote III″, &c. For $\frac{5}{60}$ he wrote ′V, or V′; for $\frac{15}{3600}$, ″XV, or XV″, &c. The practice by this Notation would be somewhat easier than by the common Notation, yet still very difficult, especially in Multiplication and Division, as appears by the

work of Barlaamus, called Logisticia, written in Greek about the year 1350; translated into Latin, and published in the year 1600.

For the excellent method of Notation now in use, called the Arabian, (because the Europeans had it from the Arabians,) we are indebted to the genius of the Eastern nations. The Indians are acknowledged to be the inventers of it; but, at what time, or how long it was before the Arabians got it, we are quite ignorant. We have sufficient reason to believe that the ancient Greeks and Romans knew nothing of it, as Maximus Planudes, the first Greek writer who treated of Arithmetic according to the Arabian Notation, acknowledges it to be his opinion, that the Indians were the inventors, from whom the Arabians got it, and the Europeans from the Arabians. Now, this writer, according to Vossius, flourished about the year of Christ 1370; or, according to Kircher, 1270, and this was long after the Arabian Notation was known in Europe. For, Dr. Wallis proves, by many good authorities, that the Europeans were acquainted with it before the year of Christ 1000, and that it was brought into England before the year 1150.

Arithmetic, at this period, we may suppose, was in a rude and imperfect state. The first and most considerable writer, after the Arabian Notation was known in Europe, was Jordanus, of Namur, who flourished about the year 1200. His Arithmetic (from which the ingenious Mr. Malcolm acknowledges he has taken several things) was published and demonstrated by Joannes Faber Stapu-

lensis, in the fifteenth century, soon after the invention of printing. The same author likewise wrote a treatise, which he called Algorismus Demonstratus, but it was never printed: the manuscript, we are informed by Dr. Wallis, is in the Savilian Library at Oxford.

To trace out the several improvements of Arithmetic in a regular gradation would be a difficult task and afford but little amusement to the reader. The most remarkable writers, before the sixteenth century, in Italy, were Lucas de Burgo (whose work is particularly recommended by Dr. Wallis) and Nicholas Tartaglia; in France, Clavius and Ramus: in Germany, Stifelius and Henischius; in England, Buckley, Diggs, and Record. In or about the year 1629, Mr. Edward Wingate's Arithmetic was printed; but the Arithmetic now extant under his name, as improved by Mr. J. Dodson, F.R.S., cannot literally be said to bear any affinity to the original work. Since Mr. Wingate wrote, the bare names of those who have written on the subject of Arithmetic, in England only, would fill a moderate volume. Many of these writers were men of scientific abilities, and it would be impossible to mention a few without doing injustice to a greater number.

It remains now to point out the most material improvements made in Arithmetic since the Arabian Notation was known in Europe. Progression, arithmetical and geometrical, the nature of powers, the extraction of roots, and the combination of numbers, &c., have received considerable improvements from several authors

at different periods. About the year 1464, Regiomontanus * introduced decimal parts in his triangular tables instead of sexagesimals, which, before his time, were used in astronomical calculations. Ramus, in his Arithmetic, printed in 1550, makes use of decimals in his calculations, as do Buckley and Record, two English authors (mentioned before) prior to Ramus; but the first treatise expressly written on the subject was by Stevinus, about the year 1582 †. Circulating, or repeating decimals ‡, were first taken notice of by Dr. Wallis, or at least, he was the first who distinctly considered the subject. But, for the greatest and most useful improvement made in the modern art of computation, we are indebted to Baron Napier, the undisputed inventor of logarithms.

In the ensuing Treatise, the Rules are given in as clear and expressive terms as possible; and those parts, which are not immediately necessary for the scholar to transcribe, or fix in his memory, are printed on a smaller type than the rest, to be consulted occasionally. Likewise, all the rules which belong to any one subject, such as PRACTICE, &c. are classed together, unmixed with any

* The real name of this writer was *John Muller;* he was called Regiomontanus, from Mons Regius, or Konigsberg, a town in Franconia, where he was born.

The nature of Decimals is explained Part I. page 89, &c. of the following treatise.

† Dr. Rees's New Cyclopædia, or Dr. Hutton's Mathematical Dict. word Decimal.

‡ For an Explanation of the Nature and Properties of circulating Decimals, see page 104, &c. of the ensuing work.

examples; then the examples follow, with references to the several propositions and rules which they are intended to exercise: by this mode of proceeding, all the rules, and the notes and observations on them, are under the eye of the scholar at once, and he of course sees in an instant what assistance he is to expect from them. The examples are very numerous, consisting of upwards of two thousand, besides a variety of Bills of Parcels, &c. These examples are in general divided into two classes; in the first class reference is made to the particular proposition which the example is intended to exercise; in the second class, the examples are promiscuously placed, and will serve as exercises for those who are farther advanced in numbers. The first question in each rule is worked at full length, for the encouragement of the learner, so that he is led gradually forward both by precept and example.

The answers to the several questions are not put down in the Complete Practical Arithmetician; a KEY to the work is published separate, containing all the answers, with the solutions at full length, wherever there is the smallest appearance of labour or difficulty. This work contains several useful notes and observations on Arithmetic, together with general Demonstrations of all the Rules, and a *Synopsis of Logarithmical Arithmetic.*

Circulating Decimals, which are so little understood, are, in the following Treatise, clearly and distinctly treated of.

Loss and Gain, a rule in which the generality of writers

have puzzled both themselves and their readers, is here rendered plain, easy, and intelligible. In Fellowship, several new rules are given. Exchange is likewise treated of in a different manner to what it usually has been, and several useful tables are introduced, which have not hitherto been inserted in books of arithmetic. These tables have, in this edition, been carefully compared with a correct set of tables in the library of *Hans Sloane*, esq. and likewise with the tables published by the principal writers on exchange, as *Kruse*, *Corbaux*, *Dubost*, &c. Those who wish for farther information on the subject of exchange than is contained in the following treatise, may consult the works above mentioned, or the *Universal Cambist*, by Dr. *Kelly*.

The nature of Ratios, and Proportions, so far as they relate to commensurable quantities is considered. These subjects are of the highest importance. The learned Whiston, in his Tacquet's Euclid, says, "*Si proportionis* "*doctrinam e Mathesi abstuleris, nihil fere præclarum* "*aut egregium relinques.*"

" The second part of the work concludes with some general observations on Numbers odd and even; Square and Cube Numbers, &c. These will serve to raise the curiosity of the learner, and give impulse to his farther enquiries.

The Bills of Parcels, Promissory Notes, &c., which in the former editions followed Duodecimals, and concluded Part I., are now classed nearly in the same order at the end of the book, forming Part III.

In this edition, the Rules for Annuities at Simple Interest have been omitted, being of no use, except as arithmetical exercises, and the principal cases of Annuities on Lives are introduced in lieu of them.

All the rules * and examples have undergone a thorough revisal, and many new ones have been added, in consequence of the distinguished approbation which this work has met with from several of the most respectable and intelligent tutors in the kingdom.

No. 1, *York-Buildings,*
New Road, St. Mary-le-bone,
London, March, 1822.

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* Algebraical demonstrations of the rules have been purposely omitted; because to a young student who is learning the *elements* of the Science, they are perfectly unintelligible.

The truth of arithmetical operations should be explained, by the teacher, from the nature of the process; for though the theory and practice of Science ought to go hand in hand, yet every experienced teacher will allow, that, in common arithmetic, the practice must in a great measure precede the theory.

The young student, who has made himself perfectly master of the *practical* parts of arithmetic, and has acquired some knowledge of algebra, will derive considerable advantage from the perusal of the *Appendix* to the *Complete Practical Arithmetician,* annexed to the *Key* to that work. This Appendix contains a *Synopsis of Logarithmical Arithmetic,* together with general demonstrations of all the principal rules in the *Complete Practical Arithmetician.*

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PART I.

EXCHANGE.

PART II.

PART III.

Explanation *of the* Characters *made use of in the following Work.*

Charact.	*Names.*	*Significations.*
$+$	Plus, or more,	the Sign of Addition, as $2+4$, signifies that 2 and 4 are to be added together.
$-$	Minus, or less,	the Sign of Subtraction, as $8-3$ signifies that 3 is to be subtracted from 8.
$\times$	multiplied into, or by,	the Sign of Multiplication, as 7×5 signifies that 7 is to be multiplied into or by 5.
$\div$	divided by	the Sign of Division, as $9\div3$, signifies that 9 is to be divided by 3; and $\frac{9}{3}$, or 3)9(, signifies the same.
$=$	equal to	the Sign of Equality, as $9=9$, signifies that 9 is equal to 9; or $\overline{5+4}-2=7$ signifies, that 5, increased by 4 and diminished by 2 is equal to 7. The line or *vinculum*, over the 5 and 4, serves as a chain to link them together, and shews, that they are to be added together, before the number 2 is subtracted.
$: :: :$	Proportion.	$2 : 4 :: 8 : 16$ signifies, that 2 is to 4 as 8 is to 16.

The other characters are explained among the Definitions in the Work.

ERRATA.

Page 83, Ex. 7, *read* $\frac{3}{4}$ of $\frac{5}{9}$
—— 89, — 19, *for* $\frac{3}{40}$ *read* $\frac{3}{10}$
—— 103, — 17, *for* 51·75 *read* 21·75 feet
—— 134, — 25, *for* $3\frac{1}{4}$ *read* $3\frac{3}{4}$
—— 252, the *equation* corrected will stand thus,

$$\frac{n+p}{p}\times r^{t} - r^{t+1} = \frac{n}{p}$$

THE

COMPLETE PRACTICAL ARITHMETICIAN.

PART I.

DEFINITIONS.

1. ARITHMETIC is the art of computing by numbers; and consists of two parts, viz. whole numbers, and fractions vulgar or decimal.

2. *Arithmetic in whole numbers* consists of entire quantities, which are not divided into parts less than an unit.

3. *Arithmetic in fractions* consists of parts of some whole quantity, or of an unit.

4. *Number* is either an *unit*, or a *collection* of units; viz. it is the name of that idea, or notion, we conceive of things considered as *one*, or *many*.

Note 1. When we consider numbers simply, without applying them to any particular subject, the idea we form of them is called *abstract*. Thus, if we speak of the number three, four, five, or any other number, abstractedly, we mean three, four, five, &c. units of any thing whatever. But, when we consider number not in its general nature, but as a number of certain particular things, as four yards, five inches,

&c. we call it a *concrete* or an *applicate* number. Example, the number four is less than five *abstractedly* considered; yet, taking the numbers in an *applicate* sense, it is not always so; thus, the quantity of four *yards* is not less than five *inches*.

5. *A whole number* is an unit, or a multiple* of one or more units.

6. *A mixed number* is a whole number with some part, or parts, annexed.

7. *An even number* is that which can be divided into two equal whole numbers.

8. *An odd number* is that which cannot be divided into two equal whole numbers.

9. *A prime number* is that which can only be divided by itself, or by an unit, without a remainder.

10. *Numbers* are said to be prime to each other when only an uuit measures, or divides, them both even.

11. *A square number* is the product of a number by itself.

12. *A cube number* is the product of a number and its square.

13. *A composite number* is that produced by multiplying two or more numbers together.

14. *A perfect number* is that which is equal to the sum of all its aliquot parts.

Note 2. There are several other numbers, which have particular names, as figurate, abundant, deficient, &c. but their chief use is in the higher parts of the mathematics.

15. *An aliquot part* is that which is contained a precise number of times in another.

16. *An aliquant part* is such as is contained in another a certain number of times, with some part, or parts, over.

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* One number is said to be a multiple of another, when the former contains the latter a certain number of times without a remainder; thus, 4 is a multiple of 2, and 8 is a multiple of 2, and of 4, &c.

17. *An integer* is any whole quantity or number; as, a pound, a yard, &c. or, 1, 2, 3, &c.

18. *Digits, or figures,* are the marks by which numbers are expressed, and are the nine following, viz. 1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, to which we may add the cipher 0, or *nought*, which is of no value when taken by itself; yet, when it is placed on the right or left hand of any figure, increases or diminishes it tenfold.

19. *The nature of all arithmetical operations* is by some quantities that are given, to find out others that are required.

20. *The principal, or fundamental, rules of Arithmetic* are Notation and Numeration.

21. *Notation* is the art of expressing numbers by figures; and teaches us to read, or write down, any number, and to have a clear and distinct idea of every figure in it.

22. *Numeration* informs us in what manner we are to exercise and accommodate numbers to the various purposes of business; it consists principally of four parts, viz. Addition, Subtraction, Multiplication, and Division.

Note 3. The operations of arithmetic in general are only of two kinds, viz. *increasing* and *diminishing;* for, multiplication is only a compendious method of performing addition, and division performs the work of many subtractions.

23. *A proposition* is something proposed to be done, or proved.

24. *An axiom* is self-evident, and cannot be rendered more plain by demonstration.

25. *A theorem* is a demonstrative proposition, wherein the nature and property of a thing is proposed to be proved.

NOTATION TABLE.

Quadrillions, or Millions four times repeated	Hundreds of Thousands of Trillions	Tens of Thousands of Trillions	Thousands of Trillions	Hundreds of Trillions	Tens of Trillions	Trillions, or Millions three times repeated	Hundreds of Thousands of Billions	Tens of Thousands of Billions	Thousands of Billions	Hundreds of Billions	Tens of Billions	Billions, or Millions of Millions	Hundreds of Thousands of Millions	Tens of Thousands of Millions	Thousands of Millions	Hundreds of Millions	Tens of Millions	Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units
																								1
																							2	1
																						3	2	1
																					4,	3	2	1
																				5	4,	3	2	1
																			6	5	4,	3	2	1
																		7.	6	5	4,	3	2	1
																	8	7.	6	5	4,	3	2	1
7.	6	5	4,	3	2	1.	9	8	7,	6	5	4,	3	2	1,	9	8	7.	6	5	4,	3	2	1

Note 1. The table above may be extended to any lenghth by continually prefixing a period of six figures towards the left hand, and writing the word quintillions, sextillions, septillions, octillions, nonillions, decillions, &c. over the unit's place of each of these periods: but the table of nine figures, which is printed on a larger type than the rest, is sufficient for common use.

Note 2. To write down any number. Rule: write down ciphers to as many periods and places as are named in the given number; then begin at the left hand, and observe, at each place, what significant figure is named; take away the cipher, and put the significant figure in its place. Ex. Write down ten million fifty thousand three hundred and one, thus:

$\left\{\begin{matrix} 00,000,000 \\ 10,050,301 \end{matrix}\right\}$ Proceed in this manner for any other number.

3. To read any number of figures when written down. Rule: divide the figures, from the right hand to the left, into threes by a comma and a period alternately, writing *m* over the figure to the left

of the first period, *b* over the figure to the left of the second period, &c. till all the figures are brought down, as in this example:

qu. q. t. b. m.
123,456.123,456.123,456.123,456.578,371.123,875.

or, instead of m, b, t, q, qu, &c. put 1, 2, 3, 4, 5, &c. to represent millions once, twice, thrice, &c. repeated; and read thus, one hundred and twenty-three thousand four hundred and fifty-six quintillions; one hundred and twenty-three thousand four hundred and fifty-six quadrillions; one hundred and twenty-three thousand four hundred and fifty-six trillions; one hundred and twenty-three thousand four hundred and fifty-six billions; five hundred and seventy-eight thousand three hundred and seventy-one millions; one hundred and twenty-three thousand eight hundred and seventy-five.

Examples.

1. Write down in words at length the following numbers:

49	437	17349	149387
75	305	10807	1078400
1075	1087	314815	30180070
378	47318	107048	108374108

2. Write down in proper figures the following numbers:

Eighty-nine. Seven hundred and fifty. Five thousand and one. Ten thousand and eighty-seven. Twenty thousand and five.

Six hundred and eighty-five thousand, three hundred and sixty.

One million five hundred thousand, and one.

Twenty-seven million, three hundred and sixty-five thousand.

Three hundred and eighty-five million, seven hundred and forty-eight thousand, three hundred, and five.

Eleven thousand, eleven hundred, and eleven.

Fifty million, fifty thousand, fifty hundred, and fifty.

SIMPLE ADDITION.

Definition.—Simple Addition is a rule by which several numbers of one denomination are collected together into one sum.

RULE.

Place the numbers under each other, viz. units under units, tens under tens, &c.; add up the figures in the row of units, and carry as many units to the next row as there are tens contained in the sum: proceed thus till the whole is finished.

For the proof.—Divide the numbers to be added into two parts, then add up each part by itself, and collect these sums together for the whole.

Note 1. If equal numbers be added to equal numbers, the whole will be equal.

2. If several numbers are to be added together, they will amount to the same sum, when placed regularly one under another, whichever line or row of figures stands uppermost.

3. Dr. Wallis, in his Arithmetic, gives the following rule to prove a simple addition sum. Add the figures in the uppermost row together, reject the nines contained in their sum, and set the excess directly even with the figures in the row. Do the same with each row, and set all the excesses of nine together in a line, and find their sum; then, if the excess of nines in this sum (found as before) be equal to the excess of nines in the total sum, the work is right.

Examples.

(1.)		(2.)		(3.)		
	3247		14934		143716	4
	....		31493		371419	7
	1498		47184		143714	2
	3471		.....		171349	7
	4734		37149		371493	0
	8714		14734		471348	0
	4374		34718			Excess of nines
Sum	26038	Sum	180212	Sum	1673039	2 Proof.
	22791		93611			
			86601		See note 3.	
Proof	26038	Proof	180212			

(4.) Add 1473, 40734, 371049, 40057, 3471473, 5734, 37492, and 4718375, together.

(5.) Collect 371434, 278949375, 67149, 3457143, 714934, 9000987, and 5734747, into one sum.

(6.) Add 5714329, 4718714, 34983714, 671493, 74987149, 6777894987, and 19, together.

(7.) Add 571493, 40007, 6493497, 4718349, 3714934, 4934938, 174934, and 147319, together.

(8.) Suppose the distance from *London* to *Biggleswade* be 45 miles, thence to *Peterborough* 36, thence to *Lincoln* 51, and thence to *Hull* 41 miles; how many miles are *Peterborough*, *Lincoln*, and *Hull*, from *London?*

SIMPLE SUBTRACTION.

Definition.—Simple subtraction teaches to deduct, or subtract, a less number from a greater of the same denomination, whereby the remainder or difference is found.

RULE.

Place the less number under the greater, so that units may stand under units, tens under tens, &c. Begin at the unit's place, and subtract each figure in the lower line from the figure above it; if the lower figure be greater than the upper, add *ten* to the upper figure, from which subtract the lower; set down the remainder, and carry *one* to the next lower figure.

For the proof.—Add the remainder and less number together, and the sum will be the greater. Or, subtract the remainder from the greater number, and the difference will be the less.

Examples.

(1.)		(2.)	(3.)	(4.)
From	9437149	473494	494871	347149
Take	1349348	193487	194985	134948
Diff.	8087801			
Proof	9437149			

(5.) From 47348 take 13456.
(6.) From 194938 take 149342.
(7) From 5007149 take 171493.
(8.) From 1493487 take 149349.

CLASS II.

(9.) From the Creation to the Flood was 1656 years; thence to the building of Solomon's Temple 1336 years; thence to Mahomet, who lived 622 years after Christ, 1630 years. In what year of the world was Christ then born, and how many years is it since the creation?

(10.) Sir Isaac Newton was born in the year 1642, and died in 1727, how old was he at the time of his decease, and how many years is it since he died?

(11.) A gentleman has two sons, the age of the elder added to his make 126 years, and the age of the younger son is equal to the difference between the age of the father and the elder son. Now, if the father be 80 years of age, how old are each of his sons?

(12.) Three boys, A, B, and C, won together 97 marbles at play; now, if the number of marbles B won be added to the number C won, they will make 60; and, if the number A won be added to the number C won, they will make 62. How many marbles did each boy win separately?

SIMPLE MULTIPLICATION.

Definition 1. *Simple multiplication* is a rule by which we increase the greater of two given numbers, of the same denomination, as often as there are units in the less; being a compendious method of performing addition.

2. The number to be multiplied is called *the multiplicand;* the number you multiply by is called the *multiplier;* and the number produced by multiplication is called the *product*. These numbers are sometimes called *factors,* because they are to constitute a *factum* or product.

The Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Proposition 1. *To multiply by a single figure, or any number not exceeding* 12.

Rule. Begin at the unit's place of the multiplicand, and multiply each figure in it by the multiplier, writing down the whole of such products as are less than 10; but, for such as exceed 10, or a number of tens, write down the excess, and carry an unit, for each 10, to the next product.

Prop. 2. *When the multiplier is the product of two, or more, numbers in the table.*

Rule. Multiply the multiplicand by one of the component parts, and that product by the other, &c. for the whole product.

Prop. 3. *When the multiplier consists of several figures.*

Rule. The multiplicand must be multiplied by each figure separately, (beginning with the right-hand figure of the multiplier,) and the first figure of every product must stand exactly under the figure you multiply by. Add these products together for the whole product.

Or, begin with the left-hand figure of the multiplier, and multiply every figure in the multiplicand by it; then multiply in a similar manner by the next figure, &c., taking care to place every succeeding product one figure farther out towards the right-hand.

Prop. 4. *When ciphers are intermixed with the figures in the multiplier.*

Rule. Omit the ciphers, and let the first figure of each product be placed under its respective multiplier.

Prop. 5. *When there are ciphers at the end of the multiplicand or multiplier.*

Rule. Neglect the ciphers, and multiply as before, then to the right-hand of the product annex as many ciphers as were omitted.

For the proof. Multiply the multiplier by the multiplicand, and if the product be the same with *that* of the multiplicand by the multiplier, the work is right.

Note 1. If two numbers are to be multiplied together, they will make the same product, whichever number you make the multiplier.

2. If several numbers, as 5, 6, 7, &c. are to be multiplied together, it is the same thing whether 5 be multiplied by the product of 6 and 7, or it be multiplied first by 6 and then by 7, &c. And, if several given numbers are to be multiplied by any number, and the sum of the products taken; it will be the same thing, if you multiply the sum of those given numbers by that multiplier.

3. The product of any two numbers can have at most but as many places of figures as are in both the multiplier and multiplicand, and at least but one less.

4. Multiplication may be proved by casting out the nines as in addition. Thus cast the nines out of the multiplier and multiplicand, and set down the remainders. Multiply the two remainders together; and, if the excess of nines in the product be equal to the excess of nines in the total product, the work is *generally* right.

Examples to Proposition 1.

(1.) Multiply	471347325
by	2
Product	942694650

(2.) Multiply 371493407 by 3.
(3.) Multiply 47048743 by 4.
(4.) Multiply 57134974 by 5.
(5.) Multiply 37180753 by 6.
(6.) Multiply 4900757149 by 7.
(7.) Multiply 3714937187 by 8.
(8.) Multiply 4708714371 by 9.
(9.) Multiply 5714937143 by 10.
(10.) Multiply 3715714936 by 11.
(11.) Multiply 149371574 by 12.

Examples to Prop. 2.

(12.) Multiply 47134987 by 56

```
         47134987
                8
        ---------
        377079896
                7
       ----------
Product 2639559272
       ----------
```

(13.) Mult. 47134784 by 21.
(14.) Mult. 37149374 by 22.
(15.) Mult. 47187413 by 24.
(16.) Mult. 7493456 by 63.
(17.) Mult. 4194734 by 72.
(18.) Mult. 3175493 by 77.
(19.) Mult. 39007149 by 84.
(20.) Mult. 71340987 by 96.
(21.) Mult. 47154734 by 132.
(22.) Mult. 704134795 by 144.

Examples to Prop. 3.

(23.) Multiply 471493475 by 4395

```
Multiply    471493475
by               4395
        -------------
           2357467375
          4243441275
         1414480425
        1885973900
        -------------
Product 2072213822625
        -------------
```

Or,

```
     471493475
          4395
 -------------
 1885973900
  1414480425
   4243441275
    2357467375
 -------------
 2072213822625
 -------------
```

Proof by multiplication.

```
          4395
     471493475
     ---------
         21975
        30765
       17580
      13185
     39555
    17580
   4395
  30765
 17580
 -------------
 2072213822625
 -------------
```

Proof by casting out the nines.

```
                 Product
                    6
Multiplicand 8      X      3 Multiplier.
                    6
```

(24.) Mult. 430714984 by 743.
(25.) Mult. 37157437 by 14972.
(26.) Mult. 47157149 by 37495.
(27.) Mult. 5714987 by 47159.
(28.) Mult. 47184749 by 371895.
(29.) Mult. 3704957 by 4718759.

Examples to Prop. 4.

(30.) Multiply	4713457
by	5704008
	37707656
	18853828..
	32994199.
	23567285
	26885596435656

(31.) Mult. 371493407 by 700505.
(32.) Mult. 57040935 by 5040648.
(33.) Mult. 40750493 by 67100805.
(34.) Mult. 371493471 by 57080507.
(35.) Mult. 4070490385 by 4090805.
(36.) Mult. 5417080574 by 3905008.

Examples to Prop. 5.

(37.) Multiply	47150000
by	3980000
	37720
	42435
	14145
Product	187657000000000

(38.) Mult. 471000 by 40700.
(39.) Mult. 507000 by 30500.
(40.) Mult. 4713000 by 6070500.
(41.) Mult. 3075600 by 30500700.
(42.) Mult. 57867000 by 4007500.

CLASS II. *Exercising all the Propositions.*

(43.) Mult. 47149 by 7.
(44.) Mult. 371594 by 12.
(45.) Mult. 5719070 by 1440.
(46.) Mult. 70409040 by 371500.
(47.) Mult. 507040500 by 4734050.
(48.) Mult. 37145674 by 3710514.
(49.) Mult. 123456789 by 1234567890.
(50.) Mult. 1234567890 by 987654321.

(51.) Required the continued product of 56,750,54730, 64007, and 587504.

(52.) Required the sum of 157 added 495 times to itself.

(53.) Let 954 be added 435 times to itself, and shew what the last sum total exceeds or falls short of four hundred and fifteen thousand.

(54.) Required the product of eleven thousand eleven hundred and eleven, by twelve thousand twelve hundred and twelve.

(55.) What is the difference between thrice six and twenty and thrice twenty-six.

(56.) There are two numbers; the greater is 19 times 508, and their difference is 15 times 112; required the sum and product of those numbers.

SIMPLE DIVISION.

Definition 1.—*Simple Division* is a rule by which we find how often one number is contained in another of the same denomination; being a short method of performing subtraction.

2. The number to be divided is called the *dividend*, the number you divide by is called the *divisor*; and hence will arise a third number, called the *quotient*, which shews how often the divisor is contained in the dividend. If the divisor does not exactly contain the dividend, a fourth number will occur, called the *remainder*, which must always be less than the divisor.

Prop. 1. *When the divisor does not exceed* 12.

Rule. Observe how often the divisor is contained in the first, or first and second figure of the dividend, and set the quotient figure under it, carry 10 for every unit remaining after subtraction to the next figure of the dividend; proceed thus, multiplying and subtracting mentally, till you have made use of all the figures in the dividend.

Prop. 2. When the divisor is a composite number.

Rule. Divide the dividend by one of the component parts, and that quotient by the other, for the required quotient. If there be a remainder to each of the quotients, multiply the last remainder by the first divisor, and to that product add the first remainder for the true one.

Prop. 3. When the divisor consists of several figures.

Rule. Find how many times it may be had in as many figures of the dividend as are just necessary; multiply the divisor by the quotient figure, subtract the product from that part of the dividend which stands above it, and, to the right hand of the remainder, bring down the next figure in the dividend, which number divide as before; and so on till all the figures in the dividend are brought down.

Prop. 4. When the dividend has ciphers on the right hand.

Rule. Cut off the ciphers from the divisor by a dash of your pen, and also cut off as many ciphers, or figures, from the dividend. But, when the division is finished, the ciphers omitted must be restored to their proper places, and the figures cut off in the dividend must be placed to the right-hand of the remainder.

Note 1. When the scholar is pretty ready at division, he may subtract each figure of the product as he produces it, and write down only the remainder.

For the proof. Multiply the quotient by the divisor, to the product add the remainder, if any, and the sum will be equal to the dividend.

2. There are several methods of proving division. If you subtract the remainder from the dividend, and divide this number by the quotient, the quotient found by this division will be equal to the former divisor.

3. Or, add the remainder, and all the products of the several quotient-figures by the divisor, together, according to the order in which they stand in the work, and the sum will be equal to the dividend.

4. Another method. Cast away the nines in the divisor and quotient, take their product, and cast away the nines, to which add the excess of nines in the remainder after division: the excess of nines in this sum will be equal to the excess of nines in the dividend, when the work is right.

5. An even number cannot divide, or measure, an odd number, nor a greater a less.

6. Half the sum of any two numbers, increased by half their difference, will give the greater number; and half their sum diminished by half their difference, will give the less number.

7. The quotient, arising from the division of the sum of two, or more, numbers, is equal to the sum of the quotients arising from the division of the parts, separately, by the same divisor.

8. If any two numbers be separately divided by 9 or 3, and the two remainders multiplied together, and that product divided by 9 or 3, the last remainder will be the same as if you divided the product of the two first numbers by 9 or 3.

9. Any number divided by 9 or 3, will leave the same remainder as the sum of its digits divided by 9 or 3. Hence, if any number is divisible by 9 or 3, the sum of its digits is likewise divisible by 9 or 3, and *vice versa*. The method of proof by casting out the nines, in the preceding rules, depends upon this theorem.

Examples to Proposition 1.

(1.) Divide 1749342345 by 2.

Dividend
Divisor 2)1749342345
Rem.
Quotient 874671172-1

(2.) Divide 471349571 by 3.
(3.) Divide 407104937 by 4.
(4.) Divide 70407143 by 5.
(5.) Divide 170049378 by 6.
(6.) Divide 493740075 by 7.
(7.) Divide 30871050743 by 8.
(8.) Divide 41375714937 by 9.
(9.) Divide 71000571479 by 10.
(10.) Divide 37407184374 by 11.
(11.) Divide 47105713475 by 12.

Examples to Prop. 2.

(12.) Divide 7149347859 by 25.

25 = 5 × 5)7149347859

5)1429869571—4 Rem.
Quotient 285973914—1 $9 = \overline{1 \times 5} + 4$.

(13.) Divide 7340473857 by 27.
(14.) Divide 749347549 by 144.
(15.) Divide 649305743 by 55.
(16.) Divide 4730715405 by 121.
(17.) Divide 3704095714 by 108.
(18.) Divide 4710437154 by 132.

(19.) Divide 1071540075 by 99.
(20.) Divide 457014374 by 96.

Examples to Prop. 3.

(21.) Divide 34674378 by 95.

Divisor.		Dividend.		Quotient. Rem.
95	)	34674378	(	364993 — 43
		285×		95
		.617		1824965
		.570×		3284937
				43 *Remainder.*
		..474		34674378 *Proof by multiplication.*
		..380×		
		...943		
		...855×		
		887		
		....855×		
		328		
		.....285×		
		43		
		34674378 *Proof by addition.*		

Proof by casting out the nines.

6
8+7 Remainder.
Divisor 5 X 7 Quotient.
6
Dividend.

34674378 Dividend.
43 Remainder.
Quotient.
364993)34674335(95 *Proof by division.*
3284937
1824965
1824965

(22.) Divide 714394756 by 1754.
(23.) Divide 47159407184 by 3574.
(24.) Divide 5719487194715 by 45705.
(25.) Divide 47157149371493887 by 17493.
(26.) Divide 671493471549375 by 47143.
(27.) Divide 571943007145 by 37149.
(28.) Divide 1714347149347 by 57143.
(29.) Divide 49371547149375 by 374567.
(30.) Divide 171493715947143 by 571007.
(31.) Divide 6754371495671594 by 678957.

Examples to Prop. 4.

(32.) Divide 14715967899 by 145000.

```
145000)14715967899(101489 62899/145000 Quotient.
       145
       ---
        215                       Or thus,
        145          145000)14715967899(101489 62899/145000
        ---                     215
         709                     709
         580                     1296
         ---                      1367
         1296                     ----
         1160                     62899 Rem.
         ----                     ----
          1367
          1305
          ----
          62899 Rem.
          ----
```

(33.) Divide 571436490075 by 36500.
(34.) Divide 1947184907OO by 73000.
(35.) Divide 7954083347594 by 47150.
(36.) Divide 1495070807149 by 371500.
(37.) Divide 671493471 4934 by 754000.
(38.) Divide 10714914714307l5 by 147500.
(39.) Divide 1471493749371 4957 by 157900.
(40.) Divide 714937494719471 5 by 1749000.

CLASS II. *Exercising all the Propositions.*

(41.) Divide 714947349 by 90.
(42.) Divide 1714937148 by 14400.
(43.) Divide 7149371478 by 121.
(44.) Divide 71900715708 by 57149.
(45.) Divide 152415787501905 21 by 123456789.
(46.) Divide 121932631112635269 by 987654321.
(47.) Divide 696161034987 21931800 by 975005700.
(48.) Divide 65645893199652417 1800 by 700489070.
(49.) Divide 7149437149547 by 3714900.
(50.) Divide 147149371 48475 by 123456.

(51.) The sum of two numbers is 348, and their difference 194, required the numbers.

(52.) What number, multiplied by 365, will produce 315725?

(53.) What number, multiplied by 95, will give the same product as 157 by 570?

(54.) What number is that, from which if a twelfth part of 1728 be deducted, and the remainder increased by the ninety-fifth part of 82175, the sum will be 1185?

(55.) What number divided by 1185, will give 497 for the quotient, and leave just a fifth part of the divisor remaining?

(56.) Required the difference between six dozen dozen and half a dozen dozen.

(57.) Subtract 759 out of 171493745 as often as you can, and shew what the last remainder exceeds or falls short of 500.

TABLES OF ENGLISH COIN, WEIGHTS, MEASURES, &c.

TABLE I. MONEY.

The lowest piece of money, used in England, is a farthing, and all accounts are kept in pounds, shillings, pence, and farthings. The pound sterling is an imaginary coin, value 20 shillings.

2 farthings - -	make	1 halfpenny
4 farthings - - -	—	1 penny.
6 pence - - -	—	half a shilling.
12 pence - -	—	1 shilling.
2 shillings and 6 pence -	—	half-a-crown.
5 shillings - -	—	1 crown.
3 seven-shillings pieces -	—	1 guinea.
10 shillings and 6 pence -	—	half-a-guinea.
21 shillings - - -	—	1 guinea.
20 shillings - -	—	1 sovereign.
10 shillings - - -	—	1 half-sovereign.

Note.

£. denotes pounds, *s.* shillings, and *d.* pence.

$\frac{1}{4}$ ······ a farthing, or the quarter of any thing.

$\frac{1}{2}$ ······ a halfpenny, or the half of any thing.

$\frac{3}{4}$ ······ three farthings, or three-quarters of any thing.

Imaginary English Coin.

A mark value	13*s.* 4*d.*	An angel value	10*s.*
A noble ····	6 8	A Carolus ····	23*s.*
A groat ····	0 4	A Jacobus ····	25*s.*

SHILLINGS AND PENCE TABLES.

		£.	s.			£.	s.
20	Shillings	1	0	130	Shillings	6	10
30	———	1	10	140	———	7	0
40	———	2	0	150	———	7	10
50	———	2	10	160	———	8	0
60	———	3	0	170	———	8	10
70	———	3	10	180	———	9	0
80	———	4	0	190	———	9	10
90	———	4	10	200	———	10	0
100	———	5	0	210	———	10	10
110	———	5	10	220	———	11	0
120	———	6	0	230	———	11	10

		s.	d.			s.	d.
20	Pence	1	8	80	Pence	6	8
24	———	2	0	84	———	7	0
30	———	2	6	90	———	7	6
36	———	3	0	96	———	8	0
40	———	3	4	100	———	8	4
48	———	4	0	108	———	9	0
50	———	4	2	110	———	9	2
54	———	4	6	120	———	10	0
60	———	5	0	130	———	10	10
70	———	5	10	132	———	11	0
72	———	6	0	144	———	12	0

TABLE II. TROY WEIGHT.

By this weight are weighed gold, silver, jewels, amber, and all liquors

24 grains	make	1 pennyweight, dwt.
20 pennyweights	—	1 ounce, oz.
12 ounces	—	1 pound, lb.

TABLE III. APOTHECARIES WEIGHT.

Apothecaries, chemists, &c. use this weight in mixing medicines; but buy and sell their drugs by avoirdupois weight.

20 grains	make	1 scruple, ℈.
3 scruple	——	1 dram, ʒ.
8 drams	——	1 ounce, ℥.
12 ounces	——	1 pound, ℔.

TABLE IV. AVOIRDUPOIS WEIGHT.

By Avoirdupois Weight are weighed such commodities as are coarse and drossy, or subject to waste; as groceries of all kinds, bread, butter, cheese, and most other common necessaries of life; pitch, tar, resin, wax, tallow, flax, &c. as are likewise all metals, silver and gold excepted.

16 drams	make	1 ounce.
16 ounces	——	1 pound.
28 pounds	——	1 quarter of an hundred weight.
4 quarters, or 112 pounds		1 hundred weight, cwt.
20 hundred weight		1 ton.

There are several sorts of silk weighed by the great pound of 24 ounces, others by the common pound of 16 ounces. Hence, to reduce great pounds to common, multiply by 3, and divide by 2; and to bring common pounds into great, multiply by 2, and divide by 3.

Note. 175 ounces troy are equal to 192 ounces avoirdupois, exactly; and 175lbs. troy are equal to 144lbs. avoirdupois. Hence.

		oz.	dwts.	grs.	
1lb. avoird.	=	14	11	16	troy.
1oz. —	=	0	18	5½	—

		oz.	drams.	
1lb. troy	=	13	2·65$\frac{1}{7}$	avoird.
1oz. —	=	1	1·55$\frac{3}{7}$	—

A firkin of butter, 56lb.
———— soap, 64lb.
A barrel of raisins, 112lb.
———— soap, 256lb.
A puncheon of prunes, 1120lb.
A fother of lead, 19½cwt. or 2184lb.
A stone, horseman's weight, 14lb.
————, butcher's meat in London, 8lb.
A stone, ditto in the country, 14lb.
A gallon of train oil 7½lb.
A truss of straw, 36lb.
———— new hay, 60lb.
———— old hay, 56lb.
A load, 36 trusses.

	lb.	oz.	dr.
A peck loaf weighs ..	17	6	1
A half-peck	8	11	0
A quartern	4	5	8

Wool-weight.

A clove, or half-stone, 7lb.
A stone, or 2 cloves, 14lb.
2 stone, or 1 todd, 28lb.
A wey, or 6½ todds, 182lb.
A sack, or 2 weys, 364lb.
A last, or 12 sacks, 4368lb.

TABLE V. CLOTH MEASURE.

Cloth measure is used by linen and woollen drapers. Hollands are measured by the English ell, and tapestry by the Flemish ell; woollens, linens, wrought silks, tape, &c. by the yard.

2¼ Inches	make	1 Nail.
4 Nails	——	1 Quarter of a yard.
3 Quarters	——	1 Flemish ell.
4 Quarters	——	1 Yard.
5 Quarters	——	1 English ell.
6 Quarters	——	1 French ell.

TABLE VI. LONG MEASURE.

This measure is used to measure distances, lengths, breadths, heights, depths, &c. of places or things.

12 Lines, or 3 barley corns,	make	1 Inch.
12 Inches - - -	——	1 Foot.
3 Feet - - -	——	1 Yard.
6 Feet, or 2 yards -	——	1 Fathom.
5½ Yards, or 11 half-yards, or 16½ feet -	——	1 Rod, pole, or perch.
4 Poles, or 100 links -	——	1 Chain.
40 Poles, or 10 chains -	——	1 Furlong.
8 Furlongs, or 80 chains	——	1 Mile.
3 Miles - - -	——	1 League.
60 Geographical miles, or 69½ statute miles,	——	1 Degree.

Note. The statute-pole is 5½ yards, but, in some counties in England, they reckon 6 yards to the pole; in the north of England 7 yards are accounted a pole, or rod. In measuring the height of horses, 4 inches make a hand.

TABLE VII. SQUARE-MEASURE.

Square measure is used to measure all kinds of superfices; such as land, paving, flooring, plaistering, roofing, slating, tiling, and every thing that has length and breadth.

Square.		*Square.*
144 Inches	make	1 Foot.
9 Feet	——	1 Yard.

Square.		*Square.*
30¼ Yards, or 272¼ feet	-	make 1 Pole, rod, or perch.
16 Poles	- -	—— 1 Chain.
40 Perches	- -	—— 1 Rood.
4 Roods, or 160 rods, or 4840 yds. or 10 chains		—— 1 Acre.
640 Acres	- -	—— 1 Mile.
100 Feet	- -	—— 1 of flooring.

Note. See Duodecimals, at the end of Part I.

TABLE VIII. CUBIC, OR SOLID, MEASURE

Is used, in mensuration, to measure all kinds of solids; or such figures as consist of three dimensions, viz. length, breadth, and depth, or thickness.

Cubic.		*Cubic.*
1728 Inches	make	1 Foot.
27 Feet	——	1 Yard.
166⅜ Yards	——	1 Pole.
64000 Poles	——	1 Furlong.
512 Furlongs	——	1 Mile.

40 feet of rough timber, or 50 feet of hewn timber, 1 ton, or load.

TABLE IX. WINE MEASURE.

By this measure all wines, brandies, rum, spirits, distilled liquors, cyder, perry, mead, vinegar, honey, oil, &c. are measured, bought, and sold.

4 Gills	- -	make 1 Pint.
2 Pints	- -	—— 1 Quart.
4 Quarts, or 2 pottles		—— 1 Gallon.
10 Gallons	- -	—— 1 Anker of brandy.
18 Gallons	- -	—— 1 Runlet.
31½ Gallons	- -	—— 1 Barrel, or half-hogshead.
63 Gallons	- -	—— 1 Hogshead.
42 Gallons	- -	—— 1 Tierce.
84 Gallons	- -	—— 1 Puncheon.

2 Hogsheads, or 126 gallons, —— 1 pipe, or butt.

2 Butts, or 4 hogsheads, or 252 gallons, —— 1 ton.

Note. In the north of England, a gill is half a pint; also the measure of a gill in London is there called a *jack*.

TABLE X. ALE AND BEER MEASURE in London.

By this measure all malt liquors are gauged, bought, and sold.

2 Pints - - -	make	1 Quart.
4 Quarts - -	——	1 Gallon.
8 Gallons - -	——	1 Firkin of *ale.*
9 Gallons - -	——	1 Firkin of *beer.*
2 Firkins, or 18 gallons -	——	1 Kilderkin.
~~32 Gallons~~ - -	——	1 Barrel of *ale.*
36 Gallons - -	——	1 Barrel of *beer.*
48 Gallons - -	——	1 Hogshead of *ale.*
54 Gallons - -	——	1 Hogshead of *beer.*
2 Hogsheads, or 96 gallons	——	1 Butt of *ale.*
2 Hogsheads, or 108 gallons	——	1 Butt of *beer.*

N.B. The above measure is used only in London for gauging and selling: in all other places in England, the following Table is the standard of ale and beer measure, according to a statute of excise made in the year 1689.

TABLE XI. ALE AND BEER MEASURE in the Country.

2 Pints - -	make	1 Quart.
282 Cubic inches, or 4 quarts	——	1 Gallon.
8½ Gallons - -	——	1 Firkin.
17 Gallons - -	——	1 Kilderkin, or half barrel.
34 Gallons - -	——	1 Barrel.
51 Gallons - -	——	1 Hogshead.

Note. Notwithstanding the above statute, common brewers, in some parts of the country, allow 36 gallons to the publicans for a barrel of ale or beer.

TABLE XII. DRY MEASURE.

Dry measure is used in measuring all *dry* commodities, as wheat, barley, beans, and other grain; fruit, roots, sand, salt, coals, oysters, &c.

2 Pints - -	make	1 Quart.
2 Quarts - -	——	1 Pottle.
2 Pottles, or 8 pints -	——	1 Gallon.
2 Gallons - -	——	1 Peck.

4 Pecks	make	1 Bushel.
4 Bushels	——	1 Coom.
2 Cooms, or 8 bushels	——	1 Quarter.
4 Quarters	——	1 Chaldron.
5 Quarters	——	1 Wey.
2 Weys, or 10 quarters	——	1 Last.

For Coals.

4 Pecks	make	1 Bushel.
3 Bushels	——	1 Sack.
36 Bushels	——	1 Chaldron.
21 Chaldrons	——	1 Score.

Note. 32 bushels make a chaldron in the country: 5 pecks make a bushel water-measure: 5 bushels make a sack of flour. The standard Winchester bushel is a cylinder of 18½ inches diameter, and 8 inches in depth, and contains 2150⅖ cubic inches.—7680 wheat, or barley-corns, are supposed to fill a pint measure.

TABLE XIII. MEASURE OF TIME.

60 Thirds	make	1 Second.
60 Seconds	——	1 Minute.
60 Minutes	——	1 Hour.
24 Hours	——	1 Day.
7 Days	——	1 Week.
4 Weeks	——	1 Month.

13 months 1 day, or 52 weeks 1 day, or 365 days, a year, for three years together: but every fourth year contains 366 days, and is called leap-year, except those centuries which cannot be exactly divided by four. This is called the *Gregorian* year, being instituted by pope Gregory in 1582, and was brought into use in England in 1752. Hence, if we consider the year to consist of 365 days 6 hours, at a medium, one day ought to be struck off the account in 130 years, the solar year being only 365 days, 5 hours, 49 minutes.

The common year is also divided into 12 calendar months.

Memorandum,—30 days has September,
April, June, and November,
February has 28 alone,
And all the rest have 31.

In a leap-year, which happens every fourth, (except in the odd centuries, as 17, 18, 19, &c.) February has 29 days.

A TABLE, shewing the Number of Days from any Day of one Month to the same Day of any other Month in the same Year.

To the same Day	From any Day of											
	Jan	Feb	Mar.	Apr	May	June	July	Aug.	Sep	Oct	Nov.	Dec.
Jan.	365	334	306	275	245	214	184	153	122	92	61	31
Feb.	31	365	337	306	276	245	215	184	153	123	92	62
Mar.	59	28	365	334	304	273	243	212	181	151	120	90
April	90	59	31	365	335	304	274	243	212	182	151	121
May	120	89	61	30	365	334	304	273	242	212	181	151
June	151	120	92	61	31	365	335	304	273	243	212	182
July	181	150	122	91	61	30	365	334	303	273	242	212
Aug.	212	181	153	122	92	61	31	365	334	304	273	243
Sept.	243	213	184	153	123	92	62	31	365	335	304	274
Oct.	273	242	214	185	153	122	92	61	30	365	334	304
Nov.	304	273	245	214	184	153	123	92	61	31	365	335
Dec.	334	303	275	244	214	183	153	122	91	61	30	365

Note. In leap year, if the end of the month of February be in the time, one day must be added on that account. To know when it is leap-year, divide the year by 4, and the remainder shews how long it is after leap-year; if nothing remains, it is leap-year, excepting the years 1700, 1800, 1900, 2100, &c.

TABLE XIV. OF NUMBER.

12 Units	- -	make	1 Dozen.
12 Dozen	- -	——	1 Gross.
12 Gross, or 144 dozen		——	1 Great gross.
20 Units	- -	——	1 Score.
5 Score	- -	——	1 Short hundred.
6 Score	- -	——	1 Long hundred.
24 Sheets	- -	——	1 Quire of paper, or parchment.
20 Quires	- -	——	1 Ream of ditto.
2 Reams	- -	——	1 Bundle of ditto.
12 Skins of parchment	-	——	1 Roll.

COMPOUND ADDITION.

Definition.—Compound Addition is a rule by which several numbers of different denominations are collected together into one sum.

RULE.

Place the numbers so that those of the same denomination may stand directly under each other. Add the first row, or lowest denomination, together, as in simple addition, and divide the sum by as many of the same denomination as make one of the next greater: set down the remainder, and carry the quotient to the next superior denomination. Proceed thus through all the denominations to the highest, which add as in simple addition.

The method of proof is the same as in simple addition.

Note. Addition of money may either be performed by the preceding rule, or by the help of the pence table.

MONEY.—*See Table* I.

(1.)

	£.	s.	d.
	174	11	4¼
	....		
	74	19	11¾
	64	13	10
	174	19	11¼
	64	18	10¾
	105	11	9¼
	74	19	10½
	44	18	11¾
Sum	779	14	7¼
	605	3	3
Proof	779	14	7¼

(2.)

	£.	s.	d.
	374	11	3¾
	74	12	7¾
	149	14	10¾
	74	14	11½
	104	13	10
	....		
	105	17	11¼
	74	19	10¾
	16	14	7¾
Sum	976	0	2
	197	12	5¾
	778	7	8¼
Proof.	976	0	2

(3.)

£.	s.	d.
174	11	4¾
39	18	10¾
714	14	7¾
64	19	11¼
108	14	9
74	14	7¼
64	13	10
174	19	4
67	12	5
149	15	9¾
74	14	7

(4.)

£.	s.	d.
147	14	9¾
77	11	4¾
10	10	10¾
7	7	4
19	4	11¼
19	3	10¼
14	10	11
74	13	9¾
104	14	9¾
74	12	10¾
16	18	5¾

(5.)

£.	s.	d.
149	14	7¼
37	11	9¾
64	14	7
104	19	11¼
64	13	10
174	19	11½
47	14	10¾
39	15	11½

(6.)

£.	s.	d.
14	11	3¼
19	18	10
77	11	3¼
49	14	7
16	18	4½
17	15	10
1	14	9½
6	18	10¾

(7.)

£.	s.	d.
14	19	4¼
17	11	10
39	18	11¼
19	14	9
19	15	11¼
18	19	10
77	19	11¼
14	11	10¼

(8.)

£.	s.	d.
14	10	4¼
77	18	3
14	13	9¾
67	12	4¾
9	11	10
18	10	5
17	19	4
19	10	4

TROY WEIGHT.—*See Table* II.

(9.)			(10.)			(11.)			(12.)		
lb.	oz.	dwt.	oz.	dwt.	gr.	lb.	oz.	dwt.	oz.	dwt.	gr.
174	11	19	174	19	23	71	11	19	74	19	23
74	10	13	714	11	14	64	8	14	64	14	17
944	9	14	714	0	18	77	0	0	74	19	11
74	11	19	74	1	22	14	3	11	66	13	9
944	10	13	948	2	21	64	2	9	74	14	11
74	11	3	74	1	12	74	1	14	14	10	3
14	9	4	715	2	14	77	2	13	19	11	14
77	10	11	714	18	16	19	2	14	17	10	13

APOTHECARIES WEIGHT.—*See Table* III.

(13.)			(14.)			(15.)			(16.)		
℔	℥	ʒ	℥	ʒ	℈	ʒ	℈	gr.	℔	℥	ʒ
47	11	7	149	7	2	749	2	19	84	11	7
94	10	6	714	3	0	607	1	18	74	10	6
74	10	4	619	2	1	714	2	17	37	5	4
75	9	3	74	6	2	400	0	0	19	4	3
69	0	2	169	5	2	74	1	16	74	1	2
57	1	2	74	1	2	715	2	14	79	2	6
18	2	1	777	6	1	64	1	18	19	2	4
74	1	2	948	5	2	174	2	19	74	9	5

AVOIRDUPOIS WEIGHT.—*See Table* IV.

(17.)			(18.)			(19.)			(20.)		
T.	cwt.	qr.	cwt.	qr.	lb.	qr.	lb.	oz.	lb.	oz.	lb.
174	19	3	174	3	27	44	27	15	17	15	15
74	14	2	714	2	24	74	26	14	27	14	11
714	13	1	149	1	14	19	14	13	16	13	9
718	16	2	719	2	16	74	19	14	74	14	14
734	15	2	407	1	23	66	27	13	70	0	0
714	14	1	149	2	17	74	19	10	64	13	10
70	13	2	714	2	18	14	18	11	74	14	11

CLOTH MEASURE.—*See Table* V.

(21.)			(22.)			(23.)			(24.)		
Yds.	qu.	n.	E.E.	qr.	n.	Ells Fr.	qr.	n.	Ells Fl.	qr.	n.
74	3	3	77	4	3	749	5	3	714	2	3
64	2	1	14	3	2	704	4	2	615	1	2
74	1	3	74	2	1	108	3	1	714	1	3
49	2	1	49	1	2	705	4	0	724	2	2
74	1	2	74	2	1	708	3	1	149	1	2
44	3	1	74	3	2	474	5	2	718	2	3
74	2	0	44	1	2	174	0	1	419	1	1
14	1	2	74	2	3	194	3	2	710	1	2

LONG MEASURE.—*See Table* VI.

(25.)			(26.)			(27.)			(28.)		
Lea.	m.	f.	F.	p.	yds.	P.	yds.	ft.	Feet	in.	b.c.
17	2	7	167	39	5½	177	5½	2	174	11	2
14	1	6	614	37	4¼	714	4½	1	49	10	1
74	1	7	714	19	3½	714	1¼	2	74	11	2
69	2	4	674	17	1¼	615	0	1	64	9	1
74	1	0	719	27	2¼	714	1¼	2	74	10	1
69	2	1	197	19	1½	719	1½	1	64	11	2
74	1	2	214	14	3¼	437	2½	1	74	10	0
94	0	3	704	19	4½	614	1½	2	64	9	1

LAND MEASURE.—*See Table* VII.

(29.)			(30.)			(31.)			(32.)		
A.	r.	p.	A.	r.	p.	A.	r.	p.	A.	r.	p.
77	3	39	714	3	39	14	3	39	174	3	39
64	2	37	619	1	18	74	1	19	714	1	27
74	1	24	714	2	27	64	2	14	618	2	12
64	2	19	619	1	34	74	1	18	719	1	14
74	1	18	719	2	37	47	2	24	734	2	11
64	2	17	719	1	24	18	1	14	715	1	24
14	1	13	615	2	14	74	2	19	639	2	13
74	2	11	74	1	18	34	1	14	714	3	24

WINE MEASURE.—*See Table* IX.

(33.)			(34.)			(35.)			(36.)		
Tuns	hhd.	gall.	Pun.	gall.	qt.	Tierce	gall.	qt.	Gall.	qt.	pt.
714	3	62	714	83	3	74	41	3	14	3	1
614	2	61	615	81	2	64	40	2	74	2	1
174	1	39	714	74	1	74	19	1	39	2	1
164	2	47	614	18	2	64	39	2	17	1	0
274	1	49	713	75	0	74	40	1	19	2	0
175	2	37	614	17	1	69	19	1	77	1	1
375	1	49	715	14	3	17	39	2	39	3	1
714	2	61	719	28	2	18	41	1	14	1	1

ALE AND BEER MEASURE.—*See Table* X.

(37.)			(38.)			(39.)			(40.)		
B.B.	fir.	gall.	A.B.	fir.	gall.	A.hhd.	gall.	qt.	B.hhd.	gall.	qt.
74	3	8	73	3	7	714	47	3	714	53	3
14	2	7	69	2	6	614	44	1	415	47	2
16	1	4	14	1	7	374	43	2	714	19	1
17	1	3	39	2	2	157	41	1	614	27	1
29	2	2	19	1	6	719	42	1	715	51	2
17	1	7	49	2	6	374	41	2	714	37	2
41	2	6	37	1	4	174	12	1	615	19	1
37	1	5	19	1	2	19	13	2	714	18	2

DRY MEASURE.—*See Table* XII.

(41.)			(42.)			(43.)			(44.)		
Ch.	b.	p.	Ch.	qr.	b.	Qr.	b.	p.	Score	ch.	b.
14	31	3	174	3	7	149	7	3	74	20	35
74	31	2	375	1	6	715	3	2	49	19	33
64	30	1	409	0	5	649	1	1	64	17	35
74	27	2	371	1	4	479	2	1	74	14	10
64	19	2	634	2	3	675	1	3	39	13	9
74	31	1	719	1	2	149	2	1	47	16	3
64	11	1	149	2	1	375	1	2	19	17	4
95	10	2	375	1	3	649	1	3	37	18	34

MEASURE OF TIME.—*See Table* XIII.

(45.)

Yrs.	m.	w.
737	12	3
347	11	2
618	10	1
374	9	2
175	3	1
714	12	3
615	10	1
714	3	1

(46.)

M.	w.	d.
64	3	6
74	1	5
34	2	3
74	1	4
63	2	1
74	1	2
64	2	1
74	1	3

(47.)

Days	hrs.	min.
714	23	59
74	14	54
64	21	55
74	13	53
69	12	14
74	12	19
37	11	17
16	12	19

(48.)

Hrs.	min.	sec.
647	59	59
137	54	54
375	56	56
714	17	19
615	54	54
714	17	13
613	34	56
624	27	39

CLASS II. *Promiscuous Examples.*

(49.) A is indebted to B £27 4*s.* 10*d.*, to C £108 11*s.* 7¼*d.*, to D £157 0*s.* 6*d.*, to E £957 11*s.* 10*d.*, to F £149 11*s.* 10*d*, to G £190 10*s.* 6*d.*, and to H. £900 5*s.* 4*d.*; what is A's whole debt?

(50.) A corn-factor has paid for wheat £49 11*s.* 10*d.*, for rye £47 13*s.* 7*d.*, for oats £104 19*s.* 10*d.*, for barley £77 11*s.* 3*d.*, for peas £88 11*s.* 9*d*, he has also paid for carriage and other incidental charges £5 11*s.* 1½*d.*, for an insurance 12 guineas; his commission on the whole amounts to 10 guineas: for what sum must he draw upon his employer to clear the account?

(51.) R of *Rotterdam* is debtor to H of *Hull* for fifty firkins of butter, 75 guineas; for 15 pieces of *Yorkshire* cloth, £215 11*s.* 10*d.*; for 24 fother of *Derbyshire* lead, £557 11*s.* 9*d.*; for cheese, £65 11*s.* 4*d.*; for bar-iron, £100 19*s.* 7*d.*; for his acceptance of a bill, drawn for £571 11*s.* 9*d.* H has also paid convoys, insurances, port-charges, &c. £27 11*s.*; for warehouse-room, cartage, &c. £7 7*s.*; the factorage of the whole amounts to 100 guineas: for what sterling money must H draw upon R to clear this account?

(52.) A collector of cash has been out with bills, and gives account that A paid him 50 guineas, B £14 11*s.* 6*d.*, C £37, D 315 quarter-guineas, E a £50 bank-note, and F 300 guineas. What money had he in charge?

(53.) A nobleman, going out of town, is informed by his steward, that his butcher's bill comes to £194 17*s*. his baker's to £49 11*s*. 6*d*., his brewer's to 95 guineas, his wine-merchant's to £107 11*s*. 3*d*., his corn-chandler's to £75, his tallow-chandler's to £27 11*s*. 6*d*., his cheese-monger's to 35 guineas, to his cabinet-maker are owing 315 guineas, also for rent, servant's wages, &c. he is indebted £140 11*s*. 6*d*.; and, if he takes 100 guineas with him, to defray his expences on the road, for what sum must he send to his banker to satisfy these demands?

(54.) A gentleman bought of a silversmith, dishes to the weight of 16lb. 11oz. 14dwt., plates 42lb. 10oz. 9dwt., spoons 14lb., salts 12lb. 9oz., waiters 11lb. 5oz. 10dwt., tankards 11lb. 10oz., and a silver tea-board, and other articles, to the weight of 14lb. 11oz. 10dwt. What weight of plate did he buy in all?

(55.) A merchant in *London* bought of a farmer in *Kent*, eight bags of hops; No. 1 weighed 3cwt. 2qr. 14lb.; No. 2, 2cwt. 1qr. 14lb.; No. 3, 4cwt. 1qr. 27lb.; No. 4, 2cwt. 3qr.; No. 5, 4cwt. 1qr. 11½lb.; No. 6, 6cwt. 1qr. 11lb.; No. 7, 7cwt. 1qr. 11¾lb.; and No. 8, weighed 5cwt. 3qr. 12lb.; the merchant, by agreement, was to pay the carriage to town; how many cwt. had he to pay for?

(56.) I bought six parcels of cloth, the first contained 37yds. 1qr.; the second, 54yds. 3qrs. 2n.; the third, 15yds. 1qr. 2n.; the fourth, 72yds. 2qr. 1n.; the fifth, 25½yds.; and the sixth, 49¾yds. How many yards did I buy in all?

COMPOUND SUBTRACTION.

Definition.—Compound Subtraction teaches us to find the difference of any two numbers of different denominations.

RULE.

Place the less number under the greater, so that those parts, which are of the same denomination, may stand directly under each other. Begin at the lowest denomination, and subtract the under number from the upper; when any of the lower denominations are greater than the upper, increase the upper number by as many as make one

of the next superior denomination, from which sum take the figure in the lower line; set down the difference, and carry 1 to the next number in the lower line, and subtract as before; and so on till you have gone through all the denominations.

The method of proof is the same as in simple subtraction.

MONEY.—*See Table* I.

(1.)

	£.	s.	d.
Borrowed	1749	11	9¼
Paid	948	12	11¾
Remains to pay	800	18	9½
Proof	1749	11	9¼

(2.)

	£.	s.	d.
Lent	4749	11	10½
Received	1494	11	10¾
Due			
Proof			

(3.)

£.	s.	d.
149	11	4½
74	10	7¾

(4.)

£.	s.	d.
647	10	7½
149	19	11¾

(5.)

£.	s.	d.
44	11	8¼
17	14	7½

(6.)

£.	s.	d.
75	11	10¼
44	19	11¾

(7.)

74	11	9¼
39	17	11½

(8.)

747	11	9¾
714	18	8½

(9.)

719	11	9¼
614	10	8¾

(10.)

613	11	7¼
149	10	4¾

(11.)

	£.	s.	d.
Borrowed	71747	11	10½
Paid at different times.	7149	11	4
	675	14	7¾
	714	19	10¼
	147	11	9
	56	19	10¼
	714	11	11½
	64	18	10¾
Paid in all			
Remains to pay			

(12.)

	£.	s.	d.
Received	71437	11	9¼
Laid out at sundry times	6174	19	10
	734	17	5½
	615	19	11¾
	375	14	10¾
	74	13	6
	19	18	11¼
	77	14	10½
Laid out in all			
Remains on hand			

(13.)

Required the balance of this acct.

Dr.			Cr.		
£.	s.	d.	£.	s.	d.
747	11	10	41	11	10
314	11	9½	74	13	9¼
647	19	10¾	64	11	6¾
374	14	7	77	13	10¼
167	15	9¼	14	15	9
317	11	8	64	15	10
			37	12	4¼

(14.)

Required the balance of this acct.

Dr.			Cr.		
£.	s.	d.	£.	s.	d.
34	11	9¾	711	10	4
75	19	11	373	13	10¾
67	14	10¾	714	19	11½
47	15	11¾	615	17	10¾
14	19	10	375	14	7
37	15	11½	14	11	6½
64	12	10¼			

TROY WEIGHT.—*See Table* II.

(15.)			(16.)			(17.)			(18.)		
lb.	oz.	dwt.	oz.	dwt.	gr.	lb.	oz.	dwt.	oz.	dwt.	gr.
14	11	9	74	12	13	175	3	10	17	10	20
11	10	14	64	14	17	159	11	14	14	11	23

APOTHECARIES WEIGHT.—*See Table* III.

(19.)			(20.)			(21.)			(22.)		
℔	℥	ʒ	℥	ʒ	℈	ʒ	℈	gr.	℔	℥	ʒ
144	10	5	27	4	1	27	1	14	74	10	5
64	11	7	14	7	2	14	0	19	65	11	6

AVOIRDUPOIS WEIGHT.—*See Table* IV.

(23.)			(24.)			(25.)			(26.)		
T.	cwt.	qr.	Cwt.	qr.	lb.	Qr.	lb.	oz.	lb.	oz.	dr.
14	12	2	17	1	25	143	22	12	174	11	10
1	14	3	14	2	27	74	19	14	59	12	13

CLOTH MEASURE.—*See Table* V.

(27.)			(28.)			(29.)			(30.)		
Yds.	qr.	n.	E.E.	qr.	n.	E.Fr.	qr.	n.	E.Fl.	qr.	n.
174	2	1	174	3	1	171	1	3	12	1	1
39	3	2	49	4	2	74	5	2	10	2	3

LONG MEASURE.—*See Table* VI.

(31.)			(32.)			(33.)			(34.)		
Lea.	m.	f.	F.	p.	yd.	P.	yd.	ft.	Ft.	in.	b.c.
21	2	4	14	34	4¼	14	3½	1	17	11	2
3	2	6	12	39	5¼	9	4½	2	14	11	1

LAND MEASURE.—*See Table* VII.

(35.)			(36.)			(37.)			(38.)		
A.	r.	p.	A.	r.	p.	A.	r.	p.	A.	r.	p.
12	1	32	112	1	31	12	1	25	19	1	20
1	3	14	74	2	37	10	3	39	14	2	21

WINE MEASURE.—*See Table* IX.

(39.)			(40.)			(41.)			(42.)		
T.	hhd.	g.	Punch.	g.	qt.	Tier.	g.	qt.	Gall.	qt.	pt.
27	2	54	147	14	2	14	1	3	24	2	2
19	3	62	79	83	3	12	41	3	18	0	1

ALE AND BEER MEASURE.—*See Table* X.

(43.)			(44.)			(45.)			(46.)		
A.B.	£	g.	B.B.	fir.	g.	A.hhd.	g.	qt.	B.hhd.	g.	qt.
14	3	5	147	1	3	271	1	2	143	1	2
12	3	7	39	3	8	49	47	3	79	52	3

DRY MEASURE.—*See Table* XII.

(47.)			(48.)			(49.)			(50.)		
Ch.	b.	p.	Ch.	qr.	b.	Qr.	b.	p.	Score	ch.	b.
74	31	3	17	3	1	147	6	2	47	1	12
47	31	2	14	3	7	94	7	3	14	20	35

MEASURE OF TIME.—*See Table* XIII.

(51.)			(52.)			(53.)			(54.)		
Yrs.	m.	w.	M.	w.	d.	D.	hrs.	m.	Hrs.	min.	sec.
17	11	2	147	2	3	167	21	50	174	50	51
14	12	3	19	2	4	19	23	54	94	59	57

CLASS II. *Promiscuous Examples.*

(55.) A horse in his furniture is worth £52 10*s.*; out of it, £24 10*s.* 6*d.*; how much does the price of the furniture exceed that of the horse?

(56.) What sum added to £11 14*s.* 9¼*d.*, will make £133 11*s.* 9½*d.*?

(57.) A tradesman, failing, was indebted to A £105 19*s.* 11*d.*, to B 150 guineas, to C £34 18*s.* 10*d.*, to D £500 19*s.*, to E £700 14*s.* 9*d.* When this happened, he had cash by him to the amount of £50, goods to the amount of £350 14*s.* 9*d.*, his household furniture was worth £24 11*s.* his book-debts amounted to £94 14*s.* 8*d.* If these things were faithfully given up to his creditors, what did they lose by him?

(58.) The great bell at *Oxford*, the heaviest in *England*, weighs 7t. 11cwt. 3qr. 4lb., *St. Paul's* bell at *London* weighs 5t. 2cwt. 1qr. 22lb., and *Tom* of *Lincoln* weighs 4t. 16cwt. 3qr. 18lb. How much are these bells, together, inferior in weight to the great bell at *Moscow*, the largest in the world, which weighs 198t. 2cwt. 1qr.?

(59.) An apprentice, who is 14 years, 11 months, 13 weeks, 14 days, 15 hours, 38 minutes old, is to serve his master till he is 21 years of age. How long has he to serve?

(60.) What are the difference of latitude and longitude between *Calcutta* in the *East Indies* (Lat. 22° 34′ N., Long. 88° 34′ E.), and *Lima* in *South America* (Lat. 12° 1′ S., Long. 76° 44′ W.)

COMPOUND MULTIPLICATION.

Definition.—Compound Multiplication is a rule by which we find the amount of any given number, of different denominations, by repeating it any proposed number of times.

Proposition 1. *When the multiplier does not exceed* 12.

Rule. Multiply the lowest denomination by it, divide the product by the number making one of the next higher denomination; set down the remainder, and carry the quotient to the product of the next higher denomination: proceed thus till all the denominations are multiplied.

Prop. 2. *If the multiplier exceeds* 12, *and is a composite number.*

Rule. Multiply successively by the component parts instead of the whole number at once.

Prop. 3. *When the multiplier cannot be produced by the multiplication of two, or more, small numbers.*

Rule. Find two, or more, numbers that compose the *nearest* number to the multiplier; then multiply by the component parts, as before, and add or subtract, the odd parts, as you find occasion.

Prop. 4. *If the multiplier be four, five, or more, hundreds.*

Rule. Multiply the given price, or quantity, by 10, and that product by 10, and so on for 10, 100, or 1000 times the price or quantity: then multiply each product by the number of thousands, hundreds, and tens, and the first line by as many as make up the number of things, or multiplier, and the sum of the products will be the answer.

Prop. 5. *If the multiplier be a whole number with parts annexed.*

Rule. When you have multiplied by the whole number, for $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, or $\frac{1}{6}$, &c. divide the top line by 2, 3, 4, 5, or 6, &c.; but, if the numerator of the fractional part be greater than 1, multiply the top line by it, and divide the product by the denominator; add this quotient to the product, or value, obtained by multiplying with the whole number.

Note. The upper figure is called the numerator, and the lower one the denominator. Thus $\frac{5}{7}$ *Numerator. Denominator.*

Examples to Proposition 1.

(1.) What cost 4 yards of cloth at 7s. 6½d. per yard?

	7	6½
		4
£1	10	2 Answer.

(2.) 5 bushels at 5s. 10d.

(3.) 6 yards at 6s. 9d.
(4.) 7 ells at 5s. 11¾d.
(5.) 8oz. at 7s. 10d.
(6.) 9lb. at 7s. 5¼d.
(7.) 10 gallons at 16s. 4¾d.
(8.) 11cwt. at 1l. 9s. 10½d.
(9.) 12 sheep at 1l. 17s. 9d.

(10.) In 9 pieces of Kersey, each 14yds. 3qrs. 2n., how many yards?

(11.) What is the weight of 12 tankards, each weighing 11oz. 10dwt. 19gr.?

(12.) In 11 pieces of cloth, each 17yds. 3qrs. 3n., how many yards?

Examples to Prop. 2.

(13.) What cost 15 gallons of wine, at 5s. 3½d. per gallon?

	5	5×3 = 15.
1	6	5½ price of 5.
	3	
£3	19	4½ price of 15. Ans.

(14.) 16hhds. at 3l. 14s. 5d.

(15.) 24 yards at 7s. 5½d.
(16.) 35cwt. at 1l. 14s. 8¼d.
(17.) 36tns. at 5l. 15s. 11¼d.
(18.) 84chal. at 1l. 16s. 9¾d.
(19.) 108 bushels at 7s. 9½d.
(20.) 132 ells at 18s. 9½d.
(21.) 144bts. at 5l. 13s. 9½d.

(22.) In 32 wedges of gold, each 2lb. 7oz. 14gr., how many pounds?

(23.) In 21 fields, each 3a. 2r. 19p., how many acres?

Examples to Prop. 3.

(24.) What cost 23 yards of cloth at 14s. 9d. per yard.

	s.	d.	
	14	9	$\overline{7\times3+2}=23.$
		7	
5	3	3	price of 7.
		3	
15	9	9	price of 21.
Add 1	9	6	price of 2.
16	19	3	price of 23.

	s.	d.	
Or thus	14	9	$\overline{6\times4}-1=23.$
		6	
4	8	6	price of 6.
		4	
17	14	0	price of 24.
Subtract	14	9	price of 1.
16	19	3	price of 23.

(25.) 31 yards at 12*s.* 7¼*d.*
(26.) 39 dozen at 6*s.* 7½*d.*
(27.) 139 pair at 4*s.* 9¼*d.*
(28.) 86lb. of silk at 19*s.* 4*d.*
(29.) 111 sacks of flour at 1*l.* 4*s.* 9*d.*
(30.) 156cwt. at 4*l.* 9*s.* 6*d.*

(31.) In 67 years, each 13m. 1 day, 6hrs., how many months?

(32.) What is the weight of 29hhds. of sugar, each 7cwt. 2qr. 18lb.?

(33.) In 67 parcels of tea, each 25lb. 7oz. 13drs., how many cwts., &c.?

Examples to Prop. 4.

(34.) What cost 394 yards at

	s.	d.	
	17	5½	per yard.
		10	
9× 8	14	7	price of 10.
		10	
87	5	10	price of 100.
		3	
261	17	6	price of 300.
78	11	3	price of 90.
3	9	10	price of 4.
343	18	7	price of 394.

(35.) 357calves at 7*l.* 10*s.* 5*d.*
(36.) 549 yards at 12*s.* 9½*d.*
(37.) 754lb. of tea at 6*s.* 10*d.*
(38.) 198lb. of indigo at 6*s.* 3¼*d.*
(39.) 754foth. at 20*l.* 5*s.* 10*d.*
(40.) 178 ells at 5*s.* 9¼*d.*
(41.) 198brls. at 1*l.* 14*s.* 9*d.*
(42.) 744chal. at 1*l.* 16*s.* 8*d.*

Examples to Prop. 5.

(43.) What cost $56\frac{1}{2}$ chaldrons

	£.	s.	d.	
at	1	14	9	per chaldron.
			7	
	12	3	3	price of 7.
			8	
	97	6	0	price of 56.
		17	$4\frac{1}{2}$	price of $\frac{1}{2}$.
	£98	3	$4\frac{1}{2}$	price of $56\frac{1}{2}$.

(44.) What cost $4\frac{5}{9}$ yards at

	s.	d.	
	7	6	per yard.
		4	
1	10	0	price of 4.
	4	2	price of $\frac{5}{9}$.
£1	14	2	price of $4\frac{5}{9}$.

s.	d.
7	6
	5
9)37	6
4	2

(45.) $1788\frac{1}{5}$ gallons at 6*s.* 4*d.*
(46.) $3714\frac{1}{8}$cwt. at 4*l.* 11*s.* 9*d.*
(47.) $7149\frac{3}{7}$ chaldrons at 1*l.* 14*s.* 9*d.*
(48.) $547\frac{5}{8}$ lasts at 5*l.* 5*s.*
(49.) $1749\frac{1}{2}$ firkins at 14*s.* $9\frac{1}{4}$*d.*
(50.) $754\frac{3}{4}$cwt. at 17*s.* $5\frac{1}{4}$*d.*

Note. Should the preceding examples be thought insufficient to complete the scholar in this useful rule, recourse may be had to the bills of parcels, Part III. Class 1.—If the teacher approve it, he may omit this proposition till the scholar has learnt compound division.

COMPOUND DIVISION.

Definition.—Compound Division teaches us to find how often one given number is contained in another of different denominations; or, to divide a given compound number into any proposed number of equal parts.

RULE.

Place the divisor to the left-hand of the dividend. Divide the highest denomination of the dividend by the divisor, and bring the remainder, if any, into the next inferior denomination, adding thereto the parts of that name in the dividend: divide this number as above, and so on till the whole is finished. If the divisor be large,

and not a composite number, divide after the manner of long division.

The method of proof is by Compound Multiplication.

Note. If the divisor be a whole number with parts annexed, multiply it by the denominator of the fractional part, adding the numerator of the fractional part to the product; then multiply the dividend by the denominator of the fractional part in the divisor, and divide as above.

Examples.

(1.) A gentleman's income is 1260*l.* 15*s.* 5*d.* a year, what is that per day, 365 days being contained in one year?

```
            £.   s.  d.      £.  s.  d.
  365  )  1260   15   5  (   3   9   1 Answer.
          1095                       10
          ----               -----------
           165               34  10  10×6
            20                       10
          ----               -----------
          3315              345   8   4
          3285                        3
          ----              ------------
            30             1036   5   0
            12              207   5   0
   [illegible]      [illegible]
   [illegible]               17   5   5
          ----              ------------
           365             1260  15   5 Proof.
           365
          ----              ------------
```

(2.) Divide 47*l.* 19*s.* 4*d.* by 3.

(3.) Div. 37*l.* 14*s.* 10*d.* by 24.

(4.) Div. 49*l.* 19*s.* 11½*d.* by 66.

(5.) Div. 34*l.* 14*s.* 9¾*d.* by 149.

(6.) Div. 477*l.* 19*s.* 10¾*d.* by 7¼.

(7.) Div. 148*l.* 11*s.* 8¼*d.* by 3⅝.

(8.) Divide 1774*l.* 19*s.* 10¾*d.* by 179.

(9.) Div. 47yds. 3qrs. 2n. by 5.

(10.) Div. 375a. 3r. 14p. by 9.

(11.) Div. 714lb. 10oz. 12gr. by 89.

(12.) Div. 374cwt. 3qr. 10lb. by 49.

(13.) Div. 374 ells E. 2qrs. 3n. by 149.

(14.) Div. 3449 ch. 21lb. 3p. by 374⅜.

(15.) Div. 47oz. 11dwt. 12gr. by 34¾.

(16.) If 60 sheep be sold for 112*l.* 10*s.*, what is the value of 1?

(17.) If 112lb. of cheese cost 2*l.* 16*s.* 8*d.*, what is that per lb.?

(18.) If 17cwt. of lead cost 15*l.* 5*s.* 7¾*d.*, what costs 1?

(19.) Bought 7 yards of cloth for 16*s.* 4*d.* what is that per yard?

(20.) If 63 oxen cost 2553*l.* 1*s.* 6*d.* what cost 1?

(21.) If 66lb. of butter cost 5*l.* 15*s.* 6*d.* what costs 1lb.?

(22.) If 528lb. of tobacco cost 23*l.* 13*s.* what costs 1lb.?

(23.) If a tun, or 252 gallons, of wine cost 60*l.* what costs 1 gallon?

(24.) A prize of 1000 guineas is to be divided among 150 sailors, what is each man's share, after deducting $\frac{1}{8}$ part for the officers?

(25.) If 125 ingots of silver, each of an equal weight, weigh 1347oz. 11dwt. 14gr. what is the weight of 1 ingot?

(26.) If 475cwt. 1qr. 14lb. be the weight of 27hhds. of tobacco, what is the weight of 1?

(27.) Bought $6\frac{1}{2}$ pieces of tapestry, containing 237 ells Flem. 2qr. 2n. what is the length of 1 piece?

CLASS II.

(28.) A common pasture, containing 54a. 1r. 35p.; another, containing $54\frac{1}{2}$ acres; and a third, containing 39a. 13p.; are to be enclosed and divided among 60 parishioners: what is each man's share, after deducting 21a. 2r. for tithes, admitting the land to be equally good?

(29.) Twenty-six wedges of gold, weighing, with a due proportion of alloy, 34lb. 3oz. 11dwt. 14gr. were brought to the mint to be coined into guineas; what is the weight of each wedge, admitting them equal, and how many guineas may be made out of the whole, supposing no loss in the metal, and that an oz. will make $3\frac{3}{4}$ guineas?

(30.) A person sold a hogshead of sugar, weighing 7cwt. 3qr. 14lb. how much pure sugar was contained in it; thirteen times the weight of the dross and hhd. being equal to the weight of pure sugar?

(31.) If a *talent* of silver be worth 357*l.* 11*s.* $10\frac{1}{2}$*d.* what is the value of a *shekel*, of which 300 make a *talent*, and what is the weight of a *talent*, a *shekel* weighing 9dwt. 3gr.?

(32.) *Camillus* the *Roman* general, after conquering the city of *Veii*, and other services done to his country, was, through the enmity and avarice of the tribunes, fined 1500 *as's*, value 4*l.* 13*s.* 9*d.* Pray what was the value of an *as* in English money?

REDUCTION.

Definition.—*Reduction* is the method of reducing numbers from one name, or denomination, to another of the same value.

RULE.

All great names are brought into small by multiplying by as many of the next less as make one of the greater, adding to the product the parts of the less name, if the number to be reduced be a compound one; and all small names are brought into great by dividing by as many of the less as make one of the next greater.

The method of proof is by reversing the question.

Note 1. To multiply a whole number by a whole number with a fraction joined to it. When you have multiplied by the whole number, as in simple multiplication, if the part be $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, or $\frac{1}{6}$, &c., divide the multiplicand by 2, 3, 4, 5, or 6, &c.; but, if the numerator of the fractional part be greater than 1, let the multiplicand be multiplied by it, and divide the product by the denominator: add this quotient to the product obtained by multiplying with the whole number.

2. To divide a whole number by a whole number with a fraction joined to it. Multiply the divisor by the denominator of the fractional part, and add the numerator to the product; let this be a new divisor; then multiply the dividend by the denominator of the fractional part for a new dividend, which divide by the new divisor to obtain the true quotient. When the dividend has a fraction joined to it, the rule for obtaining the quotient will be exactly the reverse of this.

Examples.

MONEY.—*See Table* I.

(1.) In 5*l.* 5*s.* how many shillings, pence, and farthings?

```
£.  s.
 5  5
20
----
 105 shillings.
  12
----
1260 pence.
   4
----
5040 farthings.
----
```

Here is a great name brought into a small.

(2.) In 4800 farthings how many pence, shillings, and pounds?

```
 4)4800 farthings.
   ----
12)1200 pence.
   ----
20)100 shillings.
   ----
     5 pounds.
   ----
```

Here a small name is brought into a great, by a method of operation directly the converse of the preceding.

(3.) In 19 pounds how many shillings, pence, and farthings?

(4.) In 55 guineas how many shillings, pence, and farthings?

(5.) Reduce 54*l.* 11*s.* 9¼*d.* into farthings.

(6.) Reduce 77*l.* 11*s.* 10½*d.* into halfpence.

(7.) Reduce 94*l.* 14*s.* 8*d.* into pence.

(8.) Reduce 47*l.* 14*s.* 4*d.* into two-pences.

(9.) In 34*l.* 11*s.* 9*d.* how many three-pences and pence?

(10.) In 47*l.* 19*s.* 8*d.* how many groats, pence, and farthings?

(11.) In 108*l.* 11*s.* 6*d.* how many sixpences.

(12.) How many crowns, half-crowns, shillings, sixpences, and pence, are in 54*l.*?

(13.) Reduce 74*l.* 13*s.* 9*d.* into shillings, three-pences, and farthings.

(14.) In 11520 farthings how many pence, shillings, and pounds?

(15.) In 17880 pence how many pounds?

(16.) Reduce 100800 farthings into guineas.

(17.) In 50400 halfpence how many pounds?

(18.) In 12050 shillings how many crowns and pounds?

(19.) Reduce 311040 pence into groats, shillings, crowns, and pounds.

(20.) In 102*l.* 16*s.* 3*d.* how many pieces of coin, each 7*s.* 3½*d.* in value?

(21.) In 7494 dollars, at 4*s.* 6*d.* each, how many groats, shillings, half-crowns, crowns, and pounds?

(22.) In a Jacobus, a Carolus, 5 angels, 3 marks, 5½ nobles, 6 testers, and 90 groats, how many farthings?

2. TROY WEIGHT.—*See Table* II.

(23.) In 17lb. 5oz. how many grains? *Ans.* 100320.

(24.) In 6720 grains how many ounces? *Ans.* 14.

(25.) In 14 ingots of silver, each 27oz. 10dwt., how many grains?

(26.) In 474 spoons, each weighing 3oz. 10dwt., how many pounds of silver?

(27.) How many pints, each 9oz., may be made out of 17lb. 6oz. 14dwt. of silver?

(28.) A gentleman sent a tankard to his goldsmith, weighing 50oz. 8dwt., and ordered him to make it into tea-spoons, each weighing $1\frac{1}{5}$oz. how many had he?

3. APOTHECARIES WEIGHT.—*See Table* III.

(29.) In 25lb. how many scruples and grains? *Ans.* 7200℈ 144000grs.

(30.) In 97920 grains how many ounces and pounds? *Ans.* 204℥ 17℔.

(31.) In 15℔ 1℥ 1ʒ 1℈ 2gr. how many grains?

(32.) In 174947 grains how many pounds?

(33.) An apothecary made a compound of 12℥ 1ʒ 2℈ 14gr. into *troches* of 1℈, of $1\frac{1}{3}$℈, and of 14gr.; and into *pills* of 11gr. and 13gr. each; he made an equal number of *troches* and *pills*: how many of each had he?

4. AVOIRDUPOIS WEIGHT.—*See Table* IV.

(34.) In 12 tons of iron how many lb.? *Ans.* 26880lb.

(35.) In 31360lb. of iron how many tons? *Ans.* 14 *tons.*

(36.) In 375cwt. 2qr. 15lb. of copper how many lb.?

(37.) Reduce 740900oz. into cwts. and tons.

(38.) In 39 bags of hops, each 3cwt. 1qr. 14lb., how many cwts.?

(39.) In 750 fother of lead, each $19\frac{1}{2}$cwt., how many cwts.?

(40.) In 135cwt. of raisins, how many parcels, each 90lb.?

(41.) In 570 great pounds of silk how many common?

(42.) In 525 common pounds of silk how many great?

(43.) How many pounds in 54hhds. of tobacco, each weighing $17\frac{1}{2}$cwt.?

(44.) A grocer weighed out an hhd. of sugar, containing 16cwt. 3qr. 10lb., into parcels of 6lb., of 8lb., of 12lb., of 14lb., and of 28lb., and had an equal number of each; how many of each had he?

5. CLOTH MEASURE.—*See Table* V.

(45.) In 314 yards how many nails? *Ans.* 5024 *nails.*

(46.) In 576 French ells how many yards? *Ans.* 864 *yards.*

(47.) Reduce 97yds. 3qrs. into English ells.

(48.) In 57 pieces of Holland, each 35 ells Flemish, how many nails?

(49.) In 14 bales of cloth, each 17 pieces, each piece 56 ells Flemish, how many yards?

(50.) In 394 pieces of stuff, each 23¼ yards, how many yards?

(51.) In 706 pieces of *Kersey*, each 45⅞ yards, how many yards?

6. LONG MEASURE.—*See Table* VI.

(52.) In 471 miles how many furlongs and poles? *Ans.* 3768f. 150720p.

(53.) In 123200 yards how many miles? *Ans.* 70m.

(54.) In 50 miles how many yards, feet, inches, and barley-corns?

(55.) Reduce 37m. 2furl. 37p. 5ft. 6in. into feet.

(56.) In 17400 chains how many furlongs and miles?

(57.) How many barley-corns will reach round the earth, which is 360 degrees, each 69½ miles? and how many quarters of barley are contained in such a number of barley-corns, admitting 9212 barley-corns to fill a pint; and that 512 pints will make a quarter?

(58.) How often will a *perambulator*, 2¾ yards in circumference, turn between *London* and *York*, being 198 miles?

7. LAND MEASURE.—*See Table* VII.

(59.) In 77a. 1r. 14p., how many perches? *Ans.* 12374p.

(60.) In 17280 perches how many acres? *Ans.* 108a.

(61.) If a piece of ground, containing 14a. 34p., be taken from a field of 50 acres, how many perches will the remainder contain?

(62.) A gentleman has 4 fields, the first measures 3a. 1r., the second 4½ acres, the third 5a. 30p., and the fourth 4a. 3r. 20p., and these he wishes to divide into parcels, or shares, of 3¾ roods each, for the purpose of accommodating his manufacturing tenants with small tenements: how many will he have?

8. WINE MEASURE.—*See Table* IX.

(63.) Reduce 32hhds. into quarts. *Ans.* 8064*qts.*

(64.) In 3276 gallons how many tuns? *Ans.* 13 *tuns.*

(65.) How many gallons and pints are in 75hhds.?

(66.) In 77hhds. of brandy how many half-ankers?

(67.) In 10 tuns 2hhds. 18 gallons of wine, how many pipes, puncheons, hhds., tierces, and runlets, and of each an equal number?

9. ALE AND BEER MEASURE.—*See Tables* X. *and* XI.

(68.) In 38 hogsheads of ale, in *London*, how many pints? *Ans.* 14592 *pints.*

(69.) In 38 hogsheads of ale, in the country, how many pints? *Ans.* 15504 *pints.*

(70.) Reduce 516 barrels of beer, *London* measure, into half-pints.

(71.) How many gallons of beer are contained in a back of 50 barrels, country measure?

10. DRY MEASURE.—*See Table* XII.

(72.) In 44 quarters of corn how many pecks? *Ans.* 1408 *pecks.*

(73.) In 30720 quarts how many lasts? *Ans.* 12 *lasts.*

(74.) In 50 chaldrons of coals how many pecks?

(75.) How many sacks, of 3 bushels each, are contained in 193chald. 12bush. of coals?

11. MEASURE OF TIME.—*See Table* XIII.

(76.) In 365d. 5h. 48m. 55sec. being a *solar* year, how many seconds? *Ans.* 31556935 *seconds.*

(77.) In 354d. 8h. 48m. 36½sec. being a *lunar* year, or 12 *lunar* months, how many seconds? *Ans.* 30617316½ *seconds.*

(78.) How many days, hours, minutes, and seconds, have elapsed from the creation of the world to Christmas 1818, supposing the creation to have been 4004 years before the incarnation of Christ?

(79.) If *London* was built 1108 years before Christ's nativity, how many hours is it since to Christmas 1818?

(80.) From May 18, 1818, to February 18, 1845, how many days?

CLASS II. *Promiscuous Examples.*

(81.) A butcher has 22 oxen, each weighing $238\frac{1}{2}$ stone, eight pounds to the stone, to be cut out for sea-service into pieces of 14lb. of 26lb. of 22lb. of 30lb, of 16lb. and of 15lb. and to have an equal number of each; how many pieces will these oxen produce, allowing nothing for waste?

(82.) A country gentleman ordered 58*l.* 14*s.* to be distributed among the poor inhabitants of 4 villages. Those of the place of his residence were to have 1*s.* each, those of the next 8*d.* the next were to have 6*d.* and the last 4*d.* each;—four persons (one out of each village), who shared in the bounty, were appointed to distribute the money. Now, admitting the number of indigent persons in each village to be equal, how many partook of this charity, the men who distributed the money being allowed 5*s.* $4\frac{1}{2}$*d.* each *extra.*

(83.) A gentleman sent to his goldsmith 18 ingots of silver, each weighing 3lb. 7oz. 14dwt. 21gr. with orders to make it into tankards of 18oz. 14dwt. 10gr. each, cups of 19oz. 15dwt. 11gr. each, spoons of 24oz. 10dwt. 14gr. per dozen, salts of 4oz. 12dwt. each, forks of 22oz. 11dwt. 14gr. per dozen; for every tankard he was to make one cup, a dozen spoons, one salt, and a dozen forks:—how many of each will it make, allowing 7oz. 1dwt. 14gr. for dross, and what quantity of silver will there be left?

(84.) How long would 500 people be in counting a *billion* of money, supposing each of them counted 100*l.* every minute, (without intermission), the year consisting of 365 days 6 hours?

(85.) According to the Julian account, which was used in England before the year 1752, the year consists of 365 days for three years successively, and 366 days every fourth, or $365\frac{1}{4}$ days at a mean; and the solar year, according to the best astronomical calculation, consists of 365 days 5hrs. 48m. 48sec.—Required in how many years the seasons of the year would be quite reversed, *viz.* how many years would elapse before *Christmas* would fall upon *Midsummer?*

(86.) If $44\frac{1}{2}$ guineas make 1lb. *Troy*, and 48 * halfpence make 1lb. *Avoirdupois*, what is the weight of a guinea and of a halfpenny in Troy weight?

(87.) A farmer had 5 sons, to whom he left 500*l.* in cash, and 5 bills of 84*l.* 10*s.* 6*d.* each; he ordered his debts to be paid, amounting to 120*l.*; and 20*l.* to be expended at his funeral: the residue was to be divided in this manner; the eldest was to have a fourth part, and each of the other sons to have equal shares: what was the share of each son?

(88.) The national debt is eight hundred millions, and thirty ten-pound bank notes, upon an average, weigh an ounce Avoirdupois; now, supposing this debt to consist entirely of ten-pound notes what would be the weight thereof?

(89.) The mean distance of the earth from the sun is ninety-five millions of miles, and the circumference of the earth's orbit is $3\frac{1}{7}$ times its diameter; now, as the earth goes round the sun in 365 days 6 hours, at what rate per hour does it travel?

(90.) A general distributed 307*l.* 17*s.* among 4 captains, 5 lieutenants, and 60 common soldiers: to every lieutenant he gave twice as much as to a common soldier, and to every captain three times as much as to a lieutenant: what did each receive?

THE RULE OF THREE DIRECT.

Definition.—The Rule of Three Direct teaches, by three given numbers, to find a fourth, which shall have the same ratio to the second as the third has to the first; that is, if the first be greater than the third, the second will be greater than the fourth; and, if the first be less than the third, the second will be less than the fourth.

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* In the old copper coinage three new halfpence weighed an ounce. In *Bolton's* coinage, the twopenny-pieces weigh two ounces, the penny-pieces one ounce, but a halfpenny weighs less than half an ounce; and a farthing less than a quarter of an ounce. The last coinage of 1806 is still lighter, on account of the advance in the price of copper.

RULE.

State the question by placing the numbers in such order that the first and third may be of one kind, and the second the same as the number required: then bring the first and third numbers into one name, and the second into the lowest denomination mentioned. Multiply the second and third numbers together, divide the product by the first, and the quotient will be the answer in the same denomination as the second number.

If there be a remainder after division, it is always of the same name as the lowest denomination of the middle number, and must be brought into the next inferior denomination, then divide again by the first number, &c. till you come to the lowest denomination which the middle number admits of. The several quotients, taken together, will be the answer.

The method of proof is by changing the order of the stating.

Particular Rules and Observations.

1. If the first term, and either the second or third, can be divided by any number, without a remainder, let them be divided, and the quotients used instead of them.

2. Divide the second term by the first, multiply the quotient by the third, and the product will be the answer.

3. Divide the third term by the first, multiply the quotient by the second, and the product will be the answer.

4. Divide the first term by the second, and the third by that quotient, the last quotient will be the answer.

5. Divide the first term by the third, and the second by that quotient, the last quotient will be the answer.

6. When any of the above methods can be used, they will be found more convenient than the general rule.

7. The greatest difficulty in the Rule of Three, is in stating the question, or abstracting the numbers out of the words in the question, and placing them down in their proper order: to perform which, the following observations may assist the scholar.

In all questions in the Rule of Three, there are three given terms: two of supposition, and one of demand; that of demand must always be the third number, and may be known by the words, What cost? What will? How far? How much? How many? &c. The first term must always be of the same name as the last, or term of demand; and the term sought will be of the same kind and denomination as the second term in the supposition.

8. The method of proof, as has been already observed, is by changing the order of the stating; the following example will shew in what

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manner it may be varied.—*Example.* If 2 yards of cloth cost four shillings, what will 8 yards cost at the same rate?

State it thus:

1st term.	*2d term.*	*3d term.*	*4th term.*
2 yards :	4 shillings ::	8 yards :	16 shillings, *Answer.*

Variation 1. The third term is to the fourth as the first is to the second.

2. The second term is to the first as the fourth is to the third.

3. The fourth term is to the third as the second is to the first.

Examples.

(1.) If 2cwt. 3qrs. 14lb. of sugar cost 6*l.* 14*s.* 2*d.*, what will 12cwt. 3qrs. cost?

```
 1st number.          2d number.         3d number.
2cwt. 3qr. 14lb.  :  6l. 14s. 2d.  ::  12cwt. 3qr.
4                     20                  4
----                  ----               ----
 11qr.               134 shillings       51qr.
 28                   12                  28
----                  ----               ----
322lb.               1610 pence.         408
                                         102
                                         ----
                                        1428lb.
                                         1610
                                       ------
                                        14280
                                       8568
                                      1428
                                      -------12)
                           322)2299080(7140 pence, fourth number.
Answer, 29l. 15s.             2254 2|0(59|5 shillings.
                              ----  -------
                               450 £29:15
                               322
                               ----
                              1288
                              1288
                              ----
                              ....0
```

In the above stating, when the terms are reduced according to the rule, they stand thus, 322lb. : 1610 pence :: 1428lb. : 7140 pence. Now, if 1lb. had cost 1610 pence, it is clear that 1428lb. would have cost 1428 times 1610 pence; therefore the second and third terms must have been multiplied together, which would have produced 2299080 pence for the answer.

Had 2lb. been bought for 1610 pence, the answer would have been

half the product of the second and third terms; had 3lb. been bought for 1610 pence, the answer would have been one-third of the above product, and so on; hence it is obvious, that, in all cases, the product of the second and third terms must be divided by the first term, agreeably to the rule.

(2.) If 12cwt. 3qr. of sugar be bought for 29*l*. 15*s*., what will 2cwt. 3qr. 14lb. cost? *Ans*. 6*l*. 14*s*. 2*d*.

(3.) If 6*l*. 14*s*. 2*d*. be paid for 2cwt. 3qr. 14lb. of sugar, what quantity may be bought for 29*l*. 15*s*.? *Ans*. 12cwt. 3qr.

(4.) If 29*l*. 15*s*. will buy 12cwt. 3qr. of sugar, what quantity will 6*l*. 14*s*. 2*d*. buy? *Ans*. 2cwt. 3qr. 14lb.

(5.) If a cwt. of tobacco be worth 9*l*. 16*s*., what is the worth of 1lb.?

(6.) If 1lb. of butter cost 1*s*. 8*d*. what will a firkin, or 56lb. cost? *

(7.) Bought $3\frac{1}{2}$ yards of cloth for 2*l*. 16*s*. 3*d*., what must I give for $28\frac{3}{4}$ yards at the same rate?

(8.) If I buy 56 yards of cloth for 40 guineas, how many ells Flemish can I buy for 1135*l*. 10*s*.?

(9.) A sailor entered on board a man-of-war the 14th of May, 1780, and was discharged the 11th of December, 1783, what came his wages to at 1*l*. 5*s*. per month, reckoning 28 days to a month?

(10.) How long will a person be saving 100*l*. if he lays by 1*s*. 6*d*. per week?

(11.) Bought 55 yards of holland for 11*l*. 5*s*., how many English ells can I buy for 100 guineas at the same rate?

(12.) A factor bought 30 quarters of corn for 76*l*. 17*s*. 6*d*. and 150 quarters of an inferior kind for 361*l*. 11*s*. 8*d*. to mix with it; how must he sell the mixture per bushel to gain 20*l*. by the bargain?

(13.) Bought 27 pieces of cloth, each 34 ells, at 7*s*. 6*d*. per ell, what is the value of the whole?

(14.) A creditor agrees to receive of his insolvent debtor after the rate of 10*s*. 6*d*. in the pound for a debt of 475*l*. 10*s*. how much will he receive in the whole?

(15.) If 18*l*. 14*s* $9\frac{3}{4}$*d*. were paid for the carriage of 53cwt. 2qr. 5lb., what was paid for the carriage of 1lb.?

* A great variety of easy examples, exercising the Rule-of-Three, will be found by referring to Part III. Class II. &c. of the Bills of Parcels.

(16.) A bankrupt's effects amount to 1000½ guineas. His debts amount to 2547*l*. 14*s*. 9*d*., what will his creditors receive in the pound?

(17.) The rental of a village is 4714*l*. 11*s*. 10*d*. A tax of 117*l*. 17*s*. 3½*d*. is to be made for the support of the poor;—at what rate per pound must the assessment be made to defray the expences?

(18.) A gentleman pays taxes for 350*l*. 14*s*.—The rental of the whole village is 4714*l*. 11*s*. 10*d*. upon which a tax is imposed amounting to 235*l*. 14*s*. 7*d*. What sum must this gentleman pay towards this tax?

(19.) If a tax of 9*d*. in the pound be imposed upon a village for the support of the poor, what sum must a gentleman pay towards it, who pays taxes for 350*l*. 14*s*.?

(20.) Bought 14hhds. of sugar, each weighing 7cwt. 1qr. 14lb. at 2*l*. 14*s*. 9*d*. per cwt., what do they come to?

(21.) If a pack of wool weighs 2cwt. 2qr. 14lb, what is it worth at 17*s*. 6*d*. per todd?

(22.) Bought 157 fother of lead at 5*l*. 5*s*. per cwt. paid carriage, &c. 5 guineas; what does the lead stand me in per lb.?

(23.) If an ounce of gold be worth 3*l*., what is the worth of 14 ingots, each weighing 3lb. 11oz. 15dwt. 18gr.?

(24.) Bought 76 pieces of stuff for 722*l*., at 4*s*. 9*d*. per yard; how many yards did I buy, and how many English ells did each piece contain?

(25.) Bought 4 tuns of oil for 247*l*. 11*s*.—64 gallons of which being damaged, how must I sell the remainder per gallon so as neither to gain nor lose by the bargain?

(26.) A factor bought a quantity of broad-cloth and baize for 124*l*.; the quantity of broad-cloth he bought was 117½ yards, at 17*s*. 9*d*. per yard; for every 5 yards of broad-cloth he had 1½ yard of baize:—how many yards of baize did he buy, and what did it cost him per yard?

CLASS II.

(27.) A merchant in *London* bought 59 tuns of port-wine for 50 guineas per hhd.; the freight thereof from *Oporto* to *London* cost 47*l*. 10*s*., the loading and un-

loading 7*l.* 10*s.*, custom 24*l.*, charges of the cellar 3 guineas:—what was the prime cost of a gallon of this wine?

(28.) A draper bought 5 packs of cloth, each pack containing 7 parcels, each parcel 15 pieces, and each piece 15 ells E. 2qr. 3n.—For every 5 yards he bought he gave 4*l.* 7*s.* 9*d.*, what did the 5 packs of cloth stand him in?

(29.) The globe of the earth under the *equinoctial* line is 360 degrees in circumference, each degree 69½ miles:—now, if this body turns on its axis in 23hr. 56m., at what rate per hour are the inhabitants upon the *equator* carried from West to East by this rotation, and at what rate per hour are the inhabitants of *London* carried the same way?—The latitude of *London* is 51½ N., where a degree of longitude measures 37m. 2f. 37p. 5½ft.

(30.) A tax of 225*l.* 10*s.* was laid upon four villages, A, B, C, D, for repairing the church: it has been a custom with these villages, time immemorial, that, whenever any taxes were to be levied, as often as A, B, and C, paid each 3*d.*, D paid only 2*d.* What did each village pay towards the reparation of the church?

(31.) A man bought 120 eggs at three for a penny, and afterwards 120 more at two for a penny. He immediately put them altogether into a basket, and then sold them out at five for two-pence, whether did he gain or lose?

(32.) Required the *exact* time of the day between the hours of 2 and 3, when the hour and minute hands of a clock are both together, when they make an angle of 90 degrees; or are 15 minutes apart; and at what o'clock will they be *exactly* together a second time?

(33.) A hare pursued by a greyhound, is 144 of her leaps before him at setting off; now the hare makes 4 leaps while the greyhound makes 3, but the greyhound leaps as far at twice as the hare does at thrice:—how many leaps must the greyhound take to catch the hare?

(34.) Shipped for *Jamaica* 1750 pair of stockings at 4*s.* 5*d.* per pair, and 1749 yards of *Manchester* cotton at 3*s.* 7*d.* per yard, and in return I have received 475 gallons of rum at 6*s.* 9½*d.* per gallon, and 27hhds. of sugar, each weighing 7cwt. 3qr. 15lb. neat, at 3*l.* 15*s.* 7*d.* per cwt. What is the balance between us, and in whose favour?

(35.) A gentleman's yearly income is 3780*l.* his weekly expences amount to 32*l.* 15*s.*, land-tax, repairs, &c. amount to $\frac{1}{9}$ of his annual income; the charitable donations which he distributes amount to $\frac{1}{20}$ part of the remainder, his pocket expences daily amount to $1\frac{1}{2}$ guinea,—what are his whole expences in a year, and what does he lay up at the year's end?

(36.) Laid out 571*l.* 1*s.* 8*d.* in wine, at 3*s.* 7*d.* per gallon, which having received damage, by reason of some pipes staving, I found my returns no more than 419*l.* 11*s.*, by selling what came to hand in good order, at 7*s.* 6*d.* per gallon; pray what quantity of wine was lost?

(37.) A merchant bought $22\frac{1}{2}$cwt. of pepper, and $17\frac{1}{4}$ cwt. of ginger; the pepper cost him 14*l.* 19*s.* 7*d.* per cwt. the ginger 12*l.* 17*s.* 6*d.*—What is the whole value of the pepper and of the ginger, and what must each be sold for per ounce, that he may gain 90*l.* by the pepper, and the same sum by the ginger?

(38.) Bought a puncheon of rum for 41*l.* 14*s.* 6*d.*, to which I put as much water as reduced the prime cost to 5*s.* 6*d.* per gallon; what quantity of water did I put in?

(39.) Divide the number 36 into three such parts that $\frac{1}{2}$ of the first, $\frac{1}{3}$ of the second, and $\frac{1}{4}$ of the third, may be all equal to each other?

(40.) A grocer delivered 17cwt. 3qr. 10lb. of tobacco in the roll, to be cut and dried; when it came home it weighed 16cwt. 14lb.—How much was lost in every lb.? and, admitting it cost $8\frac{1}{2}$*d.* per lb. in the roll, and $1\frac{1}{4}$*d.* per lb. cutting, what does the whole now stand him in, and what must he sell it for per lb. to gain 10 guineas by it?

(41.) If one pound of tea be equal in value to 50 oranges, and 70 oranges be worth 84 lemons what is the value of a pound of tea when a lemon is worth a penny?

(42.) The distance from *London* to *York* is 198 miles: two travellers set out at the same time in order to meet the one from *London* towards *York*, the other from *York* towards *London*. The one travelled 16 miles a day, the other 17 miles a day; how many days did they travel before they met?

(43.) A footman agreed to serve his master 12 months for 18*l.* and a livery of a certain value: at the end of 7

months he was turned away, and received his livery and 8*l.* 10*s.* in money. What was the prime cost of his livery?

(44.) Shipped off 350 casks of butter, weight 516cwt. 2qr. 14lb., which cost me 2*l.* 5*s.* per cwt., paid duty 6*d.* per cwt., cooperage 2*l.* 16*s.* 0½*d.*, boat-hire 18*s.*, porterage, &c. 2*l.* 3*s.* 7*d.*, cellarage 3*l.* 4*s.* 7*d.* What did 1cwt. of the butter stand me in when on board?

(45.) A man and his wife found, by experience, that a barrel of beer, which lasted them both 12 days, would serve the man, when alone, 20 days: how long would it serve the wife in the absence of her husband, supposing when alone they drank the same quantity each as when they were together?

(46.) There is an island 73 miles in circumference, and three footmen all start together to travel the same way round it; A travels 5 miles a day, B 8 miles a day, and C 10 miles a day: in how many days will they all come together again, and how many times will each have travelled round the island?

(47.) The distance from *London* to *York* is 198 miles: two travellers set out at the same time in order to meet: A from *London* towards *York*, and B from *York* towards *London*. When they met, which was at the end of 6 days, A had travelled 3 miles a day more than B. How many miles did each travel per day?

(48.) A hare pursued by a greyhound was 86 yards before him at starting; whilst the hare ran 5 yards, the dog ran 7 yards. How far had the dog ran when he caught the hare?

(49.) I bought 60 yards of cloth at the rate of 5 yards for a guinea, and 70 yards more at the rate of 7 yards for a guinea, and immediately sold the whole at the rate of 12 yards for two guineas. Whether did I gain or lose, and how much?

(50.) A tradesman increased his capital annually one-fourth part, and at the end of three years, one year's interest thereon at 5 per cent. amounted to 122*l.* 1*s.* 4¾*d.* ½ What sum did he begin with?

(51.) If, when port-wine is 40 guineas per hhd., a company of 60 people will spend 20 guineas therein in a cer-

tain time, what is wine a pipe, when 15 persons more will spend 65 guineas in twice the time, drinking at the same rate?

(52.) A merchant began the world with a capital of 10,000*l.*; he gained 10,000*l.* in 5 years by trading to Rnssia, and 10,000*l.* in 8 years by trading to America; but he spent 10,000*l.* every $2\frac{1}{2}$ years in gaming and extravagance. How many years did he go on at this rate before he lost all his property?

INVERSE PROPORTION.

Definition.—*Inverse*, or reciprocal, Proportion teaches by three *given* numbers, to find a fourth, which shall have the same ratio to the second, as the first has to the third; that is, if the first be greater than the third, the fourth will be greater than the second; and, if the first be less than the third, the fourth will be less than the second.

RULE.

State the question as in the direct rule. Multiply the first and second terms together, and divide the product by the third, the quotient will be the answer, and of the same denomination as you left the second number.

Note 1. Direct and inverse proportion are, properly, only parts of the same general rule; and, in a mathematical arrangement, it would be best to treat of them together. However, as inverse proportion is not of such extensive use in mercantile affairs as direct proportion, I have, according to custom, considered them separately, as being more intelligible to young students.

2. I shall here specify, by familiar examples, the difference between direct and inverse proportion in as clear and concise a manner as possible.—Observe, when the question is stated, that if the third term be *greater* than the first, and requires the fourth to be *greater* than the second; or, if the third term be *less* than the first, and requires the fourth to be *less* than the second, the proportion is direct. But, if the third term be *greater* than the first, and requires the fourth to be *less* than the second; or, if the third term be *less* than the first, and requires the fourth to be *greater* than the second, the proportion is inverse.

Ex. 1st. If 3 yards of cloth cost 18*s.* what will 24 yards cost?

If *3 yards : 18*s.* :: 24 yards : 144*s.* or £7 4*s.*

Here it is evident that 24 yards will cost more than 3 yards at the same rate; hence the proportion is *direct;* for the third term is *greater* than the first, and requires the fourth to be *greater* than the second.

Ex. 2. If 112lb. of sugar cost 56s. what will 1lb. cost?

If *112lb. : 56s. :: 1lb. : 6d.

Here 1lb. of sugar will certainly cost less than 112lb., and consequently the proportion is *direct:* for, the third term is *less* than the first, and requires the fourth to be *less* than the second.

Ex. 3. If 4 men can do a piece of work in 80 days, how many days, of the same length, will 16 men require to do the same work?

If 4 men : 80 days :: *16 men : 20 days.

Here it is plain that 16 men will do a piece of work sooner than 4 men; hence this proportion is *inverse;* for, the third term is *greater* than the first, and requires the fourth to be *less* than the second.

Ex. 4. If 21 pioneers make a trench in 18 days, how many days, of the same length, will 7 men require to make a similar trench?

If 21 pioneers : 18 days :: *7 pioneers : 54 days.

Here 7 men will evidently require a longer time than 21 men to dig a trench; hence the proportion is *inverse;* for, the third term is *less* than the first, and requires the fourth to be *greater* than the second.

Examples.

(1.) If a field of grass be mowed by 10 men in 12 days in how many days would it be mowed by 20 men?

1st number.		*2d number.*		*3d number.*
10m.	:	12d.	::	20m.
		10		
		2|0)12|0		
		6 days, *answer.*		

Note. Such is the quantity of grass that 10 men would mow it in 12 days, it is therefore obvious, that, if 20 men were employed, they would mow it in half the time.

(2.) A certain piece of grass was to have been mowed by 20 men in 6 days; an extraordinary occasion calls off half the workmen:—it is required to find in what time the rest will finish it? *Answer,* 12 *days.*

(3.) If the penny-loaf weighs 5oz. when flour is at 2s. a peck, what should it weigh when flour is sold for 2s. 6d. the peck?

(4.) Provisions in a garrison are found sufficient to last 1800 soldiers for three months; but a reinforcement being wanted, that the provisions may last for one month *only*, what number of soldiers may be added to the garrison on this emergency?

(5.) If 3yds. 2qr. of cloth of 1yd. 3qr. wide will make a suit of clothes, how many yards of stuff, of $\frac{3}{4}$ yard wide, will make a suit for the same person?

(6.) If I lend my friend 200*l.* for 12 months, on condition of his returning the favour, how long ought he to lend me 150*l.* to requite my kindness?

(7.) If a statute acre be 220 yards long, the breadth will be 22 yards; but, if the breadth of an acre be 40 yards, what will the length be then?

CLASS II.

(8.) If 720 men be placed in a garrison, and have provisions for 6 months; but hear of no relief at the end of 5 months, how many men must depart that the remaining provisions may last 5 months longer?

(9.) If 5 oxen, *or* 7 colts, eat up a certain quantity of grass in 87 days, in what time will 2 oxen *and* 3 colts eat up the same quantity of grass?

(10.) A regiment of soldiers, consisting of 1000, are to be new clothed; each coat to contain $2\frac{1}{2}$ yards of cloth of $1\frac{1}{4}$ yard wide, and to be lined with shalloon of $\frac{3}{4}$ yard wide; how many yards of shalloon will line them?

(11.) A lent his friend B 91 guineas from the 11th of December, 1817, till the 10th of May, 1818; B, on another occasion, let A have 66*l.* 13*s.* 4*d.* from September 3, 1818, to Christmas, 1819, how long ought the person *obliged* to lend his friend 40*l.* to retaliate the favour?

(12.) If a ball of 18lb. be shot from a cannon with such a force as to send it 100 feet in a second, with what velocity would a ball of 24lb. move, were it impelled by the same force?

(13.) Provisions in a garrison were sufficient to last 1800 men for 12 months, but at the end of 3 months the garrison was reinforced by 600 men, and two months after that a second reinforcement of 400 men was sent to the garrison; how long did the provisions last in the whole?

(14.) How many pounds of sugar, at $9\frac{1}{2}$ per lb., are equal in value to 24lb. of tea worth 9*s.* 6*d.* per lb.?

(15.) There are two equal parallelograms; the length of the one is 10 feet 6 inches, and its breadth 7 feet 3 inches, the breath of the other is 4 feet 2 inches; what is its length?

(16.) How many yards of paper, 27 inches wide, will hang a room that is 24 yards in circuit and 9 feet 4 inches high?

(17.) If 3 men or 4 women can do a piece of work in 34 days, how long will 2 men and 3 women be in finishing a similar piece of work?

(18.) If a board be 9 inches broad, what must be its length to contain 10 square feet?

COMPOUND PROPORTION.

Compound Proportion consists of 5, 7, 9, 11, or 13, &c. conditional terms given, to find a 6th, 8th, 10th, 12th, or 14th, &c. term respectively. When five terms are given to find a sixth, it is called the *Rule of Five*, or the *Double Rule of Three*, because all questions, in which the number of terms does not exceed five, may be answered by two statings in the Single Rule of Three.

RULE I.

1. Make as many statings in the *Rule of Three* as there are terms of supposition * or demand; using that term for

* If *five* numbers be given to find a sixth, there will be two statings, and for every two given numbers, above five, there will be one additional stating.

the middle of each stating, which is of the same name, nature, or quality, with the term required to be known.

2. Place the statings regularly one under another, so that each conditional term, or term of supposition, may stand on the left-hand of the middle term, and have a proper reference to it. The terms of demand will then stand under each other on the right-hand of the middle term, and each will refer separately to the answer correspondent to each stating.

3. From the nature of proportion the first and third terms of every stating will be of the same kind, and must be reduced to the same denomination. Examine every stating separately (using the middle term in common for each stating) by saying, if the first term give the second, does the third require more or less? if more, mark the less extreme; if less, mark the greater extreme for a divisor.

4. Multiply all the numbers together which are marked, for a divisor; and those which are not marked for a dividend, and the quotient will be the answer.

5. The work may be contracted by throwing out such numbers as occur both in the divisor and the dividend; or by dividing any two numbers in the divisor and dividend by their common measure, and using the quotients instead of the original numbers.

RULE II.

1. Set down the terms expressing the conditions of the question in one line, taking care to separate the *cause* from the *effect*.

2. Under each conditional term set its corresponding one in another line, marking the term sought, or wanting, with an asterisk (*).

3. Draw cross lines from the *cause* term, or terms, in the first part of the first line, to the *effect* term, or terms, in the second part of the second line; and, from the *effect* term, or terms, in the second part of the first line, to the *cause* term, or terms, in the first part of the second line.

4. Multiply the term, or terms, at the end of the cross-

line, where the star-term is found, into the term, or terms, at the other end of *that* line for a divisor. Then multiply all the terms together, standing at contrary ends of the other cross-line for a dividend. The quotient will be the answer, and of the same name with that term under which the asterisk is placed.

Note. *When a term is only understood, and not expressed, the place of that term must always be supplied by an unit.*

Examples.

(1.) If 7 men can reap 126 acres in 12 days, how many acres will 16 men reap in 3 days?

By Rule 1st.

m.		*acres.*		m.
†7	:	126	::	16
†12d.	:	—	::	3d.

$126 \times 16 \times 3 = 6048$ dividend.
$12 \times 7 = 84$ divisor.
The quotient is 72 acres, *Answer.*

By two statings.

7m.†	:	126a.	::	16m.	:	288a.
12d.†	:	288a.	::	3d.	:	72a.

Or thus,

7m.	:	12d.	::	16m.†	:	$5\frac{1}{4}$d.
$5\frac{1}{4}$d.†	:	126a.	::	3d.	:	72a.

(2.) If 7 men can reap 126 acres in 12 days, how many men will reap 72 acres in 3 days?

By Rule 2d.

cause.		*effect.*
If 7m.	12d.	126a.
*.	3d.	72a.

$3 \times 126 = 378$ divisor.
$7 \times 12 \times 72 = 6048$ dividend.
The quotient is 16 men, *Answer.*

By two statings.

12d.†	:	126a.	::	3d.	:	$31\frac{1}{2}$a.
$31\frac{1}{2}$a.†	:	7m.	::	72a.	:	16m.

Or thus,

12d.	:	7m.	::	3d.†	:	28m.
126a.†	:	28m.	::	72a.	:	16m.

The marks (†) point out the divisors in the single statings.

(3.) If 7 men in 12 days can reap 126 acres, in how many days will 16 men reap 72 acres? *Ans.* 3 days.

(4.) A carrier receives 15*l.* 12*s.* for the carriage of $4\frac{1}{2}$ tons 18 miles, how much will he carry 72 miles for 20 guineas?

(5.) If 100*l.* principal gain 4*l.* in 12 months, what principal will gain 20*l.* in 19 months?

(6.) The carriage of 11cwt. 2qr. for 150 miles costs 6*l.* 14*s.* 8*d.*, how much must be paid for the carriage of 15cwt. 1qr. 22lb. for 64 miles at the same rate?

(7.) If a regiment of 1878 soldiers consume 702 quarters of wheat in 336 days, how many quarters will an army of 22536 soldiers consume in 112 days?

(8.) If 100*l.* at interest for 1 year, or 365 days, gain 5*l.*, how much will 144*l.* 14*s.* 9*d.* gain in 495 days?

(9.) If 12 taylors in 7 days can finish 13 suits of clothes, how many taylors, in 19 days of the same length, can finish the clothes of a regiment of soldiers consisting of 494 men?

(10.) An ordinary of 100 men drank 20*l.* worth of wine at 2*s.* 6*d.* per bottle; how many men, at the same rate of drinking, will 7*l.* worth suffice, when wine is rated at 1*s.* 9*d.* per bottle?

(11.) If the carriage of 126lb. for 100 miles cost 6*s.*, how many pounds may I have carried 750 miles for a guinea?

(12.) If 60 bushels of oats will serve 24 horses for 40 days, how long will 30 bushels serve 48 horses at the same rate?

CLASS II.

Examples wherein the number of terms exceeds five.

(13.) If 4 compositors, in 16 days of 12 hours long, can compose 14 sheets, of 24 pages in each sheet, 44 lines in a page, and 40 letters in a line,—in how many days, of 10 hours long, may 9 compositors compose a volume, to be printed on the same letter, consisting of 30 sheets, 16 pages in a sheet, 48 lines in a page, and 45 letters in a line?—*Heath.*

By Rule I.

4*com.* :	16 *days*	::	9*com* †
12*hrs.* :	———	::	10*hrs.* †
†14*sh.* :	———	::	30*sh.*
†24*pa.* :	———	::	16*pa.*
†44*lin.* :	———	::	48*lin.*
†40*let.* :	———	::	45*let.*

$$\frac{\overset{3}{[illegible]}\times 16\times \overset{3}{\cancel{30}}\times \overset{\cancel{4}}{\cancel{16}}\times \overset{2}{\cancel{48}}\times \overset{\cancel{5}}{\cancel{45}}}{[illegible]\times \underset{7}{\cancel{14}}\times \cancel{24}\times \underset{11}{\cancel{44}}\times \underset{\cancel{8}}{\cancel{40}}} = \frac{4\times3\times16\times3\times2}{11\times7} = \frac{1152}{77} =$$

$14\frac{74}{77}$ days. *Answer.*

By Rule II.

	Producing cause.			compose.			*Produced effect.*	
1 *line.*	If 4c. :	16d. :	12h		14s. :	24p. ::	44l. :	40l.
2 *line.*	9c. :	*d. :	10h.		30s. :	16p. :	48l. :	45l.
				compose.				

$4 \times 16 \times 12 \times 30 \times 16 \times 48 \times 45 = 796262400$ dividend.

$9 \times 10 \times 14 \times 24 \times 44 \times 40 = 53222400$ divisor.

Then 796262400, divided by 53222400, gives $14\frac{511488}{532224}$ days, $= 14\frac{74}{77}$ days, *Answer.*

(14.) If 24 measures of wine, at 3*s.* 4*d.* each, serve 16 men for 6 days, how many measures, at 2*s.* 8*d.* each, will serve 48 men for 4 days?

(15.) If a garrison of 3600 men, in 35 days, at 24oz. per day each man, eat a certain quantity of bread, how many men, in 45 days, at the rate of 14oz. per day each man, will eat double the quantity?

(16.) A garrison of 3600 men has just bread enough to allow 24oz. a day to each man for 35 days; but, a siege coming on, the garrison was reinforced to the number of 4800 men: how many ounces of bread a day must each man be allowed, to hold out 45 days against the siege of the enemy?

(17.) If the carriage of 150 feet of wood, that weighs 3 stone a foot, comes to 3*l.* for 40 miles, how much will the carriage of 54 feet of free-stone, that weighs 8 stone a foot, cost for 25 miles?

(18.) If, when wine is 30*l.* per tun, 20*l.* worth will serve a ship's company of 336 men for 4 days, at a pint a day for each man—how long will 500*l.* worth serve a crew of 250 men, at $1\frac{1}{2}$ pint a day to each man, when the tun is worth but 24*l.*?

(19.) If 336 men, in 5 days of 10 hours each, dig a trench of 5 degrees of hardness, 70 yards long, 3 wide, and 2 deep, what length of trench, of 6 degrees of hardness, 5 yards wide, and 3 deep, may be dug by 240 men in 9 days of 12 hours each?

(20.) If 12 pieces of cannon, eighteen-pounders, [illegible] batter down a castle in an hour, in what time wou[illegible] twenty-four pounders batter down the same ca[illegible]th pieces of cannon being fired the same number of times, and their balls flying with the same degree of velocity?

(21.) If 12 oxen will eat $3\frac{1}{3}$ acres of grass in 4 weeks, and 21 oxen will eat 10 acres in 9 weeks, how many oxen will eat 24 acres in 18 weeks, the grass being allowed to grow uniformly?—*Newton.*

(22.) If 6 oxen *or* 10 colts can eat up 21 acres of pasture in 14 weeks, and 10 oxen *and* 6 colts can eat up 45 acres of a similar pasture in 20 weeks, *the grass growing uniformly*; how many sheep will eat up 240 acres in 40 weeks, admitting that 1134 sheep can eat the same quantity as 12 oxen *and* 22 colts?

VULGAR FRACTIONS.

DEFINITIONS.

1. *A Fraction* is a part, or a collection of several parts, of an unit, or of any whole quantity expressed by an unit.

A fraction is represented by two numbers placed one above the other, with a line drawn between them, as $\frac{3}{4}$, three-fourths of an unit, or one-fourth of three units; $\frac{5}{8}$, five-eighth of an unit, or one-eighth of five units, &c.—The lower number is called the *denominator*, and shews how many parts the unit is divided into; the upper is called the *numerator*, and shews how many of these parts are to be taken. Thus, $\frac{1}{4}$, one-fourth, shews that an unit is to be divided into four equal parts, and *one* of these parts are to be taken; $\frac{3}{4}$, three-fourths, shews that an unit is to be divided into four equal parts, and *three* of these parts are to be taken, or, which is the same thing, that the number 3 is to be divided into four equal parts, and one of these parts are to be taken.—Hence it appears that every fraction denotes a division of its numerator by its denominator, and that its value is equal to the quotient obtained by such a division.

2. *A proper fraction* is that wherein the numerator is less than the denominator, as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, &c. Hence the value of such a fraction is less than an unit.

3. *An improper fraction* is that wherein the numerator is greater than the denominator, or equal to the denomi-

nator. When the numerator of a fraction is greater than the denominator, its value is greater than an unit; if the numerator be equal to the denominator, its value is equal to an unit.

4. *A single, or simple fraction,* consists of but *one* numerator and one denominator, as $\frac{5}{6}$.

5. *A compound fraction,* or fraction of a fraction, consists of two, or more, fractions connected by the word *of*, as $\frac{1}{2}$ of $\frac{3}{4}$; $\frac{3}{4}$ of $\frac{5}{8}$ of $3\frac{1}{5}$, &c.

6. *A mixed number* is a whole number with a fraction annexed, as $17\frac{1}{2}$, $14\frac{5}{9}$, &c.

7. *A complex fraction* is a fraction having a fraction or a mixed number for its numerator or denominator, or both, as $\frac{\frac{3}{4}}{14}$, $\frac{6}{\frac{1}{5}}$, $\frac{2}{47\frac{1}{2}}$, or $\frac{\frac{3}{5}}{\frac{4}{7}}$, $\frac{42\frac{5}{8}}{87\frac{4}{5}}$, &c.

8. *The common measure* of a fraction is a number which will divide both the numerator and denominator without a remainder.

9. *Terms* of a fraction are the numerator and denominator; the numerator being the upper term, and the denominator the lower.

REDUCTION OF VULGAR FRACTIONS.

Definition.—The method of changing fractions from one form to another, without altering their value, is called *reduction:* The rules of reduction serve to prepare the fractions for addition, subtraction, &c.

Proposition 1. *To find the greatest common measure of a fraction, or of two or more numbers.*

RULE.

I. Divide the greater term by the less, and this divisor by the remainder, continually, till there is no remainder; then the last divisor will be the greatest common measure of both terms of the fraction, or of any two numbers whatever.

II. If there be more numbers than two, find the greatest common measure of two of them as above; then find the greatest common measure of the third number, and the preceding common measure, and so on all through the numbers to the last. The greatest common measure last found will be the answer.

III. If an unit be the greatest common measure of two or more numbers, these numbers are prime to each other.

Prop. 2. To abbreviate, or reduce, fractions to their lowest terms.

RULE.

Divide the terms of the given fraction by any number that will divide them without a remainder, and these quotients again in the same manner; and so on till no number greater than one will divide them. Or, divide both the terms of the fraction by their greatest common measure.

Note 1. Any number, ending with an even number, or a cipher, will divide by 2, and leave no remainder.

2. Any number ending with 5 or 0 is divisible by 5.

3. If any fraction has ciphers at the right hand of its terms, it may be abbreviated by cutting off the ciphers, as $\frac{40}{30}=\frac{4}{3}$.

4. If any number ending with 1, 3, 7, or 9, be the numerator or denominator of a fraction, and will not divide by 3, 7, or 9, that fraction is *generally* in its lowest terms. The 9th note in simple division will be found useful here.

Prop. 3. To reduce a whole number to an equivalent fraction of a given denominator.

RULE.

Multiply the whole number by the given denominator, and the product will be the numerator required.

Note. Any whole number may be expressed like a fraction by writing 1 under it for a denominator. Thus, $5=\frac{5}{1}$.

Prop. 4. To reduce a mixed number to its equivalent improper fraction.

RULE.

Multiply the whole number by the denominator of the fraction, and to the product add the numerator; this sum, written above the denominator, will form the fraction required.

Prop. 5. *To reduce an improper fraction to its equivalent whole or mixed number.*

RULE.

Divide the numerator by the denominator, and the quotient will be the whole or mixed number required.

Prop. 6. *To reduce a complex fraction to a simple one.*

RULE.

If the numerator or denominator be whole or mixed numbers, reduce them to improper fractions. Then multiply the denominator of the lower fraction into the numerator of the upper for the new numerator, and the denominator of the upper fraction into the numerator of the lower for a new denominator.

Thus, by the preceding rules,

$$\frac{4}{\frac{7}{8}} = \frac{\frac{4}{1}}{\frac{7}{8}}; \quad \frac{\frac{7}{10}}{5} = \frac{\frac{7}{10}}{\frac{5}{1}}; \quad \frac{4\frac{7}{8}}{9} = \frac{\frac{39}{8}}{\frac{9}{1}}; \quad \frac{7}{5\frac{1}{9}} = \frac{\frac{7}{1}}{\frac{46}{9}}; \quad \frac{7\frac{1}{5}}{4\frac{3}{8}} = \frac{\frac{36}{5}}{\frac{35}{8}};$$

and these are all the varieties that can possibly happen in preparing the fraction.

Prop. 7. *To reduce a compound fraction to a simple one.*

RULE.

If any of the proposed quantities be integers, mixed numbers, or complex fractions, reduce them to their proper terms. Then multiply all the numerators together for a new numerator, and all the denominators for a new denominator. Reduce this new fraction to its lowest terms.

Note. If you place the several numerators in a line, with the sign of multiplication between them; and the denominators underneath

them, in a similar manner: you may strike out such figures as are common to both the numerator and the denominator, or divide any two of them by their greatest common divisor. For it is an universal *axiom* in fractions, THAT if you multiply or divide both the numerator and denominator of a fraction by the same number, its value is not altered.

Prop. 8. *To find the proper quantity, or value, of a fraction in the known parts of an integer.*

RULE.

Multiply the numerator by the number of parts of the next inferior denomination, which makes one of the denomination of your fraction, and divide the product by the denominator, the quotient will be the value of the fraction. If there be a remainder, multiply it by the next inferior denomination, and divide by the denominator as before: proceed thus till you come to the lowest denomination.

Prop. 9. *To reduce coins, weights, measures, &c. into fractions.*

RULE.

Reduce the coin, weight, measure, &c. into the lowest name mentioned, for a numerator, under which set the number of parts contained in an unit of the integer, to which the proposed fraction is to be reduced for a denominator. Reduce the fraction to its lowest terms.

Note. This rule is exactly the reverse of the preceding.

Prop. 10. *To reduce a fraction of one denomination to the fraction of another denomination of equal value.*

RULE.

From a less to a greater denomination. Multiply the *denominator* by all the denominations, from that given to that sought: and, from a greater to a less denomination, multiply the *numerator* by all the denominations, from the denomination given to that sought.

Prop. 11. *To find the least common multiple of two or more numbers.*

RULE I.

If one or more of the given numbers be multiples of any of the others, reject those numbers of which they are the multiples. Then,

Arrange the remaining numbers in one line, and divide each of them, or the *greatest number* of them, by any number that will divide them without a remainder: set the quotients (together with the undivided numbers) in a line underneath; divide this second line as before, and so on till there are no *two* numbers that can be divided. The products of the divisors, quotients, and undivided numbers, will give the least common multiple required.

RULE II.

Find the common measure of *two* of the numbers, and divide their product by that common measure; multiply the quotient by the third number, and divide the product by the common measure of the multiplier and multiplicand. Again, multiply the last quotient by the fourth number, and divide the product by the common measure of the factors as before, and so on till the last number: the last quotient will be the least common multiple.

Prop. 12. *To reduce fractions of different denominators to others of equal value, having a common denominator.*

GENERAL RULES.

When any of the proposed quantities are integers, mixed numbers, complex or compound fractions, they must be reduced to their proper terms by the preceding rules. Then,

Rule I. Multiply each numerator into all the denominators, except its own, for a new numerator, and all the denominators together for a common denominator.

OR II. 1. Multiply all the denominators of the given fraction together for a common denominator.

2. Divide the common denominator by each of the given denominators separately, and multiply the quotients by their several numerators, the products will be the new numerators.

Or III. Find the least common multiple of all the denominators of the given fractions, and it will be the least common denominator.

2. Divide this common denominator by each of the given denominators separately, and multiply the quotients by their several numerators, the products will be the new numerators.

By the *axiom* in the note to Prop. 7th, several fractions of different denominators may easily be reduced to a commmon denominator. Thus $\frac{5}{8}$ may be reduced to the same denominator as $\frac{3}{16}$, by multiplying its terms by 2, by which it becomes $\frac{10}{16}$. Also, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{16}$, may be reduced to a common denominator by multiplying the terms of the first fraction by 4, the second by 2; and dividing those of the third by 2; thus, $\frac{1}{2}=\frac{4}{8}$, $\frac{3}{4}=\frac{6}{8}$, and $\frac{2}{16}=\frac{1}{8}$.

2. If the less denominator of two fractions divide the greater, multiply the terms of that which has the less denominator by the quotient. Let $\frac{5}{8}$ and $\frac{9}{16}$ be proposed: here $16 \div 8 = 2$, and $\frac{5}{8} \times \frac{2}{2} = \frac{10}{16}$. And hence the greater of two given fractions may easily be discovered; for, if we multiply each numerator into the other's denominator, the products will be equal when the fractions are equal; otherwise that fraction is the greater which produces the greater product by its numerator; thus, $\frac{5}{6}=\frac{10}{12}$, for $5 \times 12 = 10 \times 6$, but $\frac{5}{6}$ is greater than $\frac{7}{10}$, for $5 \times 10 = 50$, but $7 \times 6 = 42$ only.

3. When several fractions are proposed to be reduced to a common denominator, first reduce two of them to a common denominator by some of the preceding methods, and then these and a third, &c.

4. If any number of simple fractions be reduced to a common denominator, the several numerators of the new fractions will have the same ratio to each other as the original fractions: and, if these new numerators be divided by their greatest common measure, the quotients will be the least whole numbers in the same ratio. This note will be found exceedingly useful in solving all fractional questions where proportion is concerned.

Examples to Proposition 1.

(1.) Find the greatest common measure to $\frac{216}{408}$. *Or, in other words, find the greatest number that will divide* 216 *and* 408 *without a remainder. Likewise find the greatest number that will divide* 216, 408, *and* 740, *without a remainder.*

```
216)408(1                    24)740(30
    216                          72
    ---                          --
    192)216(1                    20)24(1
        192                         20
        ---                         --
         24)192(8                    4)20(5
            ---                        20
                                       --
```

Answer. The greatest common measure to 216 and 408 is 24; but the greatest common measure to 216, 408, and 740, is 4.

(2.) Find the greatest common measure to $\frac{342}{843}$.
(3.) Find the greatest common measure to $\frac{36}{42}$.
(4.) Find the greatest common measure to $\frac{243}{450}$.
(5.) Find the greatest common measure to $\frac{375}{4500}$.

Examples to Prop. 2.

(6.) Reduce $\frac{216}{408}$ to its lowest terms.

$\overset{\div 3}{\frac{216}{408}} = \overset{\div 4}{\frac{72}{136}} = \overset{\div 2}{\frac{18}{34}} = \frac{9}{17}$ answer.—Or the common measure, (by example 1.) is 24: hence $24)\frac{216}{408} = \frac{9}{17}$ as before.

(7.) Reduce $\frac{374}{1030}$ to its lowest terms.
(8.) Reduce $\frac{410}{510}$ to its lowest terms.
(9.) Reduce $\frac{345}{1745}$ to its lowest terms.
(10.) Reduce $\frac{8343}{9747}$ to its lowest terms.
(11.) Reduce $\frac{549}{7143}$ to its lowest terms.

Examples to Prop. 3.

(12.) Reduce 14 to an improper fraction, having 9 for its denominator.

$14 \times 9 = 126$ numerator: hence $14 = \frac{126}{9}$ the fraction required.

(13.) Reduce 15 to an improper fraction, having 26 for its denominator.

(14.) Reduce 34 to an improper fraction, having 91 for its denominator.

Examples to Prop. 4.

(15.) Reduce $25\frac{3}{8}$ to its equivalent improper fraction.

$25\frac{3}{8}$
8 denominator of the fraction.

203 new numerator. Then $25\frac{3}{8} = \frac{208}{8}$.

(16.) Reduce $149\frac{5}{9}$ to an improper fraction.
(17.) Reduce $375\frac{24}{99}$ to an improper fraction.
(18.) Reduce $17494\frac{543}{99999}$ to an improper fraction.
(19.) Reduce $4734\frac{57}{95}$ to an improper fraction.
(20.) Reduce $1789\frac{5}{9}$ to an improper fraction.
(21.) Place 4 sevens in such a manner that they may be equal to 78.

Examples to Prop. 5.

(22.) Reduce $\frac{375}{13}$ to its equivalent whole or mixed number.

Every fraction denotes a division of its numerator by the denominator, therefore 375 divided by $13 = 28\frac{11}{13}$, answer.

(23.) Reduce $\frac{4790}{25}$ to a whole or mixed number.
(24.) Reduce $\frac{15112}{108}$ to a whole or mixed number.
(25.) Reduce $\frac{375941}{999}$ to a whole or mixed number.
(26.) Reduce $\frac{3745174}{349}$ to a whole or mixed number.

Examples to Prop. 6.

(27.) Reduce $\dfrac{47\frac{5}{8}}{94}$ to a simple fraction.

$$\frac{47\frac{5}{8}}{94} = \frac{\frac{381}{8}}{\frac{94}{1}} = \frac{1 \times 381}{8 \times 94} = \frac{381}{752}$$ answer. (See the note to Prop. 6.)

(28.) Reduce $\dfrac{34\frac{5}{7}}{84}$ to a simple fraction.

(29.) Reduce $\dfrac{44}{147\frac{5}{9}}$ to a simple fraction.

(30.) Reduce $\dfrac{247}{\frac{5}{7}}$ to a simple fraction.

(31.) Reduce $\dfrac{\frac{147}{314}}{1789}$ to a simple fraction.

(32.) Reduce $\dfrac{394\frac{74}{99}}{894\frac{547}{719}}$ to a simple fraction.

Examples to Prop. 7.

(33.) Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $5\frac{3}{8}$ of $\dfrac{7\frac{5}{9}}{54}$ of 3 to a single fraction.

First, $5\frac{3}{8} = \frac{43}{8}$; $\dfrac{7\frac{5}{9}}{54} = \frac{68}{486}$; and $3 = \frac{3}{1}$.

Then, $\dfrac{1\times2\times43\times68\times3}{2\times3\times8\times486\times1} = \dfrac{17544}{23328} = \dfrac{731}{972}$ answer.

Or, rather,

$$\frac{1}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{3}}\times\frac{43}{\underset{2}{\cancel{8}}}\times\frac{\overset{17}{\cancel{68}}}{486}\times\frac{\cancel{3}}{1}=\frac{43\times17}{486\times2}=\frac{731}{972}$$ as before. (*See the note to* Prop. 7.)

(34.) Reduce $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{5}{9}$ of $\frac{27}{101}$ of $\frac{3}{13}$ to a single fraction.

(35.) Reduce $\frac{41}{110}$ of $\frac{3}{19}$ of $\frac{41}{108}$ of $\frac{3}{7}$ to a single fraction.

(36.) Reduce $3\frac{5}{8}$ of $\frac{5}{9}$ of $\frac{27}{301}$ of 49 to a single fraction.

(37.) Reduce $3\frac{4}{7}$ of $\frac{47}{95}$ of $\dfrac{34}{15\frac{1}{9}}$ of 108 to a single fraction.

(38.) Reduce $\frac{34}{39}$ of $\frac{1}{13}$ of $\dfrac{51\frac{3}{8}}{37\frac{3}{5}}$ of 34 to a single fraction.

Examples to Prop. 8.

(39.) Required the value of $\frac{5}{8}$ of a £.; or, which is the same thing, $\frac{1}{8}$ of £5.

```
      5
     20
    ———
 8)100(12s.
    ———
      4
     12
    ———
  8)48(6d.
    ———
```

Answ. 12s. 6d.

(40.) Required the value of $\frac{5}{8}$ of a cwt.; or, which is the ame thing, $\frac{1}{8}$ of 5cwt.

```
      5
      4
     ——
  8)20(2qr.
     ——
      4
     28
    ———
 8)112(14lb.
    ———
```

Answ. 2qr. 14lb.

(41.) What is the value of $\frac{5}{9}$ of a shilling?

(42.) Reduce $\frac{4}{5}$ of a lb. Avoirdupois to its proper quantity.

(43.) What is the value of $\frac{1}{2}$ of $\frac{6}{7}$ of a lb. Troy?

(44.) Reduce $\frac{3}{5}$ of a league to its proper quantity.

(45.) Reduce $\frac{5}{8}$ of $\frac{3}{4}$ of an acre to its proper quantity.

(46.) What is the value of $\frac{5}{6}$ of 15 yards of cloth?

(47.) What is the value of $\frac{7}{9}$ of a tun of wine?

(48.) What is the value of $\frac{3}{11}$ of a butt of beer?

(49.) What is the value of $\frac{7}{18}$ of a year?

(50.) What is the value of $\frac{5}{9}$ of a chaldron of coals?

(51.) What is the value of $\frac{2}{3}$ of 13*s*. 4*d*.?

(52.) What is the value of $\frac{3}{7}$ of 15cwt. 3qr. 14lb.?

(53.) What is the value of $\frac{3}{8}$ of a *solid* yard?

(54.) What quantity of ale is contained in $\frac{5}{9}$ of 15228 cubic inches?

Examples to Prop. 9.

(55.) Reduce 7*s*. 6$\frac{3}{4}$*d*. to the fraction of a pound.

```
 7s. 6¾d.                     20s.
 12                           12
 ———                          ———
 90                           240
  4                             4
 ———                          ———
 363 farth. numerator.        960 farth. denominator.
```

Hence $\frac{363}{960} = \frac{121}{320}$£. the fraction required.

(56.) Reduce 15*s*. 11*d*. to the fraction of a pound.

(57.) Reduce $5\frac{2}{9}$*d*. to the fraction of a shilling.

(58.) Reduce 1cwt. 2qr. 6lb. 3oz. $8\frac{8}{9}$dr. to the fraction of a cwt.

(59.) Reduce 5oz. $3\frac{1}{3}$dr. to the fraction of a lb. Troy.

(60.) Reduce 3qr. $3\frac{1}{9}$n. to the fraction of an English ell.

(61.) Reduce 147 days 15hrs. to the fraction of a year.

(62.) What part of a pound is 15*s*. $9\frac{1}{2}$*d*.?

(63.) What part of a groat is $\frac{2}{3}$ of three half-pence?

(64.) What part of 10cwt. 1qr. 12lb. is 8cwt. 1qr. 25lb. 1oz. $7\frac{3}{11}$drs.?

(65.) Reduce 4bush. $2\frac{2}{7}$ pecks of corn to the fraction of a quarter.

(66.) Reduce 1qr. 3n. to the fraction of a yard.

(67.) Reduce 2 roods 15per. to the fraction of an acre.

Examples to Prop. 10.

(68.) Reduce $\frac{2880}{7}$ of a farthing to the fraction of a pound.

Here a *small* name is brought into a *great*,

Therefore $\frac{2880}{7} \times \frac{1}{4} \times \frac{1}{12} \times \frac{1}{20} = \frac{2880}{6720}$ of a £. $= \frac{3}{7}$ of a £.

(69.) Reduce $\frac{3}{7}$ of a pound to the fraction of a farthing.

Here a *great* name is to be brought into a *small*,

Hence $\frac{3}{7} \times \frac{20}{1} \times \frac{12}{1} \times \frac{4}{1} = \frac{2880}{7}$ of a farthing.

(70.) Reduce $\frac{5}{7}$ of a penny to the fraction of a pound.

(71.) Reduce $\frac{7}{960}$ of a pound to the fraction of a penny.

(72.) What part of lb. Troy is $\frac{5}{6}$ of a dwt.?

(73.) Reduce $\frac{1}{360}$ of a lb. Troy to the fraction of a dwt.

(74.) What part of a cwt. is $\frac{4}{5}$ of a lb. Avoirdupois?

(75.) Reduce $\frac{1}{196}$ of a owt. to the fraction of a lb.

(76.) Reduce $\frac{7}{71}$ of a week to the fraction of a second.

(77.) What part of a hhd. of wine is $\frac{11}{13}$ of a gallon.

Examples to Prop. 11.

(78.) Find the least number that can be divided by 4, 7, 12, 21, and 34, without a remainder.

By Rule I.

Here 4 and 7 may be rejected, because 12 and 21 are multiples of them.

$$\begin{array}{r|rrr} 3 & 12 & 21 & 34 \\ \hline 2 & 4 & 7 & 34 \\ \hline & 2 & 7 & 17 \end{array}$$

$3 \times 2 \times 2 \times 7 \times 17 = 1428$ answer.

By Rule II.

The common measure to 4 and 7 is 1; hence $\frac{4 \times 7}{1} = 28$; $\frac{28 \times 12}{4} = 84$; $\frac{84 \times 21}{21} = 84$; $\frac{84 \times 34}{2} = 1428$ *as before.*

Note. 4 is the common measure to 28 and 12; 21 is the common measure to 84 and 21; and 2 is the common measure to 84 and 34.

(79.) What is the least number that can be divided by 4, 6, and 10, without a remainder?

(80.) Find the least number that can be divided by 2, 3, 4, 5, 6, and 7, without a remainder.

(81.) Find the least common multiple of 3, 4, 8, and 12.

(82.) Find the least number that can be divided by 1, 2, 3, 4, 5, 6, 7, 8, and 9, without a remainder.

(83.) Find the least number that can be divided by 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, and 20, without a remainder.

Examples to Prop. 12.

(84.) Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, and $\frac{1}{6}$, to a common denominator.

By Rule I.

$$\left.\begin{array}{l} 1\times3\times4\times5\times6 = 360 \\ 2\times2\times4\times5\times6 = 480 \\ 3\times3\times2\times5\times6 = 540 \\ 2\times4\times3\times2\times6 = 288 \\ 1\times5\times4\times3\times2 = 120 \end{array}\right\}\text{ new numerators.}$$

$2\times3\times4\times5\times6 = 720$ common denominator.

Hence $\frac{1}{2}=\frac{360}{720}$; $\frac{2}{3}=\frac{480}{720}$; $\frac{3}{4}=\frac{540}{720}$; $\frac{2}{5}=\frac{288}{720}$; $\frac{1}{6}=\frac{120}{720}$.

By Rule II.

$2\times3\times4\times5\times6 = 720$ common denominator.

	2)720	3)720	4)720	5)720	6)720
	360	240	180	144	120
	1	2	3	2	1
Num.	360	480	540	288	120

Hence the new fractions are $\frac{360}{720}$, $\frac{480}{720}$, $\frac{540}{720}$, $\frac{288}{720}$, and $\frac{120}{720}$, as above.

By Rule III.

$$\begin{array}{l|l} 2 & 4..5..6 \text{ denominators, rejecting 2 and 3.} \\ & 2..5..3 \end{array}$$

Then $2\times2\times5\times3 = 60$ the least common denominator.

	2)60	3)60	4)60	5)60	6)60
	30	20	15	12	10
	1	2	3	2	1
Num.	30	40	45	24	10

Hence $\frac{1}{2}=\frac{30}{60}$, $\frac{2}{3}=\frac{40}{60}$, $\frac{3}{4}=\frac{45}{60}$, $\frac{2}{5}=\frac{24}{60}$, $\frac{1}{6}=\frac{10}{60}$, fractions of the same value as above, only in their lowest terms

(85.) Reduce $\frac{1}{3}$, $\frac{4}{9}$, $\frac{5}{8}$, and $\frac{11}{16}$, to a common denominator.

(86.) Reduce $\frac{3}{7}$ of $\frac{2}{3}$ of $\frac{5}{8}$ and $\frac{3}{4}$ of $_{7}$ of $\frac{3}{5}$ to a common denominator.

(87.) Reduce $5\frac{1}{3}$, $8\frac{3}{7}$, $4\frac{1}{9}$, and $6\frac{5}{8}$, to improper fractions, having a common denominator.

(88.) Reduce $\frac{\frac{4}{7}}{6}$, $\frac{3\frac{1}{5}}{24}$, $\frac{9}{3\frac{1}{8}}$, and $\frac{5\frac{1}{3}}{3\frac{7}{8}}$, to simple fractions, having a common denominator.

(89.) Reduce $\frac{3}{5}$, $\frac{2}{3}$, $\frac{1}{7}$, and $\frac{1}{2}$, to a common denominator.

(90.) Reduce $\frac{4}{7}$, $\frac{5}{9}$, $\frac{3}{8}$, $\frac{7}{8}$, and 19, to a common denominator.

ADDITION OF VULGAR FRACTIONS.

RULE.

Reduce mixed numbers to improper fractions; complex and compound fractions to simple ones; and fractions of different denominators to a common denominator. Then the sum of the numerators written, over the common denominator, will be the sum of the fractions required.

Note 1. If the fractions are of different denominations, reduce them to their proper quantities, (by Prop. 8th in Reduction, or reduce them to the same denomination by Prop. 10th) and then add them together.

2. When several fractions are to be added together, it is commonly the best to add those two together which may most easily be reduced to a common denominator, then their sum, and a third, &c.

3. When several mixed numbers, as $4\frac{1}{4}$, &c. are to be collected into one sum, first add the fractions to the fractions, and, to the left-hand of the sum, join the sum of the whole numbers.

Examples.

(1.) Add $3\frac{5}{7}$, $4\frac{5}{8}$, and $\frac{5}{11}$ together.

First, $3\frac{5}{7}=\frac{26}{7}$, $4\frac{5}{8}=\frac{37}{8}$. Then the fractions become $\frac{26}{7}$, $\frac{37}{8}$, and $\frac{5}{11}$.

$$\left.\begin{array}{r} 26\times8\times11=2288 \\ 37\times7\times11=2849 \\ 5\times8\times7=280 \end{array}\right\}\text{ numerators.}$$

$$\frac{5417}{7\times8\times11=616}=8\tfrac{489}{616}\text{ answer.}$$

Or thus,

The sum of $\frac{5}{7}$, $\frac{5}{8}$, and $\frac{5}{11}$, when reduced to a common denominator, is, $\frac{1105}{616}=1\frac{489}{616}$. Then $3+4+1\frac{489}{616}=8\frac{489}{616}$, as before. (See the third note.)

(2.) Add $\frac{2}{5}$, $\frac{5}{8}$, and $\frac{3}{7}$, together.
(3.) Add $\frac{1}{5}$ of $\frac{3}{11}$, and $5\frac{3}{8}$, together.

(4.) Add $\frac{1}{9}$, $7\frac{5}{8}$, $\dfrac{45}{94\frac{7}{11}}$, and $\dfrac{47\frac{5}{9}}{314\frac{3}{5}}$, together.

(5.) Add $\frac{1}{9}$ of $\frac{3}{5}$, $\frac{3}{4}$ of 10, and $\frac{5}{8}$ of 12, together.
(6.) Add $\frac{3}{5}$ and $\frac{9}{10}$ of $\frac{5}{11}$ of $15\frac{3}{8}$ together.
(7.) Add $\frac{5}{9}$ of a pound, $\frac{3}{8}$ of a shilling, and $\frac{5}{7}$ of a penny, together.
(8.) What is the sum of $\frac{3}{4}$ of 1*l.* 10*s.* $\frac{1}{4}$ of 3*l.* 10*s.* and $\frac{1}{10}$ of a hundred guineas?
(9.) Add $\frac{4}{5}$ of a lb. troy to $\frac{1}{8}$ of an ounce.
(10.) Add $\frac{4}{9}$ of a ton to $\frac{5}{12}$ of a cwt.
(11.) Add $\frac{5}{6}$ of 3 ells English to $\frac{5}{12}$ of a yard.
(12.) Add $\frac{2}{5}$ of a yard, $\frac{3}{7}$ of a foot, and $\frac{5}{11}$ of a mile, together.
(13.) Add $\frac{5}{8}$ of an acre, $\frac{3}{5}$ of nineteen square feet, and $\frac{3}{7}$ of a square inch, together.
(14.) What is the sum of $\frac{3}{4}$ of a tun of wine, and $\frac{3}{5}$ of a hhd.?
(15.) Add $\frac{5}{9}$ of a chaldron to $\frac{3}{7}$ of a bushel.
(16.) Add $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, and $\frac{1}{5}$ of an hour, together.
(17.) Add $\frac{1}{5}$ of $\frac{3}{4}$ of a year, $\frac{3}{8}$ of $\frac{5}{9}$ of a day, and $\frac{7}{8}$ of $\frac{3}{5}$ of $19\frac{1}{2}$ hours, together.

SUBTRACTION OF VULGAR FRACTIONS.

RULE.

Reduce mixed numbers to improper fractions; complex and compound fractions to simple ones; and fractions of different denominators to a common denominator. Then the difference of the numerators, written above the common denominator, will give the difference of the fractions required.

Note 1. If the fractions are of different denominations, reduce them to their proper quantities, &c. as in addition, and then take their difference.

2. In subtracting mixed numbers, when the lower fraction is greater than the upper, subtract the numerator of the lower fraction from the denominator of the upper, and to their difference add the numerator of the upper fraction, carrying one to the unit's place of the lower whole number.

3. When a fraction is to be subtracted from an unit, subtract the numerator from the denominator; the remainder will be the numerator to be placed over the denominator.

4. When a proper fraction is to be subtracted from any whole number, subtract the numerator from the denominator for the numerator of the *remainder*, which must be annexed to the whole number, made less by 1.

Examples.

(1.) From $\frac{3}{4}$ take $\frac{5}{11}$.

$$\left.\begin{array}{r} 3\times 11 = 33 \\ 5\times 4 = 20 \end{array}\right\} \text{numerators.}$$

$$\begin{array}{r} \overline{13} \\ \text{— difference} \\ 4\times 11 = 44 \end{array} \qquad \text{Answer } \frac{13}{44}$$

(2.) What is the difference between $\frac{3}{5}$ and $\frac{9}{16}$?

(3.) What is the difference between $3\frac{5}{8}$ and $\frac{2}{3}$ of $\frac{5}{8}$?

(4.) What is the difference between $\dfrac{49\frac{5}{8}}{97}$ and $\dfrac{34\frac{2}{5}}{145\frac{3}{11}}$?

(5.) From $115\frac{5}{8}$ take $39\frac{7}{8}$.

(6.) Subtract $\frac{54}{713}$ from an unit.

(7.) Subtract $\frac{11}{13}$ from 365.

(8.) What is the difference between $\frac{2}{3}$ of 15 and $\frac{4}{5}$ of 72?

(9.) To what fraction must I add $\frac{3}{5}$ that the sum may be $\frac{5}{8}$?

(10.) What number is that to which if $7\frac{2}{3}$ be added the sum will be $17\frac{3}{5}$?

(11.) What number is that from which if you subtract $\frac{3}{11}$ of $\frac{5}{6}$ of an unit, and to the remainder add $\frac{3}{5}$ of $\frac{7}{8}$ of an unit, the sum will be 9?

(12.) What is the difference between $\frac{3}{4}$ of a £ and $\frac{5}{7}$ of a shilling?

(13.) From $\frac{5}{6}$ of a lb. troy take $\frac{5}{8}$ of an ounce.

(14.) From $\frac{3}{8}$ of ton take $\frac{2}{3}$ of $\frac{3}{4}$ of a lb.

(15.) From $\frac{2}{3}$ of $\frac{3}{5}$ of a hhd. of wine take $\frac{3}{5}$ of $\frac{2}{4}$ of a pint.

(16.) From $\frac{3}{5}$ of a league take $\frac{5}{8}$ of a mile.

(17.) From $\frac{5}{9}$ of $365\frac{1}{4}$ days take $\frac{3}{7}$ of $\frac{9}{10}$ of an hour.

(18.) A pound avoirdupois is equal to 14oz. 11dwts. 16 grains troy; what is the difference (in troy-weight) between the ounce avoirdupois and the ounce troy * ?

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.

Reduce mixed numbers to improper fractions, and complex fractions to simple ones. Then multiply all the numerators together for a new numerator, and all the denominators together for a common denominator.

Note 1. The work may be abbreviated by striking out such multipliers as are found both in the numerators and denominators.

2. To multiply a fraction by an integer, divide the denominator of the fraction by the integer, (if possible;) but, if that cannot be done, multiply the numerator of the fraction by it.

3. If a proper fraction be multiplied by a proper fraction, the product will be less than either the multiplier or multiplicand. And, if any number, either whole or mixed, be multiplied by a proper fraction, the product will always be less than the multiplicand. It seems rather paradoxical that the name *multiplication* should be applied to a work which really diminishes; when the word, strictly speaking, signifies the increasing of a number by repetition. But this apparent paradox will vanish, when we reflect that the *multiplication of a fraction* must necessarily increase the number of the parts into which the *whole thing* is divided, and consequently the value of each of these parts will be diminished.

Examples.

(1.) Multiply $3\frac{5}{8}$, $\dfrac{3\frac{7}{8}}{11}$, and $\frac{3}{5}$ of $\frac{9}{10}$, together.

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* Troy-weight has its name from *Troyes*, a town in the province of Champagne in France, now in the department of Aube, and was introduced into England by William the Conqueror. The English were dissatisfied with this weight, because the pound did not weigh so much as the pound in use at that time in England. Hence arose the term *Avoir du Poids*, which was a medium between the French and ancient English weights.

First $3\frac{5}{8} = \frac{29}{8}$, $\dfrac{3\frac{7}{8}}{11} = \dfrac{31}{88}$.

Then $\frac{29}{8} \times \frac{31}{88} \times \frac{3}{5} \times \frac{9}{10} = \frac{24273}{35200}$, product.

(2.) Required the product of $\frac{3}{7}$ and $\frac{11}{15}$.

(3.) What is the product of 574 by $\frac{37}{84}$?

(4.) Required the product of $\frac{375}{729}$ by 27.

(5.) Required the product of $7\frac{2}{3}$ by 25.

(6.) What is the product of $\frac{5}{8}$ of $\frac{3}{7}$, $\frac{2}{3}$ of $15\frac{1}{2}$, and $\frac{3}{11}$ of 2?

(7.) What is the continued product of $\dfrac{4}{5\frac{1}{3}}$, $\dfrac{7\frac{1}{3}}{15}$, and $\dfrac{85\frac{3}{8}}{129\frac{4}{11}}$?

(8.) What is the product of $\frac{3}{5}$ of $\frac{7}{11}$ of 15, and $\frac{14}{15}$ of $11\frac{5}{8}$?

(9.) Multiply 7ft. 9in. by 3ft. 11in. and that product by 5ft. 3in.

(10.) If a board be 12ft. 9in. long, and 5ft. 7in. broad, how many square feet does it contain?

(11.) If a closet be 17ft. $9\frac{1}{3}$in. round, and 9ft. 9in. high, how many square feet does it contain?

DIVISION OF VULGAR FRACTIONS.

RULE.

Reduce mixed numbers to improper fractions, complex and compound fractions to simple ones. Then invert the divisor, and proceed exactly as in multiplication.

Note 1. When it can be done, divide the numerator of the dividend by the numerator of the divisor, and the denominator by the denominator for the quotient.

2. To divide a fraction by an integer, divide the numerator of the fraction by the integer, if possible; but, if that cannot be done, multiply the denominator of the fraction by it.

3. If the denominators are equal, place the numerator of the dividend over the numerator of the divisor for the quotient.

4. If a proper fraction be divided by a proper fraction, the quotient will be greater than either the divisor or dividend. And, if any whole, or mixed, number be divided by a proper fraction, the quotient will be greater than the dividend; but, if a proper fraction be divided by a whole, or mixed, number, the quotient will be less than the dividend. *See the third note in multiplication.*

5. If any whole number, greater than 2, be divided by itself less 1, the quotient will be a mixed fraction; and if this mixed fraction be added to and multiplied by the whole number, the sum and product will be equal.

Examples.

(1.) Divide $\frac{3}{8}$ of $5\frac{1}{3}$ by $\dfrac{54}{119\frac{3}{7}}$.

First, $\frac{3}{8}$ of $5\frac{1}{3} = \frac{3}{8}$ of $\frac{16}{3} = \frac{2}{1}$ dividend, and $\dfrac{54}{119\frac{3}{7}} = \dfrac{378}{836} = \dfrac{189}{418}$ divisor.

Then $\dfrac{418}{189} \times \dfrac{2}{1} = \dfrac{836}{189} = 4\dfrac{80}{189}$ answer.

(2.) Divide $\frac{16}{19}$ by $\frac{3}{7}$.
(3.) Divide $\frac{18}{97}$ by 6.
(4.) Divide $\frac{25}{37}$ by 7.
(5.) Divide $\frac{36}{75}$ by $\frac{9}{25}$.
(6.) Divide $\frac{2}{11}$ by $\frac{7}{11}$.
(7.) Divide $\frac{}{7}$ of $\frac{5}{9}$ by $\frac{3}{8}$ of $\frac{15}{16}$.
(8.) Divide $15\frac{5}{9}$ by $\frac{7}{8}$ of $\frac{3}{11}$.
(9.) Divide $34\frac{1}{7}$ by $\dfrac{54\frac{1}{8}}{93\frac{5}{11}}$.
(10.) Divide $\dfrac{51\frac{3}{11}}{95}$ by $\dfrac{71}{149\frac{3}{8}}$.
(11.) Divide $\frac{7}{8}$ of $\frac{3}{5}$ of $5\frac{1}{9}$ by $\frac{1}{2}$ of $\frac{1}{5}$ of 19.
(12.) What number multiplied by $\frac{3}{4}$ will give $15\frac{3}{4}$ for the product.
(13.) What part of 108 is $\frac{5}{12}$ of an unit?
(14.) What number is that, which, if multiplied by $\frac{5}{8}$ of $\frac{3}{7}$ of $15\frac{1}{8}$, will produce only $\frac{5}{8}$ of an unit?

THE RULE OF THREE DIRECT IN VULGAR FRACTIONS.

RULE.

State the question as in the Rule of Three in whole numbers. Reduce mixed numbers to improper fractions,

complex and compound fractions to simple ones, and the first and third terms to the same denomination. Then invert the first term of the stating, and multiply the three terms together, and the product will be the answer.

Examples.

(1.) If $\frac{5}{8}$ of a yard cost $\frac{2}{5}$ of a £. what will $\frac{7}{11}$ of an ell English cost?

First $\frac{5}{8}$ of a yard $= \frac{5}{8}$ of $\frac{4}{5} = \frac{1}{2}$ of an ell.
Then, $\frac{1}{2}$ ell : $\frac{2}{5}$£. :: $\frac{7}{11}$ ell.
$\frac{2}{1} \times \frac{2}{5} \times \frac{7}{11} = \frac{28}{55}$£. $\doteq$ 10*s.* $2\frac{2}{11}$*d.* answer.

(2.) If $\frac{7}{11}$ of an English ell cost 10*s.* $2\frac{2}{11}$*d.* what will $\frac{5}{8}$ of a yard cost? *Answer* 8 shillings.

(3.) If $\frac{3}{5}$ of a lb. cost 7*s.* 9*d.* what will $54\frac{5}{8}$lb. cost?

(4.) If $\frac{3}{11}$ of $\frac{5}{8}$ of 15 ells of holland cost $2\frac{1}{11}$*l.* what will $\frac{3}{5}$ of 175 yards cost at that rate?

(5.) Bought $5\frac{1}{2}$ pieces of silk, each containing $35\frac{2}{11}$ ells English, at 5*s.* $3\frac{3}{8}$*d.* per ell, what is the value of the whole quantity?

(6.) Bought $14\frac{5}{11}$ tuns of wine at 3*s.* $3\frac{5}{8}$*d.* per quart, how much did I pay for the whole?

(7.) If $\frac{3}{5}$ of $\frac{7}{9}$ of a yard of cloth cost $\frac{2}{3}$ of $\frac{5}{8}$ of a £. what will 179 English ells cost?

(8.) At $7\frac{3}{4}$*d.* per lb. what will 11hhds. of sugar amount to, each hhd. weighing 4cwt. 3qr. $15\frac{5}{8}$lb.?

THE RULE OF THREE INVERSE IN VULGAR FRACTIONS.

RULE.

State the question as in whole numbers. Reduce mixed numbers to improper fractions, complex and compound fractions to simple ones, and the first and third terms to the same denomination. Then invert the third term of the stating, and multiply the three terms together.

Examples.

(1.) If $24\frac{2}{3}$ shillings will pay for the carriage of a *cwt.* $137\frac{3}{8}$ miles, how far may $5\frac{3}{8}$ cwt. be carried for the same money?

$$\text{First, } 137\tfrac{3}{8}\text{m.} = \frac{1099}{8}\text{m. and } 5\tfrac{3}{8}\text{cwt.} = \frac{43}{8}\text{cwt.}$$

$$\text{Then, } \frac{1}{1}\text{cwt.} \quad : \quad \frac{1099}{8}\text{m.} \quad :: \quad \frac{43}{8}\text{cwt.}$$

$$\frac{1}{1} + \frac{1099}{8} + \frac{8}{43} = \frac{1099}{43}\text{m.} = 25\frac{24}{43}\text{m. answer.}$$

(2). How many yards of matting, $\frac{3}{4}$ of a foot wide, will be sufficient to cover a floor that is $15\frac{1}{2}$ feet broad, and $27\frac{1}{4}$ feet long?

(3.) How many yards of cloth at 5*s.* 8*d.* per yard, may I give for $57\frac{3}{8}$ yards of cloth at 4*s.* 3*d.* per yard, that I may lose nothing?

(4.) What quantity of shalloon, $\frac{3}{4}$ of a yard wide, will line $11\frac{3}{4}$ yards of cloth $1\frac{1}{4}$ yard wide?

(5.) If I have $3\frac{1}{4}$ cwt. carried $15\frac{1}{4}$ miles for 4 guineas, how far ought $9\frac{1}{2}$ cwt. to be carried for the same money?

COMPOUND PROPORTION IN VULGAR FRACTIONS.

RULE.

State the question, as in whole numbers. Reduce mixed numbers to improper fractions, complex and compound fractions to simple ones, and the terms in the divisors to the same denomination as those in the dividends. Then invert the terms, which are to be multiplied together for a divisor, and take the continued product of all the terms for the answer.

Examples.

(1.) If £$3\frac{1}{2}$ be the wages of 13 men for $7\frac{1}{2}$ days, what will be the wages of 20 men for $15\frac{1}{3}$ days?

First, $3\frac{1}{2}£=\frac{7}{2}£$, $7\frac{1}{2}$d.$=\frac{15}{2}$d. and $15\frac{1}{3}=\frac{46}{3}$d.

$+\frac{13}{1}$m. : £.$3\frac{1}{2}$ :: $\frac{20}{1}$m.

$+\frac{15}{2}$d. : ——— :: $\frac{46}{3}$d.

$$\frac{1}{13}\times\frac{2}{15};\times\frac{20}{1}\times\frac{46}{3}\times\frac{7}{2}£=\frac{1288}{117}£=11\frac{1}{117}£.$$

Or thus,

First, $3\frac{1}{2}£=\frac{7}{2}£$, $7\frac{1}{2}$d.$=\frac{15}{2}$d. and $15\frac{1}{3}=\frac{46}{3}$d.

earn.

Then, If $\frac{13}{1}$m. : $\frac{15}{2}$d. $\frac{7}{2}£$.

$\frac{20}{1}$m. : $\frac{46}{3}$d. *

earn.

$$\frac{1}{13}\times\frac{2}{15};\times\frac{20}{1}\times\frac{46}{3}\times\frac{7}{2}£=\frac{1288}{117}£.=11\frac{1}{117}£.$$ *Answer.*

(2.) What is the interest of 490*l.* 15*s.* for $7\frac{3}{4}$ years, at $4\frac{1}{2}$ per cent. per annum?

(3.) If a footman travel 294 miles in $7\frac{1}{3}$ days of $12\frac{1}{2}$ hours long, in how many days, of $10\frac{2}{3}$ hours each, will he travel $147\frac{1}{8}$ miles?

(4.) Bought 5000 deals, of 15 feet long, and $2\frac{1}{2}$ inches thick, how many deals are they equivalent to, of $12\frac{1}{2}$ feet long, and $1\frac{3}{4}$ inch thick?

(5.) If $13\frac{1}{2}$ ells of cloth, $\frac{3}{4}$ yard wide, cost $5\frac{1}{2}$ guineas, what will $33\frac{1}{4}$ yards, $\frac{2}{3}$ of an ell English wide, and of the same goodness come to?

(6.) If 248 men, in $5\frac{1}{2}$ days of 11 hours each, dig a trench of 7 degrees of hardness, $232\frac{1}{2}$ yards long, $3\frac{2}{3}$ wide, and $2\frac{1}{3}$ deep; in how many days, of 9 hours long, will 24 men dig a trench of 4 degrees of hardness, $337\frac{1}{2}$ yards long, $5\frac{2}{3}$ wide, and $3\frac{1}{2}$ deep?

A PROMISCUOUS COLLECTION OF QUESTIONS, EXERCISING ALL THE PRECEDING RULES IN VULGAR FRACTIONS.

(1.) What part of 3*d.* is $\frac{4}{5}$ of 6*d.*?

(2.) A gentleman bought 3 suits of clothes, containing $7\frac{2}{3}$ yards each; the first suit cost 17*s.* per yard, the second $\frac{5}{6}$ of 17*s.* and the third $\frac{3}{4}$ of 17*s.* what did the whole cost him?

(3.) What number is that from which if $14\frac{5}{8}$ be deducted, the remainder will be $47\frac{9}{16}$?

(4.) If $\frac{3}{8}$ of a ship be worth 4000 guineas, what is the whole worth?

(5.) Suppose a ship be worth 16000*l.* of which my share is $\frac{5}{16}$; what *part* of her shall I have left if I dispose of $\frac{3}{8}$ of $\frac{5}{11}$ of $\frac{1}{2}$ of my share; and what money is that *part* worth?

(6.) What number is that, from which if you deduct $\frac{5}{8}$ of $\frac{3}{7}$, and to the remainder add $\frac{7}{11}$ of $\frac{3}{19}$, the sum will be 45?

(7.) Suppose A can do a piece of work in $6\frac{1}{2}$ days, B can do the same in $4\frac{1}{4}$ days, and C in $3\frac{1}{3}$ days; if you set them all at work together, in what time will they finish it?

(8.) I have employed 5 people, A, B, C, D, and E, upon a piece of work. Now I am told, that A, B, C, and D, can finish it in 13 days; A, B, C, and E, in 15 days; A, B, D, and E, in 12 days; A, C, D, and E, in 19 days; and B, C, D, and E, in 14 days; pray in what time may I reasonably expect to have my work done by their all working together; and, suppose I should wish to discharge 4 of them, which of them would finish the work soonest, when left to himself?

(9.) A *reservoir* has three cocks, A, B, and C, to let in water, and three others, D, E, and F, to discharge it:—now, if A be opened by *itself*, the reservoir, *when empty*, will be filled in 6 hours; if B be opened by *itself*, it will be filled in 8 hours; and, if C be opened by *itself*, it will be filled in 10 hours. Again, if D be opened by *itself*, when the *reservoir is full*, it will be emptied in 9 hours; if E be opened by *itself*, it will be emptied in 11 hours; and, if F be opened by *itself*, it will empty the reservoir in 13 hours;—in what time will the empty reservoir be *filled*, if all the cocks, A, B, C, D, E, F, are set open together, supposing the weight of the column of water in the reservoir, and the pressure of the atmosphere to be uniform during the influx and efflux of the water?

(10.) What is the difference between $\frac{3}{8}$ of $\frac{5}{6}$ of a crown and $\frac{3}{5}$ of $\frac{9}{16}$ of a guinea?

(11.) Multiply $\frac{1}{4}$ of $\frac{3}{5}$ of $5\frac{3}{8}$, $\frac{17\frac{1}{5}}{94}$, $\frac{14}{95\frac{3}{8}}$, and $\frac{5}{8}$ of 17 together, for the numerator of a fraction; and $\frac{14\frac{5}{9}}{17\frac{3}{5}}$, $\frac{4}{\frac{2}{9}}$, $\frac{\frac{7}{11}}{15\frac{1}{2}}$, and $51\frac{5}{8}$, together for a denominator, and reduce the new fraction to its proper terms.

(12.) Five boys, A, B, C, D, and E, put a number of marbles into a ring in order to play; but, a dispute happening amongst them, A snatched $\frac{2}{3}$ of the marbles out of the ring; B snatched $\frac{1}{8}$ of *those* out of his hand before he got off, and C, who was near, got $\frac{3}{8}$ of the remainder; D ran off with all A had left in the ring, except $\frac{1}{12}$ part, which E got.—A and C, not satisfied with what they got, jointly set upon D, and snatched $\frac{7}{11}$ of what he had got from him, of which number B, in the scuffle, got $\frac{1}{3}$, and E the rest; C snatched from E $\frac{1}{3}$ of the number he had then in hand, and A got $\frac{1}{11}$ of what B had left. Here D observed, that he had got just as many marbles as he put into the ring; and, if E would give A $\frac{1}{11}$ of what *he* had got, and C likewise give A $\frac{3}{13}$ of what he had in hand, then they would *all* have equal shares. Pray how many marbles were first put into the ring, supposing each boy put in an equal number, and none were lost in the scuffle?

(13.) A father had two sons; to the eldest he left $\frac{3}{7}\frac{5}{9}$ of his estate, and $\frac{3}{7}\frac{5}{9}$ of the remainder to the younger son; the residue was allotted to the widow; now, if the elder son had £500 more than the younger, pray what was left for the widow, and what was the gentleman's whole estate worth?

(14.) If a wall of $57\frac{1}{8}$ yards long, $12\frac{7}{12}$ feet high, and $1\frac{1}{2}$ brick thick, cost $342l.$ $15s.$ building, what will a wall of $34\frac{5}{8}$ yards long, $11\frac{1}{4}$ feet high, and $2\frac{1}{2}$ bricks thick, cost at the same rate per rod?

(15.) The diameter of the earth is 7970 miles, and the circumference is $3\frac{1}{7}$ times the diameter: if a man of 6 feet in height were to travel round the earth, how many yards would his head go farther than his feet?

(16.) A young man received $66l.$ $13s.$ $4d.$ which was $\frac{2}{3}$ of $\frac{1}{2}$ of his elder brother's portion, and $3\frac{1}{2}$ times his elder brother's portion was $1\frac{1}{4}$ times his father's estate; the question is, what was the value of their father's estate?

(17.) Suppose the cargo of a ship to be worth $10{,}000l.$ and that $\frac{4}{5}$ of $\frac{5}{7}$ of $\frac{9}{10}$ of the ship be worth $\frac{1}{4}$ of $\frac{4}{5}$ of $\frac{7}{12}$ of the cargo; what is the whole value of the ship and the cargo?

(18.) Required to find the least three whole numbers, such that $\frac{3}{8}$ of the first, $\frac{5}{14}$ of the second, and $\frac{7}{20}$ of the third, shall all be equal to each other?

(19.) A person left $\frac{2}{5}$ of his property to A, $\frac{3}{40}$ to B, $\frac{1}{8}$ to C, $\frac{1}{20}$ to D, $\frac{1}{40}$ to E, $\frac{1}{50}$ to F, and the rest, which was 800*l.* to his executor; what was the value of the whole property, and of each person's share?

(20.) How many deals 12 feet long and $7\frac{5}{12}$ inches broad will be required to floor a room $7\frac{1}{2}$ yards long by 5 yards wide, allowing for a vacancy $7\frac{1}{2}$ feet long by 5 feet broad?

(21.) There is an island 120 miles in circuit; 7 footmen *all start together* to travel the same way round it, and continue to travel till they all come together again: A goes 5 miles a day, B $6\frac{1}{4}$, C $7\frac{1}{3}$, D $8\frac{1}{4}$, E $9\frac{1}{2}$, F $10\frac{1}{4}$, and G $11\frac{1}{4}$. In how many days will they all be together a second time?

(22.) The hour, minute, and second hands of a watch are together at 12 o'clock, when will they all be together a second time?

DECIMAL FRACTIONS.

Definition 1. *Decimal Fractions, or Decimals,* are such as have 10, 100, 1000, &c. for their denominator; thus $\frac{1}{10}$, $\frac{25}{100}$, $\frac{225}{1000}$, &c. are decimal Fractions, and these are expressed by writing the numerator only, with a point before it on the left hand; thus, ·1, ·25, ·225, &c.

2. *When the numerator* of a decimal fraction is written without its denominator, it must always consist of as many figures as there are ciphers in the denominator, thus, $\frac{5}{10}$=·5, $\frac{5}{100}$=·05, $\frac{5}{1000}$=·005, &c. Hence the denominator of a decimal fraction is an unit with as many ciphers as there are figures in the decimal.

3. *Ciphers on the right hand* of decimals make no alteration in their value, thus, ·5, ·500, ·5000, &c. are decimals of the same value, for $\frac{5}{10}=\frac{500}{1000}=\frac{5000}{10000}=\frac{1}{2}$ by the nature of vulgar fractions.

4. *Ciphers on the left hand* of decimals decrease their value; thus, ·5, ·05, ·005, &c.=$\frac{5}{10}$, $\frac{5}{100}$, $\frac{5}{1000}$, &c.

Note 1. Decimals, as well as whole numbers, *decrease* in a ten-fold proportion towards the right-hand; therefore, decimals have the same properties as whole numbers, and are subject to the same rules.

5. *A mixed number* is composed of a whole number and a decimal, which are separated from each other by a point, thus, 115·5 signifies $115\frac{5}{10}$.

2. A mixed number, as 115·5, may be expressed thus, $\frac{1155}{10}$: also, $115{\cdot}005 = \frac{115{\cdot}005}{1} = \frac{1150{\cdot}05}{10} = \frac{11500{\cdot}5}{100} = \frac{115005}{1000}$, &c.

ADDITION OF DECIMAL FRACTIONS.

RULE.

Place all the decimal points directly under each other, so that tenths may stand under tenths, and hundredth parts under hundredth parts, &c. in the decimals; and tens under tens, hundreds under hundreds, &c. in the whole numbers. Then add them together as in whole numbers, and from the right hand of the sum point off as many figures, for decimals, as are equal to the greatest number of decimals in any of the given numbers.

Examples.

(1.) Add 5·74+3·75+94·375+·745+·005495 together.

```
 5·74
 3·75
94·375
  ·745
  ·005495
----------
104·615495 sum.
```

(2.) Add 5·714+3·456+·543+17·4957 together.
(3.) Add 3·754+47·5+·00857+37·5 together.
(4.) Add 54·34+·375+14·795+1·5 together.
(5.) Add 71·25+1·749+1759·5+3·1 together.
(6.) Add 375·94+5·732+14·375+1·5 together.
(7.) Add ·005+·0057+31·008+·00594 together.

SUBTRACTION OF DECIMAL FRACTIONS.

RULE.

Place the less number under the greater, the points under the points, tenths under tenths, hundredth parts under hundredth parts, &c. in the decimals; and the whole numbers under those of the same denomination. Then subtract as in whole numbers, placing the separating point, in the remainder, directly under those above it.

Examples.

(1.) From 57·439 take 5·93754.

```
57·439
 5·93754
────────
51·50146 difference.
```

(2.) Required the difference between 57·49 and 5·768.
(3.) What is the difference between ·3054 and 3·075?
(4.) Required the difference between 1745·3 and 173·45.
(5.) What is the difference between seven-tenths of an unit and 54 ten thousandth parts of an unit?
(6.) What is the difference between ·105 and 1·00075?
(7.) What is the difference between 150·43 and 754·355?
(8.) From 1754·754 take 375·49478.
(9.) Take 75·304 from 175·01.
(10.) Required the difference between 17·541 and 35·49.

MULTIPLICATION OF DECIMAL FRACTIONS.

RULE.

Multiply the decimals, as if they were whole numbers, and from the product cut off so many decimal places as there are both in the multiplier and multiplicand. If there are not so many places in the product, supply the defect by prefixing ciphers to the left hand.

Note 1. When any decimal is to be multiplied by 10, 100, 1000, &c.

remove the separating point so many places to the right-hand as there are ciphers; thus, ·543×10=5·43; also, ·7156×1000=715·6, &c.

3. What was observed in the third note in multiplication of vulgar fractions, respecting a proper fraction, or mixed number, is equally applicable to a pure, or mixed, decimal.

Contracted Multiplication of Decimal Fractions.

RULE.

Put the unit's place of the multiplier under that place of the multiplicand which you intend to keep in the product, and *invert* the order of all the other figures, that is, write the decimals on the left hand, and the integers, if any, on the right. In multiplying, always begin with that figure of the multiplicand which stands directly over the multiplying digit, and set the first figure in every product in a right line under each other to the right hand, observing to increase the first figure of every line with what would arise, by carrying 1 from 5 to 15, 2 from 15 to 25, 3 from 25 to 35, &c. from the product of the two figures (in the multiplicand) on the right hand of the multiplying digit.

Examples.

Ex. 1. Multiply 4·735 by ·374	Ex. 2. Multiply ·004735 by ·0374
18940	18940
33145	33145
14205	14205
1·770890 prod.	·000177089 prod.

(3.) Mult. 473·54 by ·057.
(4.) Mult. 137·549 by 75·437.
(5.) Mult. 3·7495 by ·73487.
(6.) Mult. ·04375 by ·47134.
(7.) Mult. ·371343 by by ·75493.
(8.) Mult. 49·0754 by 3·5714.
(9.) Mult. ·573005 by ·000754.
(10.) Mult. ·375494 by 574·375.

Examples under the contracted rule.

(1.) Multiply 2·38645 by 8·2175, and let there be only four places of decimals retained in the product.

Contracted way.	*Common way.*
2·38645	2·38645
5712·8	8·2175
190916	11 | 93225
4773	167 | 0515
239	238 | 645
167	4772 | 90
12	190916 | 0
19·6107	19·6106 | 52875

(2.) Let 54·7494367 be multiplied by 4·724753, reserving only five places of decimals in the product.

(3.) Multiply 475·710564 by ·3416494, retaining three decimals in the product.

(4.) Multiply 3754·4078 by ·734576, retaining five decimals in the product.

(5.) Let 4745·679 be multiplied by 751·4549, and reserve only the integers in the product.

DIVISION OF DECIMAL FRACTIONS.

RULE I.

Divide as in whole numbers, and from the right hand of the quotient point off so many figures for decimals as the decimal places in the dividend exceed those in the divisor; but, if the quotient does not contain such a number of figures as is equal to the excess, the defect must be supplied with ciphers to the left hand. If the number of decimal places in the divisor should be more than those of the dividend, annex so many ciphers to the dividend as will make them equal, and the quotient will be integers till all these ciphers are used; after which, you may continue the quotient to any assigned degree of exactness, by subjoining a cipher continually to the last remainder.

RULE II.

Make the divisor a whole number by removing the decimal point to the right hand of it, and remove the decimal point in the dividend the same number of figures towards the right-hand as the point in the divisor has been removed. If there be not a sufficient number of figures

in the dividend, supply the defect with ciphers. Then divide as in whole numbers, and the quotient will contain as many decimal places as are used in the dividend.

Contracted Division of Decimal Fractions.

RULE.

In division, the first figure in the quotient must always possess the same place with that figure of the dividend under which the unit's place of its product stands. Having thus determined the value of the quotient figures, make use of so many figures in the divisor, reckoning from the left hand towards the right, as you intend to have in the quotient. Let each remainder be a new dividend, and, for every such new dividend, leave out one figure to the right-hand of the divisor, observing to carry for the increase of the figures cut off, as in contracted multiplication.

Note. When there are not so many figures in the divisor as are required to be in the quotient, begin the division with all the figures as usual, and continue it till the number of figures in the divisor is equal to the number of figures remaining to be found in the quotient, after which use the contraction.

Examples.

(Ex. 1.) Divide ·475321 by 97·453.

97·453;)·475;3210000(·0048774
855090
754660
724890
427190
37378 rem.

(Ex. 2.) Divide 475·321 by ·97453.

·97453;)475·32100;00(487·74, &c.
855090
754660
724890
427190
37378, &c.

(3.) Divide 17·543275 by 125·7.
(4.) Divide 143754·35 by ·7493.
(5.) Divide ·000177089 by ·0374.
(6.) Divide 16 by 960.
(7.) Divide 12 by 1728.
(8.) Divide 47·5493 by 34·75.
(9.) Divide 74·3571 by ·00573.
(10.) Divide ·3754 by 75·714.

Examples under the contracted rule.

(1.) Divide 754·347385 by 61·34775, and let the quotient contain only three places of decimals.

Contracted way.	*Common way.*
—61·34775	61·34775
61·34775)754·347385(12·296	61·34775)754·34738500(12·296
14086	14086 | 988
1817	1817 | 4385
590	590 | 48350
38	38 | 353750
1	1 | 545100

(2.) Divide 59 by ·74571345, and let the quotient contain four places of decimals.

(3.) Divide 17493·407704962 by 495·783269, and let the quotient contain four places of decimals.

(4.) Divide 98·187437 by 8·4765618, and let the quotient contain ten places of decimals.

(5.) Divide 47194·379457 by 14·73495, and let the quotient contain as many decimal places as there will be integers in it.

REDUCTION OF DECIMAL FRACTIONS.

Proposition 1. *To reduce a vulgar fraction to a decimal fraction of equal value.*

RULE.

Annex ciphers to the numerator till it be equal to, or greater than, the denominator: then divide by the denominator as in division of decimals, and the quotient will be the answer.

Note. Mr. Colson, at page 162 of *Sir Isaac Newton's* Fluxions, gives the following method for reducing a fraction, having a prime number for its denominator, into a decimal. Let $\frac{1}{29}$ be proposed: then, by dividing in the common way, till the remainder becomes a single figure, we shall have $\frac{1}{29}=\cdot03448\frac{8}{29}$, for the complete quotient; and this equation multiplied by the numerator 8, will give $\frac{8}{29}=\cdot27586\frac{6}{29}$, and, if this be substituted in the first equation for $\frac{8}{29}$, we shall have $\frac{1}{29}=\cdot0344827586\frac{6}{29}$. Again, multiply the equation by 6, and it will give $\frac{6}{29}=\cdot2068965517\frac{7}{29}$; then, by substituting, as before, $\frac{1}{29}=\cdot03448275862068965517\frac{7}{29}$, &c. as far as you please.

Prop. 2. *To reduce numbers of different denominations, coins, weights, measures, &c. into decimals* *.

* The decimal tables of coin, weights, measures, &c. are calculated by the rules given in this proposition; thus in Table I. 19 shillings= £·95, &c. The use of the tables is shewn in the 9th example.

RULE I.

Reduce the given money, weight, &c. into the lowest denomination mentioned for a dividend: then reduce the integer into the same denomination for a divisor: the quotient produced by this division will be the decimal required.

RULE II.

Write the given denominations, or parts, regularly under each other, proceeding from the lowest denomination to the highest; let these be the dividends. Opposite to each dividend, on the left-hand, place such a number for a divisor as will reduce it to the next superior name, and draw a line between them. Begin to divide with the uppermost numbers, and write the quotients of each, as decimal parts, on the right hand of the dividend next below it. Divide this mixed number by its divisor, and so on till they are all used, the last quotient will be the decimal required.

Prop. 3. *To find the value of any decimal fraction in the known parts of an integer.*

RULE.

Multiply the given decimal by the number of parts contained in the next inferior denomination; and, from the right-hand of the product, point off so many figures as the given decimal consists of. Multiply the remaining decimals by the parts in the next inferior denomination, and from what results cut off as before. Proceed thus till you have brought out the least known parts of the integer, and then the several denominations, on the left-hand of the decimal points, will express the value of the decimal.

Examples to Proposition 1.

(1.) Reduce $\frac{7}{8}$ to a decimal fraction.

8)7·000(·875 answer.
60
40

(2.) Reduce $\frac{4}{7}$ to a decimal fraction.

(3.) Reduce $\frac{1}{243}$ to a decimal fraction.

(4.) Reduce $\frac{3}{4}$ to a decimal.
(5.) Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{5}{8}$ to a decimal.
(6.) Reduce $15\frac{5}{13}$ to a mixed decimal.
(7.) Reduce $\frac{5}{29}$ to a decimal.
(8.) Reduce $\frac{5}{191}$ to a decimal.

Examples to Prop. 2.

(9.) Reduce 18*s.* $9\frac{3}{4}$*d.* to the decimal of a pound.

By rule 1.

s.	s.	d.
20	18	$9\frac{3}{4}$
12	12	
240	225	
4	4	

960) 903·000000 (·940625

By Dec. Tables.

s.	d.	
0	$0\frac{3}{4}$	= ·003125
0	6	= ·025
0	3	= ·0125
18	0	= ·9
18	$9\frac{3}{4}$	= ·940625

By rule 2.

4	3 · farthings.
12	9 ·75 pence.
20	18·8125 shillings.
	·940625 £.

(10.) Reduce 7*s.* $5\frac{1}{2}$*d.* to the decimal of a pound.
(11.) What decimal part of a pound is three half-pence?
(12.) Reduce 4*s.* $7\frac{9}{11}$*d.* to the decimal of a pound.
(13.) Reduce 1 oz. 11 dwt. 3 gr. to the decimal of a pound Troy.
(14.) Reduce 24 grains to the decimal of an ounce Troy.
(15.) Reduce 5 oz. 4 dr. Avoirdupois to the decimal of a pound Troy.
(16.) Reduce 8 cwt. 1 qr. 14 lb. to the decimal of a ton.
(17.) Reduce 2 qr. 15 lb. to the decimal of a hundred-weight.
(18.) Reduce 5 lb. 10 oz. 3 dwt. 13 gr. Troy to the decimal of a hundred-weight Avoirdupois.
(19.) Reduce 1 qr. 1 n. to the decimal of a yard.
(20.) Reduce 2 qr. 3 n. to the decimal of an English ell.
(21.) Reduce 14 yds. 2 ft. $6\frac{1}{2}$ in. to the decimal of a mile.
(22.) What decimal part of an acre is 1 r. 37 poles?
(23.) What decimal part of a hogshead of wine is 2 qts. 1 pint?
(24.) Reduce 3 bush. 3 pks. to the decimal of a chaldron of 32 bushels.
(25.) What decimal part of a year is 3 w. 6 d. 7 hrs., reckoning 365 d. 6 hrs. a year?

(26.) Reduce 2·45 shillings to the decimal of a £.
(27.) Reduce 1·074 roods to the decimal of an acre.
(28.) Reduce 176·9 yards to the decimal of a mile.

Examples to Prop. 3.

(29.) Required the value of ·03125 of a pound sterling.

	·03125	
	20	
s.	0·62500	
	12	
d.	7·50000	
	4	
qrs.	2·00000	*Answer.* $7\frac{1}{2}$d.

(30.) What is the value of ·7575 of a pound sterling.
(31.) Required the value of ·75435 of a shilling.
(32.) What is the value of ·375 of a guinea?
(33.) What is the value of ·4575 of a hundred-weight?
(34.) What is the value of ·175 of a ton Avoirdupois?
(35.) What is the value of ·05875 of a pound Avoirdupois?
(36.) Required the value of ·02575 of a pound Troy.
(37.) Required the value of ·075 of a yard.
(38.) Required the value of ·475 of an English ell.
(39.) What is the value of ·04535 of a mile?
(40.) What is the value of ·6375 of an acre?
(41.) What is the value of ·574 of a hogshead of beer?
(42.) What is the value of ·4285 of a year?
(43.) Required the sum of ·475 of a pound and ·375 of a shilling.
(44.) Required the sum of ·573 of an inch and ·751 of a yard.
(45.) Required the difference between ·5 of a mile and ·375 of a furlong.
(46.) Required the sum of ·625 of a cwt. and ·20835 of a ton.
(47.) Required the sum of ·175 ton, ·195 cwt. ·145 qr. and ·15 lb.
(48.) Required he sum of ·575 lb. Troy and ·845 oz.

DECIMAL TABLES OF COIN, WEIGHT, AND MEASURE.

TABLE I.
English Coin.
1£ the Integer.

Sh.	dec.	Sh.	dec.
19	·95	9	·45
18	·9	8	·4
17	·85	7	·35
16	·8	6	·3
15	·75	5	·25
14	·7	4	·2
13	·65	3	·15
12	·6	2	·1
11	·55	1	·05
10	·5		

Pence	Decimals
6	·025
5	·020833
4	·016666
3	·0125
2	·008333
1	·004166

Farthings	Decimals
3	·003125
2	·0020833
1	·0010416

TABLE II.
Eng. Coin, or Long-Meas. 1 Shilling, or 1 Foot the Integer.

Pence or Inches	Decimals of a Shil. or Foot
6	·5
5	·416666
4	·333333
3	·25
2	·166666
1	·083333

Farthings	Decimals
3	·0625
2	·041666
1	·020833

TABLE III.
Troy-Weight.
1lb. the Integer.
Ounces the same as Pence in Table II.

Penny-weight	Decimals
10	·041666
9	·0375
8	·033333
7	·029166
6	·025
5	·020833
4	·016666
3	·0125
2	·008333
1	·004166

Grains	Decimals
12	·002083
11	·001910
10	·001736
9	·001562
8	·001389
7	·001215
6	·001042
5	·000868
4	·000694
3	·000521
2	·000347
1	·000173

1 Oz. the Integer.
Penny-weights, the same as shillings in the first Table.

Grs.	Decimals
12	·025
11	·022916
10	·020833
9	·01875
8	·016666
7	·014583
6	·0125
5	·010416
4	·008333
3	·00625
2	·004166
1	·002083

TABLE IV.
Avoird. Wt.
1cwt. the Integ.

Qrs.	Decimals
3	·75
2	·5
1	·25

Pnds.	Decimals
14	·125
13	·116071
12	·107143
11	·098214
10	·089286
9	·080357
8	·071428
7	·0625
6	·053571
5	·044643
4	·035714
3	·026786
2	·017857
1	·008928

Oz.	Decimals
8	·004464
7	·003906

DECIMAL TABLES OF WEIGHT AND MEASURE.

6	·003348
5	·002790
4	·002232
3	·001674
2	·001116
1	·000558

¼ Oz.	Decimals
3	·000418
2	·000279
1	·000139

TABLE V.

AVOIRDUP. WT.

1lb. the Integer.

Ounces	Decimals
8	·5
7	·4375
6	·375
5	·3125
4	·25
3	·1875
2	·125
1	·0625

Drams	Decimals
8	·03215
7	·027343
6	·023437
5	·019531
4	·015625
3	·011718
2	·007812
1	·003906

TABLE VI.

LIQUID MEAS.

1 Tun the Integer.

Gallons	Decimals
100	·396825
90	·357142
80	·317460
70	·277777
60	·238095
50	·198412
40	·158730
30	·119047
20	·079365
10	·039682
9	·035714
8	·031746
7	·027777
6	·023809
5	·019841
4	·015873
3	·011904
2	·007936
1	·003968

Pints	Decimals
4	·001984
3	·001488
2	·000992
1	·000496

A Hogshead the Integer.

Gallons	Decimals
30	·476190
20	·317460
10	·158730
9	·142857
8	·126984
7	·111111
6	·095238
5	·079365
4	·063492
3	·047619
2	·031746
1	·015873

Pints	Decimals
3	·005952
2	·003968
1	·001984

TABLE VII.

MEASURE.

Liquid and Dry.

1 Gall. or 1 Quarter the Integer.

Pint	Decimals	Bush.
4	·5	4
3	·375	3
2	·25	2
1	·125	1

Q. pt.	Decimals	Pecks
3	·09375	3
2	·0625	2
1	·03125	1

Decimals	Q. Pks.
·0234375	3
·015625	2
·0078125	1

Decimals	Pints
·005859	3
·003906	2
·001953	1

TABLE VIII.

LONG MEASURE.

1 Mile the Integer.

Yards	Decimals
1000	·568182
900	·511364
800	·454545
700	·397727
600	·340909

DECIMAL TABLES OF WEIGHT AND MEASURE.

500	·284091
400	·227272
300	·170454
200	·113636
100	·056818
90	·051136
80	·045454
70	·039773
60	·034091
50	·028409
40	·022727
30	·017045
20	·011364
10	·005682
9	·005114
8	·004545
7	·003977
6	·003409
5	·002841
4	·002273
3	·001704
2	·001136
1	·000568

Feet	*Decimals*
2	·0003787
1	·0001894

Inch.	*Decimals*
6	·0000947
3	·0000473
1	·0000158

TABLE IX.

TIME.

1 Year the Integer.

Months the same as Pence, in the second Table.

Days	*Decimals*
365	1·000000
300	·821918
200	·547945
100	·273973
90	·246575
80	·219178
70	·191781
60	·164383
50	·136986
40	·109589
30	·082192
20	·054794
10	·027397
9	·024657
8	·021918
7	·019178
6	·016438
5	·013698
4	·010959
3	·008219
2	·005479
1	·002739

1 Day the Integer

Hours	*Decimals*
12	·5
11	·458333
10	·416666
9	·375
8	·333333
7	·291666
6	·25
5	·208333
4	·166666
3	·125
2	·083333
1	·041666

Minutes	*Decimals*
30	·020833
20	·013888
10	·006944
9	·00625
8	·005555
7	·094861
6	·004166
5	·003472
4	·002777
3	·002083
2	·001388
1	·000694

TABLE X.

CLOTH MEAS.

1 Yard the Integer.

Quarters the same as Table IV.

Nails	*Decimals*
2	·125
1	·0625

TABLE XI.

LEAD WEIGHT.

A Fother the Inte.

Hund.	*Decimals*
10	·512820
9	·461538
8	·410256
7	·358974
6	·307692
5	·256410
4	·205128
3	·153846
2	·102564
1	·051282

Qrs.	*Decimals*
2	·025641
1	·012820

Pounds	*Decimals*
14	·0064102
13	·0059520
12	·0054945
11	·0050366
10	·0045787
9	·0041208
8	·0036630
7	·0032051
6	·0027472
5	·0022893
4	·0018315
3	·0013736
2	·0009157
1	·0004578

THE RULES OF PROPORTION IN DECIMALS.

Note. The examples following are promiscuously arranged to exercise the scholar in the Rule of Three Direct, Inverse, Compound Proportion, &c. Decimals have the same properties as whole numbers, the only difficulty being in pointing off the decimals, a repetition of the rules already given would, therefore, be superfluous.

(1.) If 3·75 yards of cloth cost 8*s.* 9*d.* what will $257\frac{1}{2}$ yards cost?

First 8s. 9d.=·4375 and $257\frac{1}{2}$ yds.=257·5

yds.		£.		yds.
3·75	:	·4375	::	257·5
		257·5		

£.   £. s. d.

3·75)112·65625(30·04166, &c.=30..0..10 answer.

(2.) If $\frac{1}{2}$ cwt. of tobacco cost 4*l.* 18*s.* how much may I buy at the same rate for ·7*l.*?

(3.) Bought 3·5 yards of cloth for 2*l.* 14*s.* 3*d.* what must I give for 27·75 yards?

(4.) Sold $75\frac{3}{4}$ chaldrons of lime, at 11*s.* $6\frac{1}{2}$*d.* per chaldron, what is the amount?

(5.) A goldsmith sold a tankard for 10·6*l.* at the rate of 5*s.* 6*d.* per oz.; what did it weigh?

(6.) If 12 men can perform a piece of work in $100\frac{1}{8}$ days, in how many days would 20 men perform the same?

(7.) In $754\frac{118}{208}$ ducats, at 4*s.* 4*d.* each, how many dollars, at 4*s.* $5\frac{3}{4}$*d.* each?

(8.) If 5400 bricks be required to pave a yard, when the bricks are ·5 foot long, and ·25 broad, how man will be required of ·75 foot long and $\frac{1}{3}$ foot broad?

(9.) If I buy 14 yards of cloth for 10 guineas, how many ells Flemish can I buy for 283·875*l.*?

(10.) If $1\frac{3}{4}$ oz. of plate cost 10*s.* $11\frac{1}{2}$*d.* what will a service, weighing 327·61875 oz. cost?

(11.) How many yards of flannel that is one English ell in width, will be sufficient to line a cloak, containing $18\frac{1}{2}$ yards of cloth $\frac{3}{4}$ yard wide?

(12.) If 248 men in $60\frac{1}{2}$ hours dig a trench, containing $13924\frac{1}{5}$ solid yards of earth, how many men in 1188

hours, will dig a similar trench, containing 26460 solid yards of earth; the earth being cast at the same distance from these men as the former?

(13.) If 2 men can do 125 rods of ditching in 65 days, in how many days can 18 men do $242\frac{4}{11}$ rods?

(14.) If $\frac{2}{3}$ of $\frac{1}{4}$ of a ship be worth 147*l.* 11*s.* 3*d.* what is the whole worth?

(15.) If a piece of Arras hanging be $6\frac{1}{4}$ yards long, and 4 yards broad, how many square ells Flemish are contained therein?

(16.) If a wedge of gold, weighing $17\frac{3}{7}$ lb. troy, be worth $679\frac{5}{7}$*l.* what is the value of $1\frac{3}{13}$ grain of that gold?

(17.) What will be the expence of tiling an out-house that is 273·5 feet long, and 51·75 feet broad, with tiles at 11*s.* $10\frac{3}{4}$*d.* per thousand, supposing every square of tiling to take up 1000 tiles?

(18) A man, with his family consisting of 4 persons, usually drink 7·8 gallons of beer in a week, how much would they drink in 22·5 weeks, if the family were to be increased by 3 persons?

(19.) I agreed for the carriage of 2·5 tons of goods 2·9 miles, for ·075 guinea, what is that per cwt. for a mile?

(20.) If a traveller perform a journey in 35·5 days, when the days are 13·625 hours long; in how many days of 11·9 hours long would he perform the same journey?

(21.) The earth turns round on its axis from west to east in 23 hours 56 minutes, and the circumference of every circle on it surface is supposed to be divided into 360 degrees. At the equator a degree is 69·07 English miles; at Madras, Barbadoes, &c. 67·21 English miles; at Madrid, Philadelphia, &c. 52·85 English miles; and at Petersburg 34·53 English miles. How many miles per hour are the inhabitants in each of these places carried from west to east by the revolution of the earth on its axis?

(22.) Goliath, the Philistine, is said to have been $6\frac{1}{4}$ cubits high, each cubit 1 foot 7·168 inches English, what was his height in English feet?

CIRCULATING, OR REPEATING DECIMALS.

Definition 1. When the denominator of a vulgar fraction, in its lowest terms, is not compounded of 2 or 5, or both, the decimal produced from such a vulgar fraction will be infinite; it is called a *repetend*, or *circulating decimal*, from a continual repetition of the same figures.

2. *A single repetend* is a decimal, where only one figure repeats, as ·222, &c. or ·3333, &c. and these may be expressed by putting a mark over the first figure. Thus ·222, &c. may be denoted by ·2′, and ·3333, &c. by ·3′.

3. *A compound repetend* has the same figures circulating alternately, as ·575757, &c. or ·57235723, &c. and these may be distinguished by marking the first and last repeating figure. Thus ·5757, &c. may be written ·5′7′, and ·57235723, &c. ·5′723′.

4. *Pure repetends* are such as have no figures in them but what belong to the repetend, as ·3′, ·5′, ·4′73′, &c.

5. *Mixed repetends* are such as have ciphers or significant figures, between the repetend and the decimal point, or such as have whole numbers to the left hand of the decimal point, as ·04′, ·07′53′, ·473′, ·357′3′, 6·5′, 4·3′75′, &c.

6. *Dissimilar repetends* are such as begin at different places from the decimal points, as ·2′53′, ·475′2′, &c.

7. *Similar repetends* are such as begin at an equal distance from the decimal points, as ·35′4′, 2·75′34′, &c.

8. *Conterminous repetends* are such as end at the same distance from the decimal points, as ·125′, ·3′54′, &c.

9. *Similar and conterminous repetends* are such as begin and end at the same place after the decimal points, as 53·27′53′, 4·63′25′, and ·46′32′, &c.

REDUCTION OF CIRCULATING DECIMALS.

Proposition 1. *To reduce a pure repetend to its equivalent vulgar fraction.*

RULE.

Make the given decimal the numerator, and let the denominator be a number consisting of so many nines as there are figures in the repetend. The terms of this fraction, divided by their greatest common measure, will give the least equivalent vulgar fraction required.

Prop. 2. *To reduce a mixed repetend to its equivalent vulgar fraction.*

RULE.

From the given mixed repetend subtract the finite figures for a numerator, and to the right hand of so many nines as there are pure repetends annex so many ciphers as there are finite decimals for a denominator. Then reduce this fraction to its lowest terms.

Note 1. Any finite decimal may be considered as infinite by making ciphers to recur; thus ·35=·3500000, &c.

2. If any circulating decimal have a repetend of any number of figures, it may be considered as having a repetend of twice or thrice that number of figures, or any multiple thereof. The number 2·35'7', having two repetends, may be considered as having a repetend of 4, 6, 8, 10, &c. places. Thus, 2·35'7'=2·35'757'=2·35'75757'=2·35'7575757', &c. Hence any number of dissimilar repetends may be made similar amd conterminous.

3. If any circulating decimal have a repetend of more than one figure, it may be transformed into another decimal, having a repetend of the *same* number of figures; thus, ·5'7'=·57'5'=·575'7', and 3·47'85'=3·478'57'=3·4785'78'=3·47857'85'.

4. When any circulating decimal has a repetend of more than one figure, it *cannot* be transformed into another decimal, having a greater or less number of figures at pleasure; but the new repetend must always contain either the *same* number of places as the original repetend, or some *multiple* thereof. Thus, ·5'7'=·57'5'=·575'7'=·5757'5'=·5757'575': Or, ·5'7'=·5'757'=·5'75757', according to the second note. But ·5'7' never can be equal to ·5'75', for then $\frac{57}{99}$ would be equal to $\frac{575}{999}$, which evidently is not the case. The truth of any of the preceding notes may be examined by turning the given repetends into their equivalent vulgar fractions, and comparing them together by the latter part of Note 2, Prop. 12, Vulgar Fractions.

5. Any series of nines, infinitely continued, is equal to unity, or one, in the next left-hand place. Thus, ·999, &c. ad infinitum,=1; ·0999, &c.=·1; also ·00999, &c =·01, and 5·999, &c.=6.

6. Any number may be multiplied by 9, 99, 999, &c. by annexing so many ciphers to the right hand of it as there are nines, and then subtracting it from itself, thus increased. Thus,

$$147\times9=1470-147=1323,\ 147\times99=14700-147=14553,$$
$$\text{and } 147\times999=147000-147=146853.$$

7. Any number, divided by 9, 99, 999, &c. will be equal to the sum of the quotients of the same number continually divided by 10, 100, 1000, &c. Thus,

$$\frac{425}{10}+\frac{42\cdot5}{10}+\frac{4\cdot25}{10}+\frac{\cdot425}{10}, \text{\&c. ad infinitum, }=\frac{425}{9}=47\cdot2', \text{ and}$$

$$\frac{425}{1000}+\frac{\cdot425}{1000}+\frac{\cdot000425}{1000}, \text{\&c. ad infinitum}=\frac{425}{999}=\cdot4'25';$$

hence, it appears that every recurring decimal is a geometrical series, decreasing, ad infinitum, and the equivalent vulgar fraction to every recurring decimal is equal to the sum of such a series.

8. If any number be divided by another prime to it, and the division continued on indefinitely, the number of repetends in the quotient will always be less than the number of units in the divisor.

9. If two or more numbers, that have repetends of equal places, be added together, the sum will have a repetend of the same number of places; for every column of periods will amount to the same sum.

10. If any circulating number be multiplied by any given number, the product will be a circulating number, containing the same number of figures in the repetend as before, for every repetend will be equally multiplied, and consequently must produce the same product.

Prop. 3. Having a vulgar fraction given, to find whether its equivalent decimal will be finite or infinite, and how many places the repetend will consist of.

RULE.

Reduce the given fraction to its lowest terms, and divide the denominator by 10, 2, or 5, as often as possible: then divide 9999, &c. by this result till nothing remains, and the number of nines made use of will be equal to the number of figures in the repetend. The repetend will always begin after so many places of figures as you perform divisions by 10, 2, or 5; and, if the whole denominator should vanish after these divisions, the decimal will be finite.

Examples to Proposition 1.

(1.) Required the least equivalent vulgar fraction to ·3′, and ·1′35′.

First, $\cdot3' = \frac{3}{9} = \frac{1}{3}$, and $\cdot1'35' = \frac{135}{999} = \frac{5}{37}$.

(2.) Required the least equivalent vulgar fractions to ·6′, ·1′62′, ·7′69230′, ·9′45′, and ·0′9′.

(3.) Required the least equivalent vulgar fractions to ·5′94405′, ·3′6′, and ·1′42857′.

Examples to Prop. 2.

(4.) Required the least equivalent vulgar fractions to 2·41′8′, ·59′25′, ·0084′97133′, and ·53′.

First, $2\cdot41'8' = \frac{2418-24}{990} = \frac{2394}{990} = \frac{133}{55} = 2\frac{23}{55}$; $\cdot59'25' = \frac{5925-5}{9990} = \frac{5920}{9990} = \frac{16}{27}$; $\cdot0084'97133' = \frac{8497133-8}{999999000} = \frac{8497125}{999999000} = \frac{83}{9768}$; and $\cdot5'3' = \frac{53-5}{90} = \frac{48}{90} = \frac{8}{15}$.

(5.) Required the least equivalent vulgar fractions to ·138′, 7·54′3′, ·043′54′, 37·54′, ·67′5′, and ·75′4347′.

(6.) Required the least equivalent vulgar fractions to ·75′, ·43′8′, ·093′, 4·75′43′, ·0098′7′, and ·45′.

Examples to Prop. 3.

(7.) Required to find whether the decimal equivalent to $\frac{249}{29304}$ be finite or infinite; if infinite, how many places the repetend will consist of, whether there will be any finite decimals to the left hand of the repetend, and how many?

First, $\frac{249}{29304}=\frac{83}{9768}$; *then* $\frac{\div 2}{9768}=\frac{\div 2}{4884}=\frac{\div 2}{2442}=1221$, *and*

1221)999999(719: *here are* 6 *nines made use of before nothing remains; and the denominator has been divided by* 2 *three times, and cannot be abridged any more; therefore the decimal will be infinite, and will consist of three finite decimals and six pure repetends. Thus,* $\frac{83}{9768}$=·0084′97133′.

(8.) Whether is the decimal equivalent to $\frac{210}{1126}$ finite or infinite?

(9.) Whether is the decimal equivalent to $\frac{4}{1160}$ finite or infinite?

(10.) Let $\frac{12}{132}$, $\frac{80}{135}$, $\frac{72}{135}$, $\frac{7}{8544}$, and $\frac{255}{792}$, be proposed.

ADDITION OF CIRCULATING DECIMALS.

RULE.

Make the repetends similar and conterminous, and to the right hand thereof set two or three of the first repeating figures, which add together as whole numbers, and carry the tens contained in the left-hand row to the right-hand row of the conterminous repetends: collect these together into one sum, like finite decimals, for the answer.

Note. The sum of the repetend, found by the preceding rule, will sometimes, though very rarely, consist of a number of nines; whenever that is the case, reject them, and make the next left-hand figure an unit more. If the decimals to be added contain only single recurring figures, after having made them end together, the sum of the right hand row may be increased by as many units as it contains nines, instead of carrying the repetend out.

Examples.

(1.) Add ·125′, 4·1′63′, 1·7′143′, and 2·5′4′, together.

Dissimilar.		*Similar.*		*Similar and conterminous.*		
·125′	=	·125′	=	·125′55555555555′	....	5555
4·1′63	=	4·163′16′	=	4·163′16316316316′	....	3163
1·7′143′	=	1·714′371′	=	1·714′37143714371′	....	4371
2·5′4′	=	2·545′4′	=	2·545′45454545454′	....	5454
		The true sum		8·548′54470131697′		*one to carry.*

(2.) Add 67·34′5′ + 9·6′51′ + ·2′5′ + 17·47′ + ·5′, together.

(3.) Add ·4′75′ + 3·754′3′ + 64·7′5′ + ·5′7′ + ·17′88′, together.

(4.) Add ·5′ + 4·37′ + 49·45′7′ + ·49′54′ + ·7′345′, together.

(5.) Add ·175′ + 42·5′7′ + ·37′53′ + ·59′45′ + 8·75′4′, together.

(6.) Add 165·1′64′ + 147·0′4′ + 4·9′5′ + 94·37′ + 4·7′123456′, together.

SUBTRACTION OF CIRCULATING DECIMALS.

RULE.

Make the repetends similar and conterminous and subtract, as if they were finite decimals; only observe, that if the repetend of the subtractor be greater than the repetend of the subtrahend, the right-hand figure of the remainder must be less by unity than it would be, if the expressions were finite.

Note. If either the subtrahend or subtractor be finite decimals, they must be made similar and conterminous with ciphers.

Examples.

(1.) From 11·47′5′ take 3·457′35′.

Dissimilar.		*Similar.*		*Similar and conterminous.*		
11·47′5′	=	11·475′7′	=	11·475′75757′	....	575
3·457′35′	=	3·457′35′	=	3·457′35735′	....	735
		The true difference		8·018′40021′		*one to carry.*

(2.) From 47·53′ take 1·7′57′.
(3.) From 17·5′73′ take 14·57′.
(4.) From 17·43′ take 12·345′.
(5.) From 1·127′54′ take ·47′384′.
(6.) From 4·75 take ·375′.
(7.) From 4·794 take ·17′44′.
(8.) From 1·457′ take ·3754.
(9.) From 1·49′37′ take ·1475.

MULTIPLICATION OF CIRCULATING DECIMALS.

GENERAL RULE.

Turn the decimals into their equivalent vulgar fractions, and find the product of these fractions: then turn the vulgar fraction, expressing the product, into an equivalent decimal fraction, and it will be the product required.

OR

Proposition 1. *When the right hand figure of the multiplicand is a single repetend, and the multiplier a finite number.*

Rule. In multiplying, increase the right-hand figure of each resulting line by as many units as there are nines in the product of the first figure in that line; and the right-hand figure of each line will be a repetend: make them all end at the same place, and then add them together.

Prop. 2. *When the multiplicand is a compound repetend, and the multiplier a finite number.*

Rule. Set the repeating figures in the multiplicand twice over, multiply the second period mentally, and carry the tens, contained in the product of the left-hand figure, to the product of the right-hand figure of the first period; then multiply the rest of the figures in the multiplicand as in common multiplication. Proceed thus with each figure in the multiplier, and every product will contain a repetend of the same number of places as the repetend in the multiplicand; lastly, make every product conterminous towards the right-hand before you add them together.

Note. It is possible for the product of the repetend to consist of a number of nines; if ever this should happen, increase the product of the right-hand figure of the first period by an unit.

Prop. 3. *When the multiplicand is a finite number, and the right-hand figure in the multiplier a single repetend.*

Rule. Multiply by the circulating figure, as if it were a finite digit; divide this product by 9, and continue the quotient till it becomes a single circulate, if the product does not divide even by 9. Proceed with the remaining figures in the multiplier (if any) as in common multiplication, taking care to set the first figure of the first product exactly under the repeating figure in the multiplier.

Prop. 4. *When the multiplicand is a finite number, and the multiplier a common repetend.*

Rule. If the multiplier be a mixed repetend, subtract the finite decimal from it, and the remainder will be a new multiplier, with which proceed as in common multiplication, and add the several products together. Then set the left-hand figure of this sum under its third, fourth, fifth, &c. figure towards the right-hand, according as the repetend consists of two, three, four, &c. places, and the rest in order after it: proceed thus, till the left-hand figure of the sum falls beyond the right-hand figure. Lastly, collect these numbers into one sum in the order they are placed, and mark as many figures for a repetend as the repetend of the multiplier consists of.

Note. If the multiplier be a pure repetend, proceed exactly in the same manner with it as with the new multiplier above.—The reason of the placing the left-hand figure of the sum under the third, &c. figures, will readily occur to any one who has consulted the 7th note in Reduction of Circulating Decimals.

Prop. 5. *When the multiplicand and multiplier are each a single circulate.*

Rule. In multiplying by the repeating digit, increase the right-hand figure of the product by as many units as it contains nines; divide this product by 9 till it recurs, and the quotient will be the product of the repetend. Proceed with the finite numbers, if any, as in Prop. 1, taking care to set the first figure of the first product exactly under the repeating figure in the multiplier.

Prop. 6. *When the multiplicand is a compound repetend, and the multiplier a single recurring digit.*

Rule. After having multiplied by the repeating figure, like a finite digit, as directed in Prop. 2, divide the product by 9, till the quotient recurs, or is sufficiently exact. Proceed with the finite numbers, if any, in the very same manner as directed in *that* Prop. taking care to set the first figure of the first product exactly under the repeating figure in the multiplier.

Prop. 7. *When the multiplicand is a single, and the multiplier a compound repetend.*

Rule. If the multiplier be a pure repetend, proceed as in Prop. 1. If it be a mixed repetend, subtract the finite decimal from it, and the remainder will be a new multiplier, with which proceed as above. Then set the left-hand figure of the sum under its third, fourth, fifth, &c. figure towards the right-hand, as directed in Prop. 4.

Prop. 8. *When the multiplicand and multiplier are each a compound repetend.*

Rule. If the multiplier be a pure repetend, proceed as in Prop. 2. If it be a mixed repetend, subtract the finite decimal from it, and the remainder will be a new multiplier, with which proceed as above. Then set the left-hand figure of the sum under its third, fourth, fifth, &c. figure towards the right-hand, as directed in Prop. 4.

Examples.

The general rule needs no example.

Examples to Prop. 1.

(1.) Mult. 4·253'
by ·257

29773' .. 33
21266'6 .. 66
8506'66 .. 66

Product 1·093106' *one to carry.*

(2.) Mult. ·3754' by 14·75.
(3.) Mult. 4·753' by 7·437.
(4.) Mult. ·30705' by ·0473.
(5.) Mult. ·14732' by ·1497.
(6.) Mult. ·37543' by ·7149.

Examples to Prop. 2.

(6) Mult. ·42′53′ by 2·57.

```
                2d period.
  ·42′53′  ..  253
   2·57
 ---------
  297′72  ..  772
 212′66′2 ..  662
 85 06′50 ..  650
 ---------
Product 1·0630′86′ two to carry.
```

(7.) Mult. ·37′54′ by 17·43.
(8.) Mult. 4·73′5′ by 7·349.
(9.) Mult. 4·1′42857′ by ·1797.
(10.) Mult. 7·14′93′ by 5·43.
(11.) Mult. ·40′705′ by 7·345.

Examples to Prop. 3.

(12.) Mult. ·437 by 3·75′

```
        ·437
     by 3·75′
     --------
     9)2185
     --------
         2427′
        3059
       1311
     --------
Product 1·64117′
```

(13.) Mult. 1·475 by 1·754′.
(14.) Mult. 173·715 by 3·7545′.
(15.) Mult. ·37504 by ·7153′.
(16.) Mult. ·57554 by ·1735′.
(17.) Mult. ·37493 by ·757′.

Examples to Prop. 4.

(18.) Mult. 4·573 by ·37′5′

```
                      4·573
·37′5′                 ·372
    3               --------
-----                  9146
·372 new mult.        32011
                     13719
                    --------
                    1·701156
                      17011 .. 56
                        170 .. 11 &c.
                          1 .. 70 &c.
                    --------
Product 1·71833′9′ one to carry.
```

(19.) Mult. 4·573 by ·3′75′.

```
          4·573
          ·3′75′
        --------
          22865
         32011
        13719
        --------
        1.714875
           1714 .. 875
              1 .. 71 &c.
        --------
Product 1·7165′91′ one to carry.
```

(20.) Mult. 4·7157 by 3·7′543′.
(21.) Mult. 47·1937 by ·007′5′.
(22.) Mult. 4·37595 by 1·7′5435′.
(23.) Mult. 371473 by ·7′5314′.

Examples to Prop. 5.

(24.) Mult. 3·456′
by ·425′

9)17283′33

1920′37 .. 03 &c.
6193′33 .. 33
13826′666 .. 66

Product 1·47100′37′ *one to carry.*

(25.) Mult. 4·57′ by 2·45′.
(26.) Mult. 3·745′ by 1·47′.
(27.) Mult. 5·7195′ by 1·788′.
(28.) Mult. 3·7532′ by ·3425′.
(29.) Mult. 714·32′ by 3·456′.

Examples to Prop. 6.

(30.) Mult. 1·4′56′ by 4·23′.
2d period.
1·4′56′ .. 456
4·23′

9)43′69′36, &c.

48′54′ .. 854
29′12′9 .. 129
58′25′82 .. 582

Product 6·165′66′ *one to carry.*

(31.) Mult. ·573′4′ by 1·573′.
(32.) Mult. ·41′32′ by 1·432′.
(33.) Mult. 3·7′534′ by ·3757′.
(34.) Mult. 7·00′430′ by 4·7005′.
(35.) Mult. 5·437′15′ by ·37053′.

Examples to Prop. 7.

(36.) Mult. 45·13′ by ·2′45′.

45·13′
·2′45′

22566′ .. 66
18053′3 .. 33 } one to carry.
9026′66 .. 66

11·05766′6666 &c.
11057666 &c.
11057 &c.
11 &c.

11·0′68735402′

This repetend is found by continuing the figures, &c.

(37.) Mult. 3·537′ by 2·4′5′.

2·4′5′
2
2·43 new mult.

3·537′
2·43

10613′ .. 33
14151′1 .. 11 } *
7075′55 .. 55

8·59680′0 &c.
85968 &c.
859 &c.

8·68363′

This repetend is found as the last.

(38.) Mult. ·47053′ by 1·73′5′.
(39.) Mult. 3·4573′ by 54·7′53′.

Examples to Prop. 8.

(40.) Mult. 7·7′2′ by ·2′97′.

	2d period.
7·7′2′	.. 72
·2′97′	
540′9′	.. 09 }
695′4′5	.. 45 } *
154′5′45	.. 45 }
2·29500′0 &c.	
2295 &c.	
2 &c.	
2·2972′97′	

(41.) Mult. ·2′97′ by 7·7′2′.

	2d period.	
·2′97′	.. 297	7·7′2′
7·65		7
14′86′	.. 486 }	7·65 n. mult.
17′83′7	.. 837 }	one to carry.
20′81′08	.. 108 }	
2·274′32′432 &c.		
22743 &c.		
227 &c.		
2 &c.		
2·2972′97′		

* The sum of the repetend, in the 2d example, Prop. 7, and in the 1st example in this Prop., consists of an infinite number of nines. See the note in Addition of Circulating Decimals.

(42.) Mult. 4·571′37′ by ·14957′3′.
(43.) Mult. 5·7149′3′ by 4·75′35′.

DIVISION OF CIRCULATING DECIMALS.

GENERAL RULE.

Turn the decimals into their equivalent vulgar fractions, and find the quotient of these fractions: then turn the vulgar fraction, expressing the quotient into an equivalent decimal fraction, and it will be the quotient required.

OR

Proposition* 1. *When the dividend is a single, or compound, repetend, and the divisor a finite number.

Rule. Divide, as if both the numbers were finite; only, instead of bringing down ciphers, bring down the repeating figure, or figures, and continue the quotient till it repeats, or is sufficiently exact.

Prop.* 2. *When the divisor is a single, or compound, repetend, and the dividend a finite number.

Rule. If the divisor be a mixed repetend, annex as many ciphers to the right hand of the dividend as there are pure repetends in the divisor. Write this dividend and divisor in the order of division; under these write

them a second time, each removed so many figures towards the right hand as there are pure repetends in the divisor; subtract each lower line from that above it, and the remainders will be a new divisor and dividend, both finite numbers. If the divisor be a pure repetend, the dividend *only* must be subtracted after the ciphers are annexed to it.

Prop. 3. *When both the divisor and dividend are single, or compound, repetends.*

Rule 1. If they are dissimilar mixed repetends, make them similar and conterminous, and subtract the finite numbers from each of them, the remainders will be a new divisor and dividend, both finite numbers.

2. If the divisor and dividend are both pure repetends, make them conterminous, and divide them like finite numbers.

Note 1. If one number be a pure, and the other a mixed, repetend, (without any whole number,) by making them similar, the pure repetend will become a mixed one, and consequently the first part of the preceding rule will discover the true quotient.

2. If one number be a pure, and the other a mixed, repetend, (composed of a pure repetend and a whole number only,) make them conterminous, and subtract the whole number from the pure repetend belonging to it, for a new divisor or dividend; then proceed as if both the numbers were finite.

Examples.

The general rule needs no example.

Examples to Prop. 1.

(1.) Divide 56·6′ by 137.

true quotient.

137)56·6′666666(·4′136253′.

Rem. 5

(2.) Divide 24·31′8′ by 1·792.

quotient.

1·792)24·31′8′181818(13·570413, &c.

Rem. 1722.

(3.) Divide 7·54′03′ by 14·25.

(4.) Divide 4·3173′ by ·075.

(5.) Divide 14·937′5′ by 1·788.

(6.) Divide 43·5′75′ by ·00456.

Examples to Prop. 2.

(7.) Divide 8·5968 by ·24′5′.

true quotient.

·24′5′)8·596800(35·024
2 85968

·243)8·510832 *new dividend.*

(8.) Divide 2·295 by ·2′97′.

quotient.

·2′97′)2·295000(7·7195′4′
2295

·297)2·292705 *new dividend.*

Rem. 162

(9.) Divide 47·345 by 1·7′59′.
(10.) Divide 35178 by 3·75′3′.
(11.) Divide 17·342 by ·4′5678′.
(12.) Divide ·37453 by 1·4′235′.

Examples to Prop. 3.

(13.) Divide 13·51′69533′ by 4·2′97′.

First, 4·2′97′=4·29′72′=4·29′72972′.

4·29′72972′
42

New divisor 4·2972930

13·5′169533′
135

13·5169398 *new dividend.*

true quotient.

4·2972930)13·5169398000(3·14′5′

Rem. 1953315

(14.) Divide ·4′5′ by ·1′18881′:

First, ·4′5′=·45′4545′

quotient.

·118881)·4545450000000(3·8235294, &c.

Rem. 13986

(15.) Div. ·4′75′ by ·37′53′.
Here ·4′75′=·47′54′
true quotient.
·37′53′)·47′54′(1·26′
3 4

·3750) ·4750 *new dividend.*

250 rem.

(16.) Div. 3·7′53′ by ·2′4′.
Here 3·7′53′=3·7′53753′
and ·2′4′=·2′42424′
·2′42424′)3·7′53753′(15·484, &c.
3

·242424)3·753750 *new dividend.*

56784 rem.

(17.) Divide ·357′2′ by 49·5′735′.
(18.) Divide ·1′54′ by ·5′.
(19.) Divide ·3′ by ·57′6′.
(20.) Divide 4·5′732′ by ·7′.

PRACTICE.

Definition.—*Practice* has its name from its daily use amongst merchants and tradesmen, being an easy and concise method of working most questions that occur in trade and business, and is only a contraction of the Rule of Three, when the first term is an unit.

A Table of the aliquot Parts of Money.

Of a Pound.						*Of a Shilling.*	
s.	*d.*	*£.*	*s.*	*d.*	*£.*	*d.*	
10	0	$= \frac{1}{2}$	1	3	$= \frac{1}{16}$	6	$= \frac{1}{2}$
6	8	$= \frac{1}{3}$	1	0	$= \frac{1}{20}$	4	$= \frac{1}{3}$
5	0	$= \frac{1}{4}$		10	$= \frac{1}{24}$	3	$= \frac{1}{4}$
4	0	$= \frac{1}{5}$		8	$= \frac{1}{30}$	2	$= \frac{1}{6}$
3	4	$= \frac{1}{6}$		$7\frac{1}{2}$	$= \frac{1}{32}$	$1\frac{1}{2}$	$= \frac{1}{8}$
2	6	$= \frac{1}{8}$		6	$= \frac{1}{40}$	1	$= \frac{1}{12}$
2	0	$= \frac{1}{10}$		5	$= \frac{1}{48}$	$\frac{3}{4}$	$= \frac{1}{16}$
1	8	$= \frac{1}{12}$		4	$= \frac{1}{60}$	$\frac{1}{2}$	$= \frac{1}{24}$
1	4	$= \frac{1}{15}$		$3\frac{3}{4}$	$= \frac{1}{64}$	$\frac{1}{4}$	$= \frac{1}{48}$

Rule 1. ***When the price is less than a penny.*** Multiply the given quantity by the number of farthings contained in the price, and the product will be farthings, which reduce to pence, shillings, and pounds.

Rule 2. ***When the price is an aliquot part of a shilling.*** Divide the quantity by the aliquot part, and that quotient by 20.

Rule 3. ***When the price is pence and farthings, and they no aliquot part of a shilling.*** Divide the given quantity by some aliquot part of a shilling, then consider what part of this aliquot part the rest is, and divide the quotient thereby; this quotient, added to the former, will be the answer in shillings, which divide by 20.

Rule 4. ***When the price is more than one shilling, but less than two.*** Let the given number stand for shillings, and work for the pence and farthings by the preceding rules.

Rule 5. ***When the price is any number of shillings less than*** 20. Multiply the quantity by half the price, double the first figure in the product for shillings, and the rest of the product will be pounds.

Rule 6. *When the price is shillings and pence.* If they are an aliquot part of a pound, divide the quantity by that aliquot part, and the quotient will be the answer. If they are not an aliquot part, multiply the quantity by the shillings, and take parts for the rest.

Rule 7. *When the price is pounds and shillings.* Multiply the quantity by the pounds, and proceed with the shillings as in the foregoing rules.

Rule 8. *When the price is pounds, shillings, pence, and farthings.* Multiply the quantity by the pounds, and work for the rest by the preceding rules.

Note. When the given quantity consists only of 1, 2, or 3, figures, proceed by the 1st, 2d, 3d, or 4th, Prop. of Compound Multiplication.

Rule. 9. *If there be a fraction in the given quantity,* work for the whole number by some of the preceding rules, and find the produce of the fraction by multiplying the price by the numerator, and dividing the product by the denominator; then add them together for the answer.

Note. If the price be pounds, shillings, and pence, or pounds, shillings, pence, and farthings, and if the quantity of things does not exceed 1000, proceed by the 5th Prop. in Compound Multiplication.

A Table of the aliquot Parts of Weights and Measures.

AVOIRDUPOIS WEIGHT.	
Of a Ton.	*Of $\frac{1}{2}$ Cwt. or 56lb.*
cwt.	*lb.*
10 = $\frac{1}{2}$	28 = $\frac{1}{2}$
5 = $\frac{1}{4}$	14 = $\frac{1}{4}$
4 = $\frac{1}{5}$	8 = $\frac{1}{7}$
$2\frac{1}{2}$ = $\frac{1}{8}$	7 = $\frac{1}{8}$
2 = $\frac{1}{10}$	*Of a $\frac{1}{4}$ Cwt. or 28lb.*
	lb.
	14 = $\frac{1}{2}$
	7 = $\frac{1}{4}$
Of a Cwt.	4 = $\frac{1}{7}$
	$3\frac{1}{2}$ = $\frac{1}{8}$
qr.	*Of a Pound.*
2 or 56lb. = $\frac{1}{2}$	*oz.*
1 or 28 = $\frac{1}{4}$	8 = $\frac{1}{2}$
16 = $\frac{1}{7}$	4 = $\frac{1}{4}$
14 = $\frac{1}{8}$	2 = $\frac{1}{8}$

Table continued.

TROY WEIGHT.

Of an Ounce.

dwt.	gr.		
10	0	=	$\frac{1}{2}$
6	16	=	$\frac{1}{3}$
5	0	=	$\frac{1}{4}$
4	0	=	$\frac{1}{5}$
3	8	=	$\frac{1}{6}$
2	12	=	$\frac{1}{8}$
2	0	=	$\frac{1}{10}$
1	16	=	$\frac{1}{12}$

Of a Dwt.

gr.		
12	=	$\frac{1}{2}$
8	=	$\frac{1}{3}$
6	=	$\frac{1}{4}$
4	=	$\frac{1}{6}$
3	=	$\frac{1}{8}$
2	=	$\frac{1}{12}$

LAND MEASURE.

Of an Acre.

r.	p.		
2	0	=	$\frac{1}{2}$
1	0	=	$\frac{1}{4}$
	32	=	$\frac{1}{5}$
	20	=	$\frac{1}{8}$
	16	=	$\frac{1}{10}$
	8	=	$\frac{1}{20}$

CLOTH MEASURE.

Of a Yard.

qr.	n.		
2	0	=	$\frac{1}{2}$
1	0	=	$\frac{1}{4}$
	2	=	$\frac{1}{8}$
	1	=	$\frac{1}{16}$

Of an English Ell.

qr.	n.		
2	2	=	$\frac{1}{2}$
1	1	=	$\frac{1}{4}$
1	0	=	$\frac{1}{5}$
	2	=	$\frac{1}{10}$
	1	=	$\frac{1}{20}$

Of a Flemish Ell.

qr.	n.		
1	2	=	$\frac{1}{2}$
1	0	=	$\frac{1}{3}$
	3	=	$\frac{1}{4}$
	2	=	$\frac{1}{6}$
	1	=	$\frac{1}{12}$

Of a French Ell.

qr.	n.		
3	0	=	$\frac{1}{2}$
2	0	=	$\frac{1}{3}$
1	2	=	$\frac{1}{4}$
1	0	=	$\frac{1}{6}$
	3	=	$\frac{1}{8}$
	2	=	$\frac{1}{12}$
	1	=	$\frac{1}{24}$

Rule 10. *When the given quantity is of several denominations.* Multiply the given price by the highest denomination as in Compound Multiplication, and take parts of the given price for the inferior denominations of the given quantity, and the sum will be the true value.

Examples to Rule I.

(1.) What cost 4715 yards of tape, at $\frac{1}{4}$*d.* per yard?

4|4715

12)1178$\frac{3}{4}$*d.*

20)98-2

£4 : 18 : 2$\frac{3}{4}$

(2.) 871 at $\frac{1}{4}$*d.*
(3.) 425 at $\frac{1}{2}$*d.*
(4.) 5714 at $\frac{3}{4}$*d.*

Examples to Rule II.

(5.) 425 yards at 1*d.*

1*d.*|$\frac{1}{12}$|425

2|0)3|5-5

£1 : 15 : 5

(6.) 3749 at 1*d.*
(7.) 496 at 1$\frac{1}{2}$*d.*
(8.) 3741 at 2*d.*
(9.) 574 at 3*d.*
(10.) 1749 at 4*d.*
(11.) 1731 at 6*d.*

Examples to Rule III.

(12.) 354 at 1$\frac{1}{4}$*d.*

1*d.*|$\frac{1}{12}$|354

$\frac{1}{4}$|$\frac{1}{4}$| 29 : 6
7 : 4$\frac{1}{2}$

2|0)3|6 : 10$\frac{1}{2}$

£1 : 16 : 10$\frac{1}{2}$

(13.) 5714 at 1$\frac{1}{4}$*d.*
(14.) 142 at 1$\frac{3}{4}$*d.*
(15.) 1749 at 2$\frac{1}{4}$*d.*
(16.) 134 at 2$\frac{1}{2}$*d.*
(17.) 5794 at 2$\frac{3}{4}$*d.*
(18.) 1749 at 3$\frac{1}{4}$*d.*
(19.) 574 at 3$\frac{1}{2}$*d.*
(20.) 1749 at 3$\frac{3}{4}$*d.*
(21.) 749 at 4$\frac{1}{4}$*d.*
(22.) 1749 at 4$\frac{1}{2}$*d.*
(23.) 3749 at 4$\frac{3}{4}$*d.*
(24.) 173 at 5*d.*
(25.) 146 at 5$\frac{1}{4}$*d.*
(26.) 3741 at 5$\frac{1}{2}$*d.*
(27.) 1493 at 5$\frac{3}{4}$*d.*
(28.) 749 at 6$\frac{1}{4}$*d.*
(29.) 1741 at 6$\frac{1}{2}$.
(30.) 349 at 6$\frac{3}{4}$*d.*
(31.) 547 at 7*d.*
(32.) 374 at 7$\frac{1}{4}$*d.*
(33.) 5491 at 7$\frac{1}{2}$*d.*
(34.) 1649 at 7$\frac{3}{4}$*d.*
(35.) 1498 at 8*d.*
(36.) 749 at 8$\frac{1}{4}$*d.*
(37.) 4719 at 8$\frac{1}{2}$*d.*
(38.) 1747 at 8$\frac{3}{4}$*d.*
(39.) 4954 at 9*d.*
(40.) 7143 at 9$\frac{1}{4}$*d.*
(41.) 494 at 9$\frac{1}{2}$*d.*
(42.) 374 at 9$\frac{3}{4}$*d.*
(43.) 471 at 10*d.*
(44.) 3751 at 10$\frac{1}{4}$*d.*
(45.) 4967 at 10$\frac{1}{2}$*d.*
(46.) 4971 at 11*d.*
(47.) 5794 at 11$\frac{3}{4}$*d.*

Examples to Rule IV.

(48.) 4756 at 12$\frac{1}{4}$*d.*

$\frac{1}{4}$|$\frac{1}{48}$|4756
99-1

2|0)485|5 : 1

£242 : 15 : 1

(49.) 321 at 12$\frac{1}{2}$*d.*
(50.) 479 at 12$\frac{3}{4}$*d.*
(51.) 574 at 13*d.*
(52.) 675 at 13$\frac{1}{4}$*d.*
(53.) 4949 at 13$\frac{1}{2}$*d.*

(54) 574 at 13¾*d.*
(55) 495 at 14*d.*
(56) 5714 at 14¼*d.*
(57) 371 at 14½*d.*
(58) 4714 at 14¾*d.*
(59) 3719 at 15*d.*
(60) 174 at 15¼.
(61) 4749 at 15½.
(62) 374 at 15¾.
(63) 498 at 16*d.*
(64) 3714 at 16¼*d.*
(65) 5714 at 16½*d.*
(66) 494 at 16¾.
(67) 3751 at 17*d.*
(68) 494 at 17¼*d.*
(69) 375 at 17½*d.*
(70) 5794 at 17¾*d.*
(71) 4954 at 18*d.*
(72) 371 at 18¼*d.*
(73) 579 at 18½*d.*
(74) 3751 at 18¾*d.*
(75) 479 at 19*d.*
(76) 371 at 19¼*d.*
(77) 471 at 19½*d.*
(78) 579 at 19¾*d.*
(79) 471 at 20*d.*
(80) 3741 at 20¼*d.*
(81) 494 at 20½*d.*
(82) 379 at 20¾*d.*
(83) 4981 at 21*d.*
(84) 375 at 21¼*d.*
(85) 3741 at 21½*d.*
(86) 495 at 21¾*d.*
(87) 5947 at 22*d.*
(88) 5931 at 22¼*d.*
(89) 482 at 22½*d.*
(90) 541 at 22¾*d.*
(91) 7194 at 23*d.*
(92) 5497 at 23¼*d.*
(93) 714 at 23½*d.*
(94) 4984 at 23¾*d.*
(95) 4935 at 23½*d.*

Examples to Rule V.

(96) 425 at 6*s.*
3
£.127|5
2
10*s.*

(97) 425 at 7*s.*
3½
212½
1275
£.148|7½
|2
15*s.*

(98) 475 at 2*s.*
(99) 379 at 3*s.*
(100) 1794 at 4*s.*
(101) 1788 at 5*s.*
(102) 1789 at 6*s.*
(103) 414 at 7*s.*
(104) 5413 at 8*s.*
(105) 7194 at 9*s.*
(106) 344 at 10*s.*
(107) 794 at 11*s.*
(108) 427 at 12*s.*
(109) 149 at 13*s.*
(110) 371 at 14*s.*
(111) 495 at 15*s.*
(112) 3741 at 16*s.*
(113) 794 at 17*s.*
(114) 494 at 18*s.*
(115) 371 at 19*s.*

Examples to Rule VI.

(116) 3754 pair of gloves at 2*s*. 6*d*. per pair.

2*s*. 6*d*. | $\frac{1}{8}$ | 3754

£.469 5

(117) 3520 bushels at 3*s*. 6*d*.

3

6*d*. | $\frac{1}{2}$ | 10560

1760

2|0)1232|0

£.616

(118) 660 at 2*s*. 6*d*.
(119) 663 at 4*s*. 10*d*.
(120) 924 at 13*s*. 4*d*.
(121) 712 at 6*s*. 8*d*.
(122) 512 at 7*s*. 6*d*.

(123) 1749 at 5*s*. 8*d*.
(124) 3741 at 4*s*. 6*d*.
(125) 493 at 3*s*. 2*d*.
(126) 741 at 5*s*. 9*d*.

Examples to Rule VII.

(127) 7341 at 2*l*. 6*s*.

7341

2

14682 value at 2*l*.

2202 6 value at 6*s*.

£.16884 6 answer.

(128) 435 at 2*l*. 7*s*.

2

870 value at 2*l*.

130 10 value at 6*s*.

21 15 value at 1*s*.

£.1022 5 answer.

(129) 754 at 4*l*. 2*s*.
(130) 371 at 5*l*. 3*s*.
(131) 149 at 9*l*. 4*s*.
(132) 374 at 10*l*. 5*s*.
(133) 191 at 12*l*. 6*s*.
(134) 174 at 3*l*. 7*s*.
(135) 512 at 5*l*. 8*s*.
(136) 140 at 7*l*. 9*s*.
(137) 360 at 2*l*. 10*s*.

(138) 344 at 2*l*. 11*s*.
(139) 192 at 3*l*. 12*s*.
(140) 351 at 4*l*. 13*s*.
(141) 412 at 5*l*. 14*s*.
(142) 372 at 2*l*. 15*s*.
(143) 741 at 1*l*. 16*s*.
(144) 314 at 1*l*. 17*s*.
(145) 471 at 1*l*. 18*s*.
(146) 374 at 19*l*. 19*s*.

Examples to Rule VIII.

(147) 4514 at 2*l*. 17*s*. 7$\frac{1}{2}$*d*.

2

9028 value at 2*l*.

3611 4 ditto at 16*s*.

225 14 ditto at 1*s*.

6 | $\frac{1}{2}$ | 112 17 ditto at 6*d*.

1$\frac{1}{2}$ | $\frac{1}{4}$ | 28 4 3 do. at 1$\frac{1}{2}$*d*.

£.13005 19 3 Answer.

(148) 471 at 5*l*. 14*s*. 9$\frac{1}{4}$*d*.
(149) 3714 at 2*l*. 13*s*. 11$\frac{3}{4}$*d*.
(150) 415 at 4*l*. 11*s*. 10$\frac{1}{2}$*d*.
(151) 341 at 5*l*. 13*s*. 9$\frac{1}{2}$*d*.
(152) 7494 at 10*l*. 17*s*. 10$\frac{1}{2}$*d*.
(153) 34121 at 11*l*. 14*s*. 8$\frac{3}{4}$*d*.
(154) 7251 at 14*l*. 11*s*. 5$\frac{1}{2}$*d*.

Examples to Rule IX.

(155) 3749⅜ at 3*l*. 15*s*. 6*d*.

3*l*.	15*s*.	6*d*.	the price
		3	
8)11	6	6	three times ditto.
1	8	3¾	3-8ths of ditto.

	3749			
	3			
	11247			
	2624	6		
6*d*. | ½ |	187	9		
	93	14		
⅛ add	1	8	3¾	
£.14153		17	9¼	Answ.

Note. ⅛ of 3 times the price is the same as 3 times ⅛ of the price, or ⅜, by the nature of fractions.

(156) 371⅕ at 3*l*. 14*s*. 7½*d*.
(157) 4917⅛ at 4*l*. 18*s*. 10½*d*.
(158) 1375$\frac{5}{9}$ at 2*l*. 19*s*. 11¾*d*.
(159) 4759$\frac{1}{11}$ at 4*l*. 15*s*. 9¾*d*.
(160) 574⅗ at 19*l*. 11*s*. 6*d*.
(161) 1749$\frac{1}{12}$ at 4*l*. 19*s*. 10¼*d*.

Examples to Rule X.

(162) What is the value of 18cwt. 1qr. 21lb. of tobacco, at 6*l*. 19*s*. 11*d*. per cwt.?

		£.	s.	d.	
1 qr. is ¼ cwt.		6	19	11	
				2	
		13	19	10	
				9	
		125	18	6	value of 18 cwt.
7 lb.	¼	1	14	11¾	value of ¼ cwt.
3½lb.	½		8	8¾	·75 ditto of 7lb.
½lb.	$\frac{1}{7}$		4	4¼	·875 ditto of 3½lb.
				7¼	·982$\frac{1}{7}$ ditto of ½lb.
		£.128	7	2½	·607$\frac{1}{7}$ answer.

(163) 19cwt. 3qr. 11lb. of hops, at 4*l*. 11*s*. 9*d*. per cwt.
(164) 19cwt. 3qr. 19lb. of sugar, at 2*l*. 4*s*. 8*d*. per cwt.
(165) 11cwt. 1qr. 16lb. of soap, at 3*l*. 7*s*. per cwt.
(166) 9cwt. 3qr. 10lb. of treacle, at 1*l*. 18*s*. 9*d*. per cwt.
(167) 9ton 13cwt. 3qr. 19lb. at 14*l*. 15*s*. 9*d*. per ton.
(168) 3qr. 19lb. 10oz. at 11*l*. 12*s*. 5½*d*. per cwt.
(169) 74oz. 2dwt. 12gr. of silver, at 4*s*. 11½*d*. per oz.
(170) A pair of chased silver salts, weight 7oz. 11dwt. at 8*s*. 11½*d*. per oz.

(171) 571oz. 14dwt. 16½gr. at 3*l.* 11*s.* 9¼*d.* per oz.

(172) What is the rent of 725a. 2r. 19p. of land, at 2*l.* 11*s.* 9*d.* per acre?

(173) 51a. 3r. 15p. at 4*l.* 10*s.* per acre.

(174) 97a. 14p. at 3*l.* 11*s.* 10*d.* per acre.

(175) 514yds. 3qrs. 2n. at 17*s.* 9½*d.* per yard.

(176) 125 ells Eng. 1qr. 1n. at 1*l.* 11*s.* 9½*d.* per ell.

(177) What cost 17 French ells 1qr. 3n. of Brussels lace, at 3*l.* 19*s.* 11¼*d.* per ell?

(178) 349 Flem. ells 1qr. 3n. of holland, at 1*l.* 11*s.* 6*d.* per ell.

(179) 475yds. 3qr. 2n. at 1*l.* 14*s.* 9¼*d.* per ell English.

(180) 375⅝ ells English, at 18*s.* 11¾*d.* per yard.

Note. If more examples be wanted, recourse may be had to the Bills of Parcels, Part III. Class II.

TARE AND TRET.

Definitions.

1. *Tare and Tret* are practical rules for deducting certain allowances made by merchants and tradesmen in selling their goods by weight.

2. *Gross Weight* is the whole weight of any sort of goods, together with the box, barrel, bag, &c. that contains it.

3. *Tare* is an allowance made to the buyer for the weight of the box, barrel, bag, chest, wrappers, &c.

4. *Tret* is an allowance of 4lb. in 104lb. for waste, dust, &c.

5. *Cloff*, or *draught*, is an allowance of 2lb. for every 3cwt. made by the seller to the buyer, that the weight may hold good, when sold by retail.

6. *Suttle* is when part of the allowance is deducted from the gross.

7. *Neat Weight* is what remains after all allowances are deducted.

Proposition 1. *When the tare is at so much in the whole gross weight, to find the neat weight.*

RULE.

Subtract the tare from the gross, and the remainder will be the neat weight.

Prop. 2. *When the tare is at so much per box, bag, barrel, &c. to find the neat weight.*

Rule. Multiply the number of boxes, bags, &c. by the tare, and subtract the product from the gross.

Prop. 3. *When the tare is at so much per hundred weight, to find the neat weight.*

Rule. If the tare be an aliquot part of an *cwt.* divide the gross weight by the aliquot part, and the quotient will be the tare to be deducted from the gross. If the tare be not an aliquot part of a *cwt.* first take some aliquot part of a *cwt.* and then part of *that* part, &c. according to the nature of the question, the sum of the quotients belonging to these parts will be the whole tare, which deduct from the gross.

Prop. 4. *The gross weight of any sort of merchandize given, to find the neat weight, when tret is allowed with tare.*

Rule. Find the tare, as before, and subtract it from the gross, the remainder will be the *suttle.* Then, divide the suttle by 26, and the quotient will be the tret, which deduct from the suttle.

Prop. 5. *The gross weight of any sort of merchandize given, to find the neat weight, when tare, tret, and cloff, are allowed.*

Rule. Find the neat weight by the last rule, and call that the *second* suttle. Then divide the second suttle by 168, and the quotient will be the cloff, which deduct from the second suttle.

Note 1. The above rule will only give the neat weight when cloff is 2lb. for every 3cwt., which is generally the case. The 168 is found by dividing 3cwt. or 336lb by 2; hence it will be very easy to find a divisor when any other allowance of cloff is made. At the Custom-house, the following allowances are made upon goods imported, viz. 1lb. upon goods not weighing less than 1cwt.—2lb. from 1 to 2cwt.—

3lb. from 2 to 3cwt.—4lb. from 3 to 10cwt.—7lb. from 10 to 18cwt.—and 9lb. from 18 to 30cwt., and upwards.

2. There are other allowances, such as *break*, which is sometimes at so much per hhd. bag, &c.; and *damage*, which is so much in the whole for any part of the merchandize which may have received injury.

Examples to Proposition 1.

(1.) What is the neat weight of 6 hhds. of tobacco, each weighing 12cwt. 3qr. 11lb. gross, tare in the whole 854lb.?

cwt.	qr.	lb.	
12	3	11	
		6	
77	0	10	whole gross weight.
7	2	14	tare.
69	1	24	neat weight.

28)854

4)30 14

7 2 14

(2.) Required the neat weight of 27 bales of silk, each weighing $349\frac{1}{2}$lb. gross; tare in the whole 3cwt. 1qr. 15lb.

(3.) Required the neat weight of 29hhds. of tobacco, each weighing 14cwt. 3qr. 17lb. gross; tare in the whole 1547lb.

(4.) In 43 bags of cotton, each weighing 3cwt. 1qr. $11\frac{1}{2}$lb. gross, tare in the whole $77\frac{1}{2}$lb. what is the neat weight?

(5.) What is the neat weight of 4hhds. of sugar weighing as follow, viz.

No.		cwt.	qr.	lb.		lb.
1	....	5	3	11	Tare	25
2	....	4	1	10	....	18
3	....	7	2	14	....	37
4	....	9	1	24	....	53

Examples to Prop. 2.

(6.) What is the neat weight of 12hhds. of tobacco, each weighing 5cwt. 3qr. 14lb. gross; tare per hhd. 97lb.?

cwt.	qr.	lb.	
5	3	14	
		12	
70	2	0	whole gross weight.
10	1	16	tare.
60	0	12	neat weight.

lb.	
97	
12	
28)1164	
4)41	16
10	16.

(7.) Required the neat weight of 19 casks of indigo, each weighing 4cwt. 1qr. 19lb. gross; tare per cask 37lb.

(8.) Required the neat weight of 47hhds. of tobacco, weighing 147cwt. 1qr. 11lb. gross; tare 75lb. per hhd.

(9.) In 19 bags of pepper, each 84¾lb. gross; tare per bag 4¼lb. how many lbs. neat?

(10.) In 75 bales of silk, each weighing 254lb. gross, tare per bale 14lb. how many lbs. neat?

(11.) What is the neat weight of 354 barrels of figs, each weighing 124lb. gross; tare 11lb. per barrel?

Examples to Prop. 3.

(12.) What is the neat weight of 7 barrels of figs, each weighing 2cwt. 1qr. 12lb. gross; tare 21lb. per cwt.?

		cwt.	qr.	lb.	
		2	1	12	
				7	
14lb.	$\frac{1}{8}$	16	2	0	gross.
7lb.	$\frac{1}{2}$	2	0	7	tare at 14lb. per cwt.
		1	0	3½	ditto at 7lb. per cwt.
		3	0	10½	whole tare.
		13	1	17½	neat weight.

(13.) Required the neat weight of 29 barrels of pot-ash, each weighing 1cwt. 3qr. 18lb. gross; tare 12lb. per cwt.

(14.) Required the neat weight of 15 casks of argol, weighing gross 97cwt. 2qr. 15lb. tare 15lb. per cwt.

(15.) Required the neat weight of 19 barrels of anchovies, each weighing 35lb. gross; tare 11½lb. per cwt.

(16.) Required the neat weight of 17hhds. of tobacco, each weighing 4cwt. 3qr. 14lb. gross; tare 19lb. per cwt.

Examples to Prop. 4.

(17.) In 7hhds of sugar, weighing gross 47cwt. 2qr. 4lb. tare in the whole 10cwt. 2qr. 14lb; tret 4lb. per 104, how much neat weight?

cwt.	qr.	lb.	
47	2	4	gross.
10	2	14	tare.
26)36	3	18	*suttle.*
1	1	19	tret.
35	1	27	neat.

(18.) How much neat weight is contained in 12cwt. 3qr. 19lb. gross; tare in the whole 37lb.; tret 4lb. per 104?

(19.) Required the neat weight of 19 chests of sugar, each weighing 7cwt. 3qr. 19lb. gross; tare 12lb. per cwt. tret 4lb. per 104.

(20.) Suppose 19½lb. per cwt. tare, and 4lb. per 104lb. tret, were allowed on 19 casks of prunes, each 4cwt. 1qr. 14lb. gross, what would be the neat weight?

Examples to Prop. 5.

(21.) Required the neat weight of 45hhds. of tobacco, weighing gross 224cwt. 3qr. 20lb.; tare 25cwt. 3qr.; tret 4lb. per 104lb.; cloff 2lb. for every 3cwt.

cwt.	qr.	lb.	
224	3	20	gross.
25	3	0	tare.
26)199	0	20	*suttle.*
7	2	18	tret.
168)191	2	2	*second* suttle.
1	0	15$\frac{12}{28}$	cloff.
190	1	14$\frac{2}{28}$	neat.

(22.) In 7hhds. of tobacco, each weighing gross 5cwt. 3qr. 17lb.; tare 11lb. per cwt. tret 4lb. per 104; cloff 2lb. for every 3cwt. how much neat weight?

(23.) The neat weight of 5 casks of currants is re-

quired, each weighing 7cwt. 3qr. 11lb. gross; tare 2qr. 11lb. per cask; tret 4lb. per 104lb., and cloff 2lb. per 336lb.

CLASS II. *exercising all the Propositions.*

(24.) Bought 19cwt. 1qr. 27lb. gross of tobacco in leaf, at 5*l.* 0*s.* 4*d.* per cwt. neat, and 12cwt. 3qr. 19lb. gross in rolls, at 5*l.* 17*s.* 8*d.* per cwt.; the tare of the former was 140lb., and the latter 48½lb.; what did the tobacco stand me in?

(25.) Bought 17¼hhds of sugar, each 10cwt. 1qr. 14lb.; tare 7lb. per cwt.; tret 4lb. per 104lb.; what is the value at 1*l.* 12½*s.* per cwt. neat?

(26.) Bought 7 hogsheads of treacle, each weighing 4cwt. 3qr. 17lb. gross; tare 17lb. per cwt.; break 8lb. per hhd.; and damage in the whole 99¼lb.; what is the value at 1*l.* 17*s.* 6*d.* per cwt. neat?

(27.) In 29 parcels, each weighing 3cwt. 3qr. 14lb. gross; tare 8lb. per cwt.; tret 4lb. per 104lb.; and cloff 2lb. per 3cwt.: how much neat weight, and what is the value at a guinea and a half per cwt. neat?

(28.) The neat value of a hhd. of Barbadoes sugar was 4*l.* 14*s.* 6*d.*; the custom and fees 2*l.* 11*s.* 4*d.*; freight 1*l.* 1*s.* 6*d.*; factorage 5*s.* 9*d.*; the gross weight was 11cwt. 1qr. 15lb.; tare 11½lb. per cwt.: pray what was the sugar rated at per cwt. neat in the bill of parcels?

(29.) In 7hhds. of oil, each weighing 3cwt. 2qr. 14lb. gross; tare 21lb. per cwt.; how many gallons neat, and what is the value at 5*s.* 4*d.* per gallon?

(30.) I have imported 87 jars of Lucca oil, each containing 57 gallons; what came the freight to at 5*s.* 3*d.* per cwt. neat, reckoning 1lb. in 11lb. for tare, and 7½lb. of oil to a gallon?

Note. More examples may be had by turning to Part III. Class III. of the Bills of Parcels, or to Class VI., entitled Invoices, Accounts of Sales, &c.

INTEREST.

Definition 1. *Interest* is the premium, or money, which one person allows to another for the use of any sum of money for a determinate space of time.

2. *The principal* is the money lent.

3. *The rate per cent.* is a certain sum, agreed on between the borrower and the lender, to be paid for the use of every £100 in the principal for a year. The *greatest* legal interest, in England, is £5 per cent.

4. *The amount* is the principal and its interest added together.

SIMPLE INTEREST.

Definition. Simple Interest is the money arising from the principal only, though such interest should remain unpaid for any number of years; thus, if the interest of £100 for 1 year be £4, it will be £8 for 2 years, &c., or £2 for half a year, £1 for a quarter of a year, &c.

Proposition 1. *To find the interest of any sum of money, having the principal, the time of its continuance in years, and the rate per cent., given.*

Rule. Multiply the principal by the rate *per cent.*, that product, divided by 100, will give the interest for one year. Then, if the interest for one year be multiplied by the number of years given in the question, the product will be the interest for *that* time. Or, multiply the principal by the rate per cent., and that product by the time; the last product, divided by 100, will give the interest required.

Note 1. If there be any parts annexed to the whole years, as $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, &c. after you have found the interest for the number of years, add $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, &c. of one year's interest to it.

2. If the rate of interest have any part or parts annexed to it, as $\frac{1}{4}$, or $\frac{3}{4}$, &c., after you have multiplied the principal by the whole number, take the respective part, or parts, of the principal, which add to the product, and proceed for the given time as above.

3. If the rate per cent. be an aliquot part of 100, or if it can be

divided into convenient aliquot parts, take the same part or parts, of the principal for the interest of one year.

Prop. 2. *To find the interest of any sum of money, having the principal, the time of its continuance in days, and the rate per cent. given.*

Rule. As 365 days are to the interest of the given sum for a year, so are the days given to the interest required.

Or, reduce the principal into the lowest denomination contained in it, then multiply it by the number of days, and *that* product by the rate *per cent.* for a dividend: let this dividend be divided by 36500 *, and the quotient will be the answer in the same denomination as the principal was reduced to.

Note. If the interest of a sum of money be required for any number of weeks, reduce them into days, and proceed as above; or, as 52 weeks are to the interest of the given sum for a year, so are the weeks given to the interest required, nearly.

Prop. 3. *To find the interest of any sum of money, having the principal; the time of its continuance in years, and months, or years, months and days; and the rate per cent. given.*

Rule. Find the interest for the years by the first rule, work for the months by the aliquot parts of a year, and for the days by the aliquot parts of a month, reckoning 12 months to a year, and 30 days to a month.

Note. Though the rule to Prop. 3, be not precisely accurate, yet it will be found not less useful than the others which are so; for, in some cases, it is customary to consider the time elapsed different ways. Thus, in the courts of law, interest is always calculated in years, quarters, and days; but, in calculating the interest on the public bonds of the South-sea and India Companies, and in the Bank of England, &c. the time is generally taken in calendar months and days; and on Exchequer bills in quarters of a year and days.

Prop. 4. *When the amount, time, and rate per cent. are given to find the principal.*

Rule. As the amount of £100, at the rate and for the time given, is to £100, so is the amount given to the principal.

* When the rate of interest is 5 per cent. reduce the principal into the lowest denomination contained in it, then divide by 7300, and the quotient will be the answer.

Prop. 5. *When the amount, principal, and time, are given to find the rate per cent.*

Rule. As the principal is to its interest, for the whole time, so is £100 to *its* interest for the same time; divide this interest by the time, and the quotient will be the rate *per cent.*

Prop. 6. *When the principal, rate per cent. and amount, are given to find the time.*

Rule. As the interest of the principal for one year, at the given rate, is to one year, so is the whole interest to the time required.

Examples to Proposition 1.

(1.) What is the interest of 357*l.* 10*s.* for 3 years, at 5 *per cent. per annum.*

357*l.* 10*s.* principal.
5 rate per cent.

£.17,87 10 — d.
20 — 17 17 6
3

s. 17,50 — 53 12 6 *Ans.*
12

d. 6,00

Or thus,
5*l.* is $\frac{1}{20}$ | 357*l.* 10*s.*

Interest for 1 year £.17 17 6
3

Interest for 3 years £.53 12 6

Or thus,
£.357 10 principal.
5 rate per cent.

1787 10
3 time.

£.53,62 10
20

s. 12,50
12

d. 6,00

Answ. £.53 12 6

(2.) Required the interest of 349*l.* 10*s.* for 7 years, at 4 per cent. per annum.

(3.) Required the interest of 429*l.* 11*s.* 6*d.* for 6 years, at 5 per cent. per annum.

(4.) What is the interest of 625*l.* 15*s.* for 3½ years, at 4 per cent. per annum?

(5.) What is the interest of 494*l.* 13*s.* 9*d.* for 5¾ years, at 5 per cent. per annum?

(6.) Required the interest of 700 guineas, for 9 years, at 4½ per cent. per annum.

(7.) Required the interest of 420*l.* for $7\frac{1}{4}$ years, at $3\frac{1}{2}$ per cent. per annum.

(8.) Required the interest of 500*l.* 15*s.* for $5\frac{1}{2}$ years, at $4\frac{1}{4}$ per cent. per annum.

(9.) Required the interest of 97*l.* 18*s.* 6*d.* for $3\frac{1}{4}$ years, at $4\frac{3}{4}$ per cent. per annum.

Examples to Prop. 2.

(10.) Required the interest of 357*l.* 10*s.* for 65 days, at 5 per cent. per annum.

The interest for 1 year, by the first example, is 17*l.* 17*s.* 6*d.*
Then, 365 days : 17*l.* 17*s.* 6*d.* :: 65 days : 3*l.* 3*s.* $7\frac{3}{4}d.\frac{65}{73}$

Or thus,

The principal reduced to the lowest term mentioned in it, is 7150 sh. which multiply by 5, the rate per cent. and then by 65, the number of days, and the last product will be 2323750 sh. for a dividend, which divide by 36500, after the manner of compound division, and the quotient will be 63*s.* $7\frac{3}{4}d.\frac{65}{73}$, or 3*l.* 3*s.* $7\frac{3}{4}d.\frac{65}{73}$. Or, multiply 7150, the shillings in the principal, by 65, and divide the product (464750) by 7300, as in compound division, the quotient will be 63*s.* $7\frac{3}{4}d.\frac{65}{73}$. *Answer.*

(11.) Required the interest of 194*l.* 11*s.* 6*d.* for 315 days, at $4\frac{1}{2}$ per cent. per annum.

(12.) What is the interest of 700*l.* for 149 days, at $4\frac{3}{4}$ per cent. per annum?

(13.) Required the interest of 494*l.* 12*s.* 10*d.* for 29 weeks, at 5 per cent, per annum.

(14.) Required the interest of 347*l.* 10*s.* for 18 weeks, at 4 per cent. per annum.

(15.) Required the interest of 540*l.* 10*s.* from Jan. 1, 1821, to Sept. 22, in the same year, at 4 per cent. per annum.

(16.) What is the interest due on an *Exchequer* bill of 400*l.* value, at $3\frac{1}{2}$ per cent. per annum, for $2\frac{1}{2}$ years, and 59 days?

(17.) Required the interest due upon an *Exchequer* bill of 100*l.* value, for 294 days, reckoning the interest at 3*d.* per day.

Examples to Prop. 3.

(18.) Required the interest of 342*l.* 10*s.* for 8 years, 4 months, and 15 days, at 4 per cent. per annum.

342*l*. 10*s*.	4m.	$\frac{1}{3}$	13*l*. 14*s*. interest for 1 year.	
4			3	
13|70 0			41 2	interest for 3 years.
20	15d.	$\frac{1}{8}$	4 11 4	interest for 4 months.
			11 5	interest for 15 days.
14|00			46 4 9	*Answer*.

(19.) Required the interest of 500 guineas for 5 years, 9 months, and 27 days, at $4\frac{3}{4}$ per cent. per annum.

(20.) What is the interest due upon an India bond of 500*l*. value at $3\frac{3}{4}$ per cent. per annum, from May 15, 1819, to September 22, 1821?

(21.) Sold an India bond of 100*l*. value, with interest due thereon, for 2 months, 17 days, at 4 per cent. per annum, premium 10*s*. what is its value?

(22.) A gentleman left his daughter, by will, 875*l*. 10*s*. to be paid her when she is 21 years of age, with interest at 5 per cent. per annum. Now she was 18y. 7m. 3d. at her father's decease, reckoning 12 months to a year, and 30 days to a month. Pray what will be the amount of her fortune when she comes of age?

Examples to Prop. 4.

(23.) What principal, put to interest for seven years at 5 per cent. per annum, will amount to 465*l*. 8*s*. 3*d*.?

5*l*. interest of 100*l*. for 1 year.
7 time.

35 interest of 100*l*. for 7 years.
100

135 amount of 100*l*. at 5 per cent. per annum, for 7 years.

135*l*. : 100*l*. : : 465*l*. 8*s*. 3*d*. : 344*l*. 15*s*., *answer*.

(24.) What principal, put to interest for 5 years, will amount to 570*l*. 16*s*. 6*d*. at 4 per cent. per annum?

(25.) What principal, put to interest for $3\frac{1}{4}$ years, at $4\frac{1}{2}$ per cent. per annum, will amount to 205*l*. 11*s*. $7\frac{1}{4}d.\frac{3}{4}$?

(26.) What principal, put to interest for $4\frac{2}{3}$ years, will amount to 350*l*. 12*s*. 6*d*. at $3\frac{1}{4}$ per cent. per annum?

Examples to Prop. 5.

(27.) At what rate per cent. will 475*l.* 13*s.* 9*d.* amount to 570*l.* 16*s.* 6*d.* in 5 years time?

570*l.*	16*s.*	6*d.*	amount.
475	13	9	principal.
95	2	9	interest.

475*l.* 13*s.* 9*d.* : 95*l.* 2*s.* 9*d.* :: 100*l.* : 20*l.*

This 20*l.* divided by 5, the number of years, gives 4*l.* the rate per cent.

(28.) At what rate per cent, will 344*l.* 15*s.* amount to 465*l.* 8*s.* 3*d.* in 7 years time?

(29.) At what rate per cent. will 175*l.* 18*s.* amount to 205*l.* 11*s.* $7\frac{3}{4}d.\frac{4}{5}$ in $3\frac{1}{2}$ years?

(30.) At what rate per cent. will 300*l.* amount to 350*l.* 12*s.* 6*d.* in $4\frac{1}{2}$ years?

Examples to Prop. 6.

(31.) In what time will 344*l.* 15*s.* amount to 465*l.* 8*s.* 3*d.* at 5 per cent. per annum?

344*l.* 15*s.* principal.
 5 rate per cent.

£17|23 15
 20

s. 4|75
 12

d. 9|00

465*l.*	8*s.*	3*d.*	amount.
344	15	0	principal.
120	13	3	whole interest.

17*l.* 4*s.* 9*d.* : 1 year :: 120*l.* 13*s.* 3*d.* : 7 years, answer.

(32.) In what time will 475*l.* 13*s.* 9*d.* amount to 570*l.* 16*s.* 6*d.* at 4 per cent. per annum?

(33.) In what time will 175*l.* 18*s.* amount to 205*l.* 11*s.* $7\frac{3}{4}d.\frac{4}{5}$, at $4\frac{1}{2}$ per cent. per annum?

(34.) In what time will 300*l.* amount to 350*l.* 12*s.* 6*d.* at $3\frac{3}{4}$ per cent. per annum?

CLASS II. *Promiscuous Examples.*

(35.) A young gentleman, whose father has been dead 12 years, is informed by his guardians that after paying all the just debts of his father, there remained the net sum

of 17466*l.* 5*s.* for which they have allowed him 5 per cent. simple interest, except 100*l.* which was deducted annually for his education; if the gentleman be now 21 years of age, pray what is the amount of his fortune?

(36.) Lent my friend 20*l.* October 20, 1813; on the 22d of May, 1815, I borrowed of him 150*l.* and on July 30, in the same year, 150*l.* more. On July 21, 1816, I paid him 15*l.* 18*s.*—on August 21, 40*l.*—on October 21, 50*l.*—on February 13, 1817, I paid 9l. 12*s.*—on June 18, 111*l.*—and on January 13, 1818, 80*l.* How stood our account at that period, allowing 5 per cent. simple interest for the money?

(37.) Lent 500 guineas at 4½ per cent. per annum, which by the 25th of September, 1818, was raised by the interest to 700*l.* 15*s.* Pray on what day and in what year did I lend the money?

(38.) If 100*l.* in 11 years gain 38*l.* 10*s.* in what time would any other sum gain as much interest as will make its amount 5 times the principal?

(39.) What difference is there between the interest of 500*l.* for 4¾ years, at 5 per cent. and half that sum for twice the time, at half the same rate per cent?

(40.) Lent *Hilton Morrison* per bill, (dated August 1, 1819) payable 2 months after date, 957*l.* 18*s.* which I received as follows, *viz.* October 5, 94*l.* 17*s.* November 27, 47*l.* 19*s.* 6*d.* December 15, 100 guineas, January 1, 1820, 55*l.* 11*s.* 4*d.*; March 15, 101*l.* 14*s.*; May 12, 105 guineas; August 19, 140*l.* 2*s.* 6*d.*; Septemper 11, 50*l.* 6*d.* and on March 15, 1821, I received the balance of the principal. Pray what interest ought I to claim at 4 per cent.?

BROKERAGE.

Definition.—*Brokerage* is an allowance of so much *per cent.* made to persons called Brokers; who, from their knowledge of merchants and the different branches of commerce, are generally employed in buying or selling goods for others.

Proposition 1. *To find what allowance must be made*

to a broker for buying or selling goods, having the rate per cent. and value of the goods, &c. given.

Rule 1. Divide the given sum by 100, and take parts from the quotient with the rate *per cent.*

Or, 2. Divide the given sum by the aliquot parts of a pound contained in the rate per cent. The result divided by 100 will give the brokerage.

Or, 3. As £100 is to the rate per cent. so is the given sum to the brokerage.

Note. The allowances made to brokers are generally at 2s. or 2s. 6d. per cent.; but, should the brokerage so far accumulate, from repeated negociations, as to exceed 20s. per cent., it must be calculated by the following rule of commission, or by the second rule given above.

Examples.

(1.) Suppose I employ a broker to sell goods for me to the amount of 715*l.* 15*s.* what is his allowance at 3*s.* 9*d.* per cent.?

By Rule I.

```
         l.  s.
     l. 7|15 15
           20
     s. 3|15
           12
     d. 1|80
            4
     f. 3|20

          l.  s.  d.
2 6 | 1/8 | 7   3   1¼ ·2
1 3 | 1/2 |    17  10¾ ·9
                8  11¼ ·45
      l. 1   6  10    ·35 answer.
```

By Rule II.

```
              l.   s.
2 6 | 1/8 | 715  15
1 3 | 1/2 |  89   9   4½
             44  14   8¼
      l. 1|34     4   0¾
            20
      s. 6|84
            12
      d. 10|08
             4
            |35
```

By Rule III.

100*l.* : 3*s.* 9*d.* :: 717*l.* 15*s.* : 1*l.* 6*s.* 10*d.* $\frac{7}{20}$ answer.

(2.) When a broker sells goods to the amount of 7134*l.* 15*s.* 10*d.* what may he demand for brokerage, if he be allowed 5*s.* 9*d.* per cent.?

(3.) Suppose I employ a broker to sell goods for me to the amount of 1057*l.* 17*s.* what may he demand for brokerage, if I allow him 4*s.* 7*d.* per cent.?

(4.) What is the brokerage of 3759*l.* 17*s.* 6*d.* at 19*s.* 9$\frac{1}{4}$*d.* per cent.?

CLASS II.

(5.) If a broker sells goods to the value of 750*l.* 19*s.* at an allowance of $\frac{5}{8}$*l.* per cent. how much is due to him?

(6.) Required the brokerage of 2947*l.* 15*s.* 6*d.* at $\frac{3}{4}$*l.* per cent.

COMMISSION.

Definition.—Commission is an allowance made by merchants to their factors, or agents, in foreign countries, for buying or selling goods, and is generally at a certain rate *per cent.* according to the custom of the country where the factors reside.

Proposition. To find what allowance must be made to a factor at any rate per cent. having the sum given, from which his commission is to be taken.

Rule 1. Multiply the sum by the rate per cent.; the product, divided by 100, will give the commission.

Or 2. As £100 is to the rate per cent. so is the given sum to the commission.

Note. If the rate per cent. be less than 20*s.* proceed by the rules for brokerage, or by the second rule given above.

Examples.

(1.) If I empower my factor to purchase goods for me to the amount of 500*l.* 14*s.* what does his commission come to at 2$\frac{1}{2}$ per cent.?

	500*l.* 14*s.*	
	$2\frac{1}{2}$	
	1001	8
	250	7
l.	12|51	15
	20	
s.	10|35	
	12	
d.	4|20	

Answ. 12*l.* 10*s.* $4\frac{1}{5}$*d.*

Or thus,

£. $2\frac{1}{2}$ | $\frac{1}{20}$ | 500*l.* 14*s.*

£. 12 10 $4\frac{1}{5}$

Or,

100*l.* : 2*l.* 10*s.* :: 500*l.* 14*s.* : 12*l.* 10*s.* $4\frac{1}{5}$*d.* answer.

(2.) My factor informs me, that he has bought goods, on my account, to the amount of 757*l.* 14*s.* what comes his commission to at $3\frac{3}{4}$*l.* per cent.?

(3.) My factor informs me, that he has sold goods, on my account, to the amount of 500*l.* 17*s.* what comes his commission to at $1\frac{3}{8}$ per cent.?

(4.) Consigned goods to my factor, as per invoice, to the amount of 1175*l.* 14*s.* what does his commission come to at $4\frac{3}{8}$ per cent.?

CLASS II.

(5.) If I allow my factor $7\frac{5}{8}$ per cent. for commission, what may he demand for purchasing goods for me to the amount of 977*l.* 18*s.*?

(6.) What does the commission of 7497*l.* 15*s.* come to at $12\frac{7}{8}$ per cent.?

INSURANCE.

Definition. *Insurance* is a security given in consideration of a premium of so much per cent. paid down by the proprietors of goods, &c. to the insurers, whereby they engage to answer for the loss or damage of ships, houses, goods, &c. by storms, fires, or other accidents.

Proposition 1. *To find what premium must be given for an Insurance of property, to any amount at any rate, per cent.*

Rule 1. Multiply the value of the property by the rate *per cent.* the product, divided by 100, will give the pre-

mium to be paid down. If the rate *per cent.* be less than 20*s.* divide the value of the property by 100, and take parts from the quotient with the rate *per cent.*

Or, II. As £100 is to the rate per cent. so is the given sum to the premium.

Prop. 2. *To find what sum ought to be insured, to recover the value of the property, and all expences attending the insurance.*

Rule. Add the premium per cent., the brokerage, the policy, and other incidental charges together, and deduct the sum from £100.

Then, as the remainder is to £100, so is the value of the property to the sum which ought to be insured, in order to recover the value and the expenses incurred.

Examples to Proposition 1.

(1.) What premium must be paid for an insurance of goods to the amount of 500*l.* 14*s.* at $2\frac{1}{2}$ per cent.?

Answer 12*l.* 10*s.* $4\frac{1}{5}$*d.*

This example is the same as the first in Commission, and must be worked in the same manner.

(2.) What premium must be paid for insuring goods to the amount of 715*l.* 15*s.* at 3*s.* 9*d.* per cent.?

Answer 1*l.* 6*s.* 10*d.*

This example is the same as the first in Brokerage, and must be worked in the same manner.

(3.) What premium must be given as a pledge for the insurance of an East-India ship and cargo, valued at 475751*l.* 18*s.* when the rate of insurance is $17\frac{1}{4}$ per cent.?

(4.) Shipped off goods for *Jamaica* to the value of 4794*l.* 18*s.* when the rate of insurance was $11\frac{5}{8}$ per cent., what premium must be paid in London for an insurance to recover the same value in case of failure of the voyage?

CLASS II.

(5.) When the insurance of goods to a certain port is $16\frac{1}{4}$ per cent. what premium must be given as a pledge for the security of goods to the amount of 7000 guineas?

(6.) Suppose I insure goods to the amount of 300*l.* 18*s.* what premium must I pay at the rate of 2*s.* 6*d.* per cent.?

(7.) My factor at *Barbadoes* consigns goods to me, amounting to the value of 579*l.* 15*s.* 6*d.* what premium must I pay for an insurance of those goods at $11\frac{3}{8}$ per cent.?

Examples to Prop. 2.

(8.) Suppose I want to insure goods worth 600*l.* at a premium of 8*l.* per cent., and that the stamp for the policy cost 7*s.* 6*d.* per cent., brokerage ½ per cent., and other incidental charges 1*l.* 10*s.*; what sum ought I to insure for, to recover the value of my property and all the expences attendant thereon?

	£	s.	d.
Premium....	£8	0	0
Policy......	0	7	6
Brokerage...	0	10	0
Charges, &c..	1	10	0
	£10	7	6

£	s.	d.
£100	0	0
10	7	6
£89	12	6

89*l.* 12*s.* 6*d.* : 100*l.* :: 600*l.* : 669*l.* 9*s.* $1\frac{1}{4}$*d.* answer.

(9.) If my expences per cent. be 7*l.* 10*s.* premium, policy 5*s.* brokerage 25*s.* and other charges 27*s.* what sum ought I to ensure to recover all the expenses and the value of the property, supposing that property to be worth 20,000*l.*?

PURCHASING OF STOCK.

Definition. *Stock* is a general name for the capitals of our trading companies, and the money borrowed by government, at so much per cent. to defray the expenses of the nation.

Proposition 1. *To ascertain the value of any quantity of stock at any given rate per cent.*

Rule. If the current price of the stock to be transferred be under par, viz. less than £100, multiply the stock by the rate *per cent.*, the product, divided by 100, will give the purchase. If the price of the stock be above par, multiply the quantity to be transferred by

such part of the rate *per cent.*, as exceeds 100; divide this product by 100 as before, to which add the given stock for the whole purchase.

Or, As £100 stock is to the rate *per cent.*, or current price, so is the stock to be transferred to its current value.

☞ The broker is always allowed 2*s.* 6*d.* or ⅛ per cent. on the capital for buying and selling.

Prop. 2. *Any sum of money being given, to find how much stock that sum will purchase.*

Rule. As the rate per cent. or current price of £100 stock, is to £100; so is the given sum to the quantity of stock it will purchase.

Prop. 3. *Given the current price of a nominal* £100. *and the rate of interest upon it; to find the interest upon a real* £100.

As the current price of £100, is to the rate of interest it bears; so is £100, to the rate per cent.

Note. The principal trading companies in England are the East-India and the South-sea companies. Every capital stock, or fund, of a company is raised for some particular purpose, and limited by parliament to a certain sum; it therefore follows, that, when that sum is completed, no stock can be bought of the company; yet the shares already purchased may be transferred from one person to another. The government annuities, and other securities of money, which have at any time been raised by the authority of parliament for the public service, are to be considered as national debts, contracted on the credit of some certain tax; various interests for which debts are half-yearly paid to the different stock-holders from the produce of the taxes; and must continue to be so paid till these debts are redeemed or paid off, by the same authority by which they were contracted. This plan of raising money for the exigencies of the state, commenced soon after the Revolution in 1688, and is the easiest and best method of raising money, both for the subject and the state, when managed with economy and prudence. We know, from experience, that taxes, laid on such articles as could well support the weight of them, have produced considerable surplusses; that is, they have amounted to more than the absolute security engaged for; hence foreigners as well as natives have been induced to advance their money on so safe a foundation. These surplusses, after payment of the interest they stand charged with, are carried to a seperate and distinct account, known by the name of the *Sinking Fund.* This fund was to be kept *most sacredly* for the valuable purpose of lessening, or *sinking*, and paying off gradually, the national debt, by an act of George I. anno 1716; and had not that act been rendered ineffectual by subsequent acts, the national debt, and consequently the taxes raised to pay off its interest, could never have amounted to that enormous height, which

we now find them. The prices of stocks are continually fluctuating above and below *par;* for, if there be more buyers than sellers, a person, who is indifferent about selling, will not part with his share without a considerable profit; and, on the contrary, if many are disposed to sell, and few inclined to buy, the value of such shares will naturally fall. So, when a person, who is unacquainted with transactions of this nature, reads in the papers the price of stocks, where Bank stock is marked perhaps $278\frac{1}{2}$, India stock $232\frac{1}{4}$, South-sea stock $87\frac{1}{4}$-$\frac{3}{8}$, he is to understand that £100, of those respective stocks sells at such a time for those several sums. In comparing the prices of the different stocks one with another, it must be remembered, that the interest due on them from the last payment is taken into the current price, and the seller never receives any separate consideration from it, except in the case of India bonds, where the interest due is calculated to the day of sale, and paid by the purchaser over and above the premium agreed for. But, as the interest on the different stocks is paid at different times, this, if not rightly understood, would lead a person into considerable mistakes in his calculation of their value; some always having a quarter's interest due on them more than others, which makes an appearance of a considerable difference in their price, when, in reality there is none; thus, for instance, on the 8th of July, 1818, the 3 per cents. reduced sold for $78\frac{1}{4}$, and the 3 per cent. consols. for only $77\frac{1}{2}$, though each of them produce the *same* annual sum of 3 per cent.; but the 3 per cents. reduced had a quarter's more interest due on them than the 3 per cent. consols., which amounts to 15*s.* the exact difference.

Examples to Proposition 1.

(1.) What must be given for 750*l.* 16*s.* in the 3 per cent. annuities, when $64\frac{1}{8}$ will buy 100*l.*?

|⅛| 750*l.* 16*s.*
8

6006 8
8

48051 4
93 17

l. 481|45 1
20

s. 9|01
Answ. 481*l.* $9\frac{1}{100}$*s.*

Or thus,

100*l.* : $64\frac{1}{8}$*l.* :: 750*l.* 16*s.* : 481*l.* $9\frac{1}{100}$*s.*

(2.) What is the purchase of 540*l.* 16*s.* Bank-stock, at $112\frac{5}{8}$ per cent.?

Here the rate exceeds 100*l.* by $12\frac{5}{8}$.

540*l.* 16*s.*
$12\frac{5}{8}$

6489 12
$\frac{5}{8}$=338 0

l. 68|27 12
20

s. 5|52

d. 6|24=$\frac{6}{25}$.

l. 68 5 $6\frac{6}{25}$.
540 16 0 add.

609 1 $6\frac{6}{25}$ answ.

Or thus,

100*l.* : $112\frac{5}{8}$*l.* :: 540*l.* 16*s.* : 609*l.* 1*s.* $6\frac{6}{25}$*d.*

(3.) What is the purchase of 7575*l.* 15*s.* Bank, at $125\frac{5}{8}$ per cent.?

(4.) Required the purchase of 900*l.* South-Sea Stock, at $89\frac{1}{4}$ per cent.

(5.) What must be given for 1759*l.* 18*s.* 9*d.* India stock, when $196\frac{1}{4}$*l.* will purchase 100*l.*?

Examples to Prop. 2.

(6.) Suppose I have 5000*l.* what nominal sum will that purchase in the 3 per cents. at $78\frac{1}{4}$ per cent.?

$78\frac{1}{4}l. : 100l. :: 5000l. : 6389l.\ 15s.\ 6\frac{1}{4}d.\frac{25}{313}$ *Answ.*

(7.) A person has 700*l.* by him, what sum will that purchase in the Irish 5 per cents. at $96\frac{1}{4}$ per cent.?

(8.) How much stock in the 5 per cents. will 450*l.* purchase, when $104\frac{5}{8}$*l.* will buy 100*l.*?

Examples to Prop. 3.

(9.) Suppose a person purchase in the 3 per cents. at $78\frac{1}{4}$*l.* what interest per cent. does he get for his money?

$78\frac{1}{4}l. : 3l. :: 100l. : 3l.\ 16s.\ 8d.\frac{40}{313}$. *Answ.*

(10.) A person purchased in the Irish 5 per cents. at $96\frac{3}{8}$*l.* what interest per cent. did he make of his money?

(11.) When the 5 per cents. are at $104\frac{3}{4}$*l.* what does a purchaser make per cent. of his money?

CLASS II.

(12.) Bought 5000*l.* capital stock in the 3 per cent. consolidated annuities, and paid brokerage $\frac{1}{8}$ per cent. on the capital, what was the purchase at $85\frac{1}{8}$ per cent?

(13.) What is the value of 759*l.* 10*s.* South-sea Old annuities, at $64\frac{1}{4}$ per cent. brokerage $\frac{1}{8}$ per cent.?

(14.) Suppose when Bank-stock, which bears 7 per cent. interest, due on the 5th of April, sells for 154*l.* per cent. India stock, bearing $10\frac{1}{2}$ per cent. interest, payable Jan. 5, sells for 231*l.* which stock will produce me the greater interest for my money, and what will that interest be, supposing that I purchase in April after the dividend on the Bank-stock has been received *?

* In an *Epitome of the Stocks and Public Funds,* a table is published called an Equation Table, shewing in which fund it will be the most advantageous to purchase. The example above is taken from this table, where the author shews that when Bank stock sells for 154l. per cent. India

(15.) Suppose that the 3 per cents. consols sell for $70\frac{7}{8}l.$ per cent. when the 3 per cents. reduced sell for $71\frac{1}{2}$ per cent.; for instance, on January 20, which fund will be the most advantageous to purchase in, the interest on the consols being due the 5th January, and on the 3 per cents. reduced on the 5th of April?

(16.) If I buy 10,000*l.* capital in the India stock, in January, immediately after the dividends have been received at $209\frac{1}{2}l.$ per cent. what will it cost me, allowing the broker $\frac{1}{8}$ per cent. on the capital for buying? and what do I make per cent. of my money, India stock bearing $10\frac{1}{2}$ per cent.

(17.) Suppose I have 600*l.* what nominal sum, in the Navy 5 per cents. will that purchase, at $103\frac{3}{8}l.$ per cent. allowing the broker $\frac{1}{8}$ per cent. on the capital, or sum purchased?

(18.) On June 8th, 1818, I sold out 1000*l.* consols. at $77\frac{1}{2}$, and, with the sum received, purchased in the Navy 5 per cents. at 106; what is my annual gain, in point of interest, my broker being allowed $\frac{1}{8}$ per cent. on the capital, in each transaction?

(19.) Which is the most advantageous, with respect to annual income, land bought at 25 years purchase *, or Bank-stock bought at 168*l.* per cent. the Bank-stock bearing 7 per cent. interest?

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stock should sell for 231l. per cent. and that each will produce 4l. 10s. 10d. per cent. interest, and therefore they are equally advantageous. Now this would be true, were the interests payable at the same time; but as that is not the case, the whole table, and all similar tables appear to be founded on error, and can tend only to mislead the public. To illustrate this remark—in the example before us, the India stock has a quarter's interest due on it at the time of purchasing, and therefore, in point of interest, is preferable to the Bank-stock; and had the purchase been made in February at the same rate, the Bank stock would have had the advantage.

* Divide 100l. by the rate per cent., and the quotient will give the number of years purchase; that is, the number of years in which an estate will bring in the purchase-money: divide 100l. by the number of years purchase, and the quotient will give the rate per cent.

DISCOUNT.

Definition. *Discount*, or *Rebate*, is an allowance made for the payment of any sum of money before it becomes due: and the present worth of any sum, or debt, is such a sum as, if put to interest for the time, and at the rate for which the discount is to be made, would amount to the sum, or debt, due.

Proposition. Any sum, due some time hence, being given to find its present value to the creditor, discounting at any rate per cent.

Rule. As the amount of 100*l.* for the given rate and time, is to 100*l.* so is the given sum to its present worth. The difference between the given sum and its present value will give the discount.

Or, as the amount of 100*l.* for the given rate and time, is to the interest of 100*l.* for *that* time, so is the given sum to the discount. The difference between the given sum and its discount will give the present value.

Note. The preceding rule is built upon this basis, viz. that the *present worth* of any sum of money, due some time hence, put to interest for the time, for which the discount is to be made, should amount to the sum, or debt, due: and that the *discount*, put to interest for the same time, should amount to the interest of the sum due for that time.

2. Thus, the *present worth* of 100l. due one year hence, discounting at the rate of 5 per cent., is 95l. 4s. 9$\frac{1}{7}$d., and the *discount* of 100l. for one year, at the rate of 5 per cent., is 4l. 15s. 2$\frac{6}{7}$d., according to the rule. Now, if the creditor should put the present money allowed him (viz. 95l. 4s. 9$\frac{1}{7}$d.) to interest, at the rate of 5 per cent. for one year, it will amount to 100l. exactly, and therefore he is not injured: again, if the debtor puts the discount allowed *him* (viz. 4l. 15s. 2$\frac{6}{7}$d.) to interest, at the rate of 5 per cent. for one year, it will amount to 5l., the exact sum which he might have made of the 100l. had he kept it in his hands till it became due.

3. When goods are sold to any amount, payable at different times, at the same or different rates per cent., calculate the present worth of each payment separately, as a debt independent of the other payments, and the sum of these will be the present value of the goods to the seller.

4. It is customary with bankers and merchants, in discounting bills, to calculate the interest of the sum drawn for in the bill, from the time of their discounting it to the time it becomes due, including three days of grace; by this practice they make the discount more than it ought to be.

The customary Rule for Discount.

Find the interest of the sum to be discounted at 5 per cent. from the day on which it is discounted to the day on which it becomes due, including 3 days beyond that date, upon a bill, and this interest will be the discount. Subtract this interest from the sum to be discounted, and the remainder will be the present worth.

Or, for each pound sterling, reckon *one penny* per calendar month, when the discount is at 5 per cent.

5. Thus, the discount, upon a bill of 15,000l., due 57 days after date, is 123l. 5s. 9$\frac{3}{73}$d., being the interest of 15,000l. for (57 + 3 days of grace) 60 days. See Prop. 2, page 131.

6. When goods are bought or sold on which discount is to be made for present payment at any rate per cent. if no time be specified, the interest of the value of the goods for a year is the discount.

Examples.

(1.) What are the present worth and discount of 550*l.* 10*s.* for 9 months, at 5 per cent. per annum?

6m.	$\frac{1}{2}$	£5		interest of 100l. for 1 year.
3m.	$\frac{1}{2}$	2	10	ditto for $\frac{1}{2}$ year.
		1	5	ditto for $\frac{1}{4}$ year.
		3	15	ditto for $\frac{3}{4}$ year.
		100	0	
		£103	15	amount of 100l. for $\frac{3}{4}$ of a year.

103l. 15s. : 100l. :: 550l. 10s. : 530l. 12s. 0$\frac{1}{2}$d. $\frac{26}{83}$, the present worth; which, deducted from 550l. 10s., gives 19l. 17s. 11$\frac{1}{4}$d. $\frac{57}{83}$ for the discount.

A merchant or banker would make the discount 20l. 12s. 10$\frac{1}{2}$d.

Or thus,

103l. 15s. : 3l. 15s. :: 550l. 10s. : 19l. 17s. 11$\frac{1}{4}$d. $\frac{57}{83}$, the discount; which, deducted from 550l. 10s., gives 530l. 12s. 0$\frac{1}{2}$d. $\frac{26}{83}$ for the present worth.

A merchant or banker would make the present worth 529l. 17s. 1$\frac{1}{2}$d.

(2.) Required the present worth of 594*l.* 14*s.* 9*d.* due 8 months hence, allowing a discount of 5$\frac{3}{4}$ per cent. per annum.

(3.) Sold goods to the value of 915*l.* 17*s.* payable 7 months hence; what must I allow for present payment, at 8 per cent. per annum?

(4.) How much ready money should I have for a note of 75*l.* which would be due 19 months hence, if I allow a discount of 5 per cent. per annum?

CLASS II.

(5.) What is the discount of 15,000*l.* for 57 days, at 5 per cent. per annum?

(6.) Sold goods to the value of 800*l.* 16*s.* payable as follows, viz. $\frac{1}{4}$ at two months, $\frac{1}{6}$ at 3 months, $\frac{1}{7}$ at 9 months, $\frac{3}{8}$ at 11 months, and the rest at 12 months; what must be discounted for present payment, at 5 per cent. per annum?

EQUATION OF PAYMENTS.

Definition. When several bills are payable at different times, bearing no interest till after the term of payment, the finding a time, at which, if they are all paid together, neither the holder nor the receiver will suffer loss, is called *equating*, or reducing the times of payment to one.

Proposition. ***To find the equated time at which several bills, payable at different times, may be paid at once, without loss either to the holder or receiver, allowing simple interest.***

Rule. If the times of payment be of different denominations, they must each be reduced to the same denomination. Then, multiply each payment by the time at which it becomes due; and divide the sum of the products by the sum of the payments, the quotient will be the time required.

Note 1. As this Rule of Equation of Payments has been the occasion of more disputes than all the rules of arithmetic put together, the reader will not be displeased to find here the several suppositions on which its principal defenders have founded their demonstrations.

2. Mr. *Cocker* supposes the equated time will be true, ‘ When the sum of the interests of the several bills which are payable *before the equated time*, from the times which they respectively become due to *that time;* is equal to the sum of the interests of the bills payable

after the equated time, from *that time* to the times at which they respectively become due.' But the argument by which he attempts to prove the truth of the rule is, according to Mr. *Malcolm*, very erroneous.

3. Mr. *Hutton* supposes the equated time to be true, 'When the interest of the sum of the debts or bills, from the time of the question to the equated time, is equal to the sum of the interests of the several debts or bills from the time of the question to the several terms of payment; and then, by an example, shews that the rule agrees with this supposition.

4. Mr. *R. Burrow*, in his Diary for the year 1777, reduces the subject, 'To find in what time the whole sum of the single payments will produce the same amount as that which arises from the sum of all the single payments, together with the *interest* of each payment from the time of its becoming due to the time of the last payment;' and then gives an algebraical demonstration, which shews that the rule is true according to this supposition.

5. That the rule is universally true, according to *any* of these suppositions, or that, if it be true according to one of them, it must necessarily be true according to the whole, may easily be demonstrated.

6. The following is KERSEY'S RULE.—*Find the present worth of each debt or bill, discounting from the time at which it is payable*, (by the rule of Discount,) *then find* (by Prop. 6. of Simple Interest) *in what time the sum of these present worths will amount to the sum of the debts or bills, and that is the time sought.* There are other rules given by different authors, as *Sir Samuel Moreland's*, *Ward's*, &c.; but, upon a close attention to their principles, they will be found exactly the same as one or other of the rules already given: indeed, the foundation of *Burrow's* demonstration seems to have been taken from *Moreland's* rule. *Malcolm's* rule will be given in the second part of this treatise: its requiring an extraction of the square-root makes it inadmissible in this place.

Examples.

(1.) A owes B 110*l.* whereof 50*l.* is to be paid at two years' end, 40*l.* at $3\frac{1}{2}$ years' end, and 20*l.* at $4\frac{1}{2}$ years' end; at what time may B receive the whole at once, without prejudice to either party?

50	multiplied	by 2	gives	100
40	—	by $3\frac{1}{2}$	—	140
20	—	by $4\frac{1}{2}$	—	90

110 sum of the payments. 330 sum of the products.

Then, 330 divided by 110 gives 3 years, the answer.

Illustration.

Suppose the interest of money to be at 5 per cent. and that 3 years is the *true equated time* as found above. It is evident that A *gains* the interest of 50l. for one year, which is 2l. 10s., by extending the term of payment to 3 years instead of 2; and that he *loses* the interest of 40l. for half a year, and the interest of 20l. for 1½ year, by paying 40l. half a year before it comes due, and 20l. 1½ year before it becomes due; which interests, added together, make 2l. 10s., so that his gain and his loss, on this consideration, appear to be equal. But, we must recollect, that B is not intitled to the *interest* of 40l. for half a year, and of 20l. for 1½ year, but to the *discount* of each of these sums for those times; so that the rule cannot be *precisely accurate*, though it be near enough to the truth for any practical purpose to which it can be applied.

(2.) I am to pay 500*l.* at three different payments, viz. 100*l.* at 2 months, 200*l.* at 4 months, and the rest at 6 months; but the person who is to receive the money has agreed to take a *single* note for the payment of the whole at once, for what length of time must the note be given?

(3.) A debt of 700*l.* is to be discharged thus: 150*l.* present, 300*l.* at 6 months, 200*l.* at 9 months, and the rest at 12 months; what is the equated time for the payment of the whole?

(4.) A merchant buys goods to the amount of 750*l.* 350*l.* of which is to be paid at 3 months, and the rest at 9 months; to prevent farther trouble, it is agreed to pay the whole at once, and to prolong the time of the first payment in proportion to the shortening the time of the second; at what time must the whole be discharged without prejudice to either?

(5.) A debt of 500*l.* 15*s.* is payable as follows: 150*l.* at 2 months, 147*l.* 17*s.* at 74 days, 137*l.* 18*s.* at 95 days, and the rest at 5 months. It is to be discharged at one payment; what is the equated time, reckoning 30 days to a month?

CLASS II.

(6.) A traveller received 1200 in 4 bills, all payable at Newcastle-upon-Tyne; viz. 600*l.* due at 4 months, 300*l.* at 5 months, 200*l.* at 7 months, and 100*l.* at 10

months: he agreed to pay the banker there, a reasonable commission, and the expense of the stamps, provided he would give him a *single* bill on London for the payment of the whole at once; for what length of time after date ought this bill to be drawn?

(7.) A debt is to be discharged thus, $\frac{1}{3}$ present, $\frac{1}{6}$ at 25 days, $\frac{1}{9}$ at 3 months, and the rest at 4m. 17d. what time may the whole be paid at once?

(8.) Three legacies are left by a gentleman, in his will, payable by his executors, to *one* person, or his heirs. The first legacy of 500*l*. 18*s*. is payable in $\frac{1}{2}$ a year, the 2d of 900*l*. 17*s*. 6*d*. is payable in 1 year 114 days, and the 3d of 1700*l*. 18*s*. $4\frac{3}{4}$d. is payable in $2\frac{1}{2}$ years. The legatee and executors have agreed, that the payment of these sums shall be made at once; at what time must that be, that neither party may be injured, allowing simple interest?

COMPOUND INTEREST.

Definition.—Compound Interest is that which is produced not only from the sum of money lent as the principal, but also from the interest, which, (when unpaid,) as it becomes due, is added to the principal.

Proposition. To find the interest of any sum of money, unpaid, for any equal number of payments at any rate per cent.

Rule I. Find the amount of the given principal for the time of the first payment by Simple Interest; then consider this amount, as the principal for the second payment, and find its amount as before. Proceed thus through all the payments, always considering the last amount as the principal of the next payment; then, if the given principal, or money lent, be deducted from the last amount, the remainder will be the interest required.

Or, Rule II.

Reduce the given sum into farthings, which multiply by the rate per cent. and cut off two figures from the right hand of each successive product, (or place each successive product two figures farther towards the right-hand,) and the last result will be farthings.

Note. The above rules will be true, whether the payments are made yearly, half-yearly, quarterly, monthly, or by any other aliquot part of a year: thus, for half-yearly payments, take half the rate per cent., and twice the number of years;—for quarterly payments, take ¼ of the rate per cent., and four times the number of years, &c. But the given time must be complete years, half-years, or quarters; thus, you cannot find the interest of a given sum payable yearly, for 4¼ years, 4½ years, &c. by the above rules, as directed by several authors. The truth of this remark will easily appear to those who are acquainted with logarithmical arithmetic, and the involution of numbers to fractional powers.—*For other rules, see Compound Interest by Decimals.*

Examples.

(1.) What is the compound interest of 357*l.* 10*s.* for 3 years, at 5 per cent. per annum?

5*l.* is $\frac{1}{20}$)	357*l.*	10*s.*		principal.
	17	17	6	interest for the first year.
$\frac{1}{20}$)	375	7	6	amount for ditto.
	18	15	4½	interest for the 2d year.
$\frac{1}{20}$)	394	2	10½	amount for ditto.
	19	14	1½-$\frac{9}{16}$	interest for the 3d year.
	413	17	0-$\frac{9}{16}$	amount for ditto.
	357	10	0	*principal.*

Answer, 56*l.* 7*s.* 0*d.*-$\frac{9}{16}$ whole interest, which is 2.14.6-$\frac{9}{16}$ more than the simple interest of the same sum. *See Example 1, Simple Interest.*

(2.) What is the compound interest of 700*l.* 18*s.* for 4 years, at 5 per cent. per annum?

(3.) What is the compound interest of 1057*l.* 17*s.* 6*d.* for 6 years, at 4 per cent. per annum?

(4.) Required the amount of 500*l.* 17*s.* for 5 years, at $4\frac{1}{2}$ per cent. compound interest?

(5.) What will 700*l.* amount to in 7 years, at $4\frac{3}{4}$ per cent. per annum, compound interest?

CLASS II.

(6.) Find the several amounts of 500*l.* payable yearly, half-yearly, and quarterly, for 4 years, at 5 per cent. per annum. *Answ.* 607*l.* 15*s.* $0\frac{1}{4}$*d.* *for yearly*, 609*l.* 4*s.* $0\frac{1}{4}$d. *for half-yearly, and* 609*l.* 18*s.* $10\frac{1}{2}$*d.* *for quarterly payments.*

(7.) What is the amount of 715*l.* for 6 years, the interest payable half-yearly, at $4\frac{1}{2}$ per cent. per annum?

(8.) What is the compound interest of 740*l.* 18*s.* for $9\frac{1}{2}$ years, by quarterly payments, at 4 per cent. per annum?

FELLOWSHIP, OR PARTNERSHIP.

Definition. *Fellowship, or Partnership,* is a general rule by which the accounts of merchants, &c. trading in company, with a joint stock, are adjusted; so that every partner may have his due share of the gain, or sustain a proportional part of the loss, according to the money he has advanced in the stock, and the time of its continuance therein.

SINGLE FELLOWSHIP, OR PARTNERSHIP FOR ANY EQUAL TIME.

Definition. *Single Fellowship, or Partnership for any equal time,* is when different stocks are employed for any certain equal time. The effects of bankrupts are by this rule properly divided among their creditors, legacies adjusted in deficiencies of assets, &c.—It likewise teaches us to divide any given number into unequal parts, proportional to certain other given numbers.

Proposition. *Having each man's particular stock and the whole gain or loss given, to find each man's part of the gain or loss.*

Rule. As the whole stock is to the whole gain or loss, so is each man's particular stock to his particular share of the gain or loss.

Method of proof. Add all the shares together, and the sum will be equal to the given gain or loss when the work is right.

Note 1. When there are many partners concerned, the following rule, which is best performed by decimals, will be found useful.—Divide the whole gain, or loss, by the whole stock, and the quotient will be a common multiplier, by which multiply every man's particular stock, and the several products will give each man's share of the gain or loss.

2. *Proposition. To divide any given number into any number of unequal parts proportional to certain other given numbers.*

Rule. Make the sum of the numbers to which the required parts must be proportional, the first term; the number to be parted, or divided, the second; and each of the given numbers, to which the required ones must be proportional, the several third terms of so many statings in the Rule of Three, the fourth terms of which will be the respective parts required.

Examples.

(1.) Three merchants, A, B, and C, enter upon a joint adventure; A puts into the common stock 250*l.* 10*s.* B, 300*l.* 15*s.* and C. 410*l.* 18*s.* After all expences were paid, a clear gain of 327*l.* 11*s.* 6*d.* was to be divided amongst them; what was each man's share?

250l. 10s. A's stock.
300l. 15s. B's stock.
410l. 18s. C's stock.

£962 3 *sum*, or the whole stock.

l.	*s.*		*l.*	*s.*	*d.*		*l.*	*s.*		*l.*	*s.*	*d.*	rem.
962	3	:	327	11	6	::	250	10	:	85	5	$8\frac{1}{2}$	— 3338, A's share.
962	3	:	327	11	6	::	300	15	:	102	7	$10\frac{1}{4}$	— 666, B's share.
962	3	:	327	11	6	::	410	18	:	139	17	$10\frac{1}{2}$	—15239, C's share.
										327	11	6	proof.

(2.) Two merchants traded together; A put into the stock 500*l.* 17*s.* 10*d.* and B 700 guineas; they gained 300*l.* 15*s.* what is each person's share thereof?

(3.) Four merchants, A, B, C, and D, entered into partnership with a stock of 5075*l.* 18*s.* of which A contributed 574*l.* 16*s.* B 947*l.* 18*s.* 6*d.* C. 3044*l.* 17*s.* and D the rest; they gained 1358*l.* 18*s.* what was each merchant's share thereof in proportion to his stock?

(4.) The money and effects of a bankrupt, after every unavoidable expence is deducted, amount to 7174*l.* 14*s.* At this time he is indebted to A 540*l.* 14*s.* to B 770*l.* 18*s.* to C 4005*l.* 14*s.* to D 975*l.* 18*s.* 9*d.* and to E 3060 guineas, how must it be divided amongst them, and what will they receive in the pound?

(5.) Six merchants, A, B, C, D, E, and F, sustained a loss of 79750*l.* by shipwreck on a foreign coast—A put on board, as part of the cargo, to the value of 7754*l.* 17*s.* B 15749*l.* 14*s.* C 3497*l.* 16*s.* D 5754*l.* 18*s.* 10*d.* E 3775*l.* 19*s.* and F 37497*l.* 19*s.* 8*d.* whereof there was a *salvage* in the cargo of 18750*l.* which was sold in the country for 7347*l.* clear gain; what was each merchant's loss?

(6.) Three merchants, A, B, and C, freight a ship with wine; A put on board 500 tuns, B 340, and C 94: by a storm at sea they were obliged to cast 150 tuns overboard; what loss does each sustain?

(7.) Let the number 1680 be divided into 6 such parts as shall be to each other, as 1, 2, 3, 4, 5, and 6, respectively.

(8.) Three merchants entered into partnership, with a stock of 1789*l.* 4s. their several stocks were in proportion, as 7, 8, and 9; they gained 500*l.*; required each person's stock and gain?

CLASS II.

(9.) There was a mixture of 3 different kinds of wine, in which, for every 3 gallons of one kind, there were 4 of another, and 7 of a third; what quantity of each kind is in a mixture of 292 gallons?

(10.) A father left his estate of 10090*l.* among 3 sons, in such manner, that, for every 2*l.* that A gets, B shall have 3, and C 5; how is the estate divided?

(11.) An old lady left 229*l.* 13*s.* 4*d.* to be divided amongst three of her nieces, A, B, and C, thus: as often

as A had $5\frac{3}{9}l$. B had $4\frac{3}{7}l$.; and as often as B had $4\frac{1}{9}l$. C had $3\frac{2}{3}l$.; pray what money did the old lady leave to each of them?

(12.) Divide 500*l.* amongst 4 people, thus; give A $\frac{1}{2}$, B $\frac{1}{3}$, C $\frac{1}{4}$, and D $\frac{1}{5}$.

(13.) Two persons traded together; the difference of their stocks was 51*l.* 11*s.* 6*d.* A's gain was 57*l.* 18*s.* and B's 29*l.* 14*s.* required each person's stock?

(14.) Three merchants, A, B, and C, freight ships to *Lisbon*, with sugar to the value of 15778*l.* 2*s.* 6*d.* A bought 250cwt. 1qr. 22lb. at 2*l.* 16*s.* per cwt. B paid 2*l.* 6*s.* 8*d.* per cwt. for his; but meeting with a storm at sea, the sailors were under the necessity of casting out part of the ship's lading.——A's proportional part cast overboard was equal to the $\frac{1}{100}$ part of their whole *cargo*, and $3\frac{3}{4}$ times the *whole* quantity cast over-board was equal to $3\frac{1}{3}$ times the *whole freight* of A and B. When they came to land, A sold his remaining part for 4 guineas per cwt. and found himself a loser of 10 per cent. besides charges. B advanced the remaining part of his commodity 20 per cent. and C gained 4*s.* 8*d.* per cwt. by the quantity he saved.—What did each merchant lose by this voyage, the charge thereof amounting to 500 guineas.

DOUBLE FELLOWSHIP, OR PARTNERSHIP FOR UNEQUAL TIMES.

Definition. *Double Fellowship* is that which supposes the several stocks, advanced for the purposes of trade, to be continued for unequal times, or to be increased or diminished at pleasure, with the consent of the several partners, at any time during the continuance of such partnership.

Proposition. Given each man's stock, the time of its continuance, and the whole gain or loss, to find each man's part of the gain or loss.

Rule. Multiply each man's stock by the time of its continuance. Then, as the sum of all the products is to the whole gain or loss, so is each man's product to his part of the gain or loss.

Method of proof as in single fellowship.

Note. The truth of this rule may be shewn thus; when the times are equal, the shares of the gain or loss are evidently as the stocks, as in Single Fellowship; and, when the stocks are equal, the shares are as the times; wherefore, when neither are equal, the shares must be as their products.

2. When there are many partners concerned, divide the whole gain or loss by the sum of the products of each man's stock and time, and the quotient will be a common multiplier, by which multiply the products of each man's stock and time separately, to obtain his share of the gain or loss.—This rule is best adapted to decimals.

3. The following rules and observations will be found very useful in solving difficult questions in Compound Fellowship: and, on this account, will doubtless be acceptable to the generality of readers.

4. *Prop.* 1. *Given each man's stock and time, and one man's gain, (or loss,) to find each man's particular gain, (or loss,) and consequently the whole.*

Rule. As the product of that man's stock and time, whose gain or loss is given, is to his gain or loss, so is the product of any other man's stock and time to his gain or loss.

5. *Prop.* 2. *Given each man's gain (or loss) and time, and the whole stock, to find each man's particular stock.*

Rule. Multiply each man's gain or loss into all the times except his own. Then, as the sum of the products is to the whole stock, so is each man's product to his stock.

6. *Prop.* 3. *Given each man's gain, (or loss) and time, and one man's stock, to find each man's particular stock.*

Rule. As that man's gain (or loss) whose stock is given, is to the product of his stock and time, so is any other man's gain (or loss) to the product of *his* stock and time. These products, divided by their respective times, will give their separate stocks.

7. *Prop.* 4. *Given each man's stock and gain (or loss) and the sum of their times, to find their particular times.*

Rule. Reduce each man's stock and gain (or loss) into one denomination, and multiply each man's gain (or loss) into all the stocks except his own. Then, as the sum of the products is to the sum of the times, so is each man's product to his time.

8. *Prop.* 5. *Given each man's stock and gain (or loss) and one man's time, to find the particular times of all the rest.*

Rule. Reduce each man's stock and gain (or loss) into one denomination, and multiply each man's gain (or loss) into all the stocks, except his own. Then as that man's product, whose time is given is to *that* time, so is any other man's product to his time.

Examples.

(1.) Three merchants, A, B, and C, enter into partnership; A puts in 89*l.* 5*s.* for 5 months, B 92*l.* 15*s.* for

7 months, and C 38*l.* 10*s.* for 11 months: with this stock they traffic, and gain 86*l.* 16*s.* Required each person's share of the gain in proportion to his stock, and the time of its continuance.

89l. 5s.	multiplied	by 5	gives	446l.	5s.	A's product.
92 15	——	by 7	—	649	5	B's product.
38 10	——	by 11	—	423	10	C's product.
	Sum of the products			1519	0	

1519	:	86 16	::	446 5	:	25	10	A's gain.
1519	:	86 16	::	649 5	:	37	2	B's gain.
1519	:	86 16	::	423 10	:	24	4	C's gain.
						£86	16	proof.

(2.) Three merchants, A, B, and C, engage in partnership; A puts in 547*l.* 19*s.* 6*d.* for 7 months, B 475*l.* 18*s.* for 9 months, and C 1747*l.* 14*s.* for 4 months: they trade, and gain 225*l.* Required each person's share thereof?

(3.) Four farmers, A, B, C, and D, jointly hired a pasture of a neighbour for 20 guineas, into which A turned 7 oxen for 13 days, B 9 oxen for 14 days. C 11 oxen for 25 days, and D 15 oxen for 37 days; how much must each farmer pay for his share of the pasture?

(4) A family of 10 persons took a large house for ½ a year, for which they were to pay 26*l.* 2*s.* 6*d.* for that time. Now, at the end of 14 weeks, they took in 4 lodgers, and 3 weeks after 4 more; and so on for every 3 weeks (during the term) they took in 4 more lodgers. What must one of each class pay per week of the rent?

(5.) Three merchants enter into partnership, and trade as follows: A put in 150*l.* and at the end of 7 months took out 50*l.*: 5 months after that he put in 170*l.*:—B put in 205*l.* and at the end of 5 months 110*l.* more, but took out 150*l.* four months after:—C put in 300 guineas, and, when 6 months had elapsed, he drew out 150*l.* but 9 months after he put in 500*l.*—Their partnership continued 18 months, at the end of which time they gained 450*l.* Required each person's share thereof?

CLASS II *exercising the notes, &c.*

(6.) Three merchants traded together as follows: A put in 500*l.* for 3 months, B 350*l.* for 5 months, and C

400*l.* for 2 months, by which he received 29*l.* 12*s.* 7$\frac{1}{9}$*d.* profit. What must A and B receive for *their* respective stocks, and what did they gain in the whole?

(7.) Three merchants traded together in this manner: A's money continued 8 months, for which he received 44*l.* 4*s.* gain; B's continued 6 months, for which he had 42*l.* 16*s.* 9$\frac{3}{5}$*d.*; and C's 12 months, by which he was entitled to receive 79*l.* 11*s.* 2$\frac{2}{5}$*d.*—Their whole stock was 227*l.* hence is required each person's particular stock?

(8.) A, B, and C, are in company, and put in together 3822*l.* A's money was in 3 months, B's money was in 5 months, and C's money was in 7 months; they gained 234*l.* which was so divided, that $\frac{1}{2}$ of A's gain was equal to $\frac{1}{3}$ of B's gain, and $\frac{1}{3}$ of B's gain was equal to $\frac{1}{4}$ of C's gain; what did each merchant gain and put in?

(9.) X, Y, Z, in company, make one common stock of 4262*l.* X's money was in 4 months, Y's 6 months, and Z's 9 months. They gained 420*l.* which was to be divided in the following manner, viz. $\frac{1}{2}$ of X's gain to be equal to $\frac{1}{3}$ of Y's, and $\frac{1}{3}$ of Y's gain to be equal to $\frac{1}{4}$ of Z's. Quere, what each person gained and put in?

(10.) Four merchants, A, B, C, and D, trade together; A clears 76*l.* 4*s.* in 6 months, B 57*l.* 10*s.* in 5 months, C 100 guineas in 12 months, and D (with a stock of 200 guineas) 78*l.* 15*s.* in 9 months. Required each man's particular stock?

(11.) Three persons, A, B, and C, traded together; A's stock was 89*l.* 5*s.* B's 92*l.* 15*s.* and C's 38*l.* 10*s.* Their respective gains were 25*l.* 10*s.*, 37*l.* 2*s.* and 24*l.* 4*s.*; also, if the times that each person's stock was employed in trade be added together, the sum will be 23 months, pray how long was each man's stock in trade?

(12.) A merchant (B) in trade, with a capital of 5000*l.* after a certain time, agreed to take a friend (C) into partnership, who was to have a share in the profits according to the money he advanced, and the time of its continuance. Now C put 1400*l.* into the stock, and they traded together in this manner, till, willing to enlarge their sphere of trade, they admitted another person (D) as a partner with a stock of 1800*l.*—At the end of 3 years (or 36 months) reckoning from the time that B

commenced business, B's gain was found to be 1125*l.* C's 210*l.* and D's 202*l.* 10*s.* quere, how long were C's and D's money employed in trade, and what did each merchant gain per cent. for his money?

(13.) Two merchants, A and B traded together with a stock of 315*l.*; A's money was employed 12 months, and B's only 8: when they came to divide the profits of their traffic, they had equal shares,—Pray what money did each person put into the stock?

(14.) A certain village is possessed by three proprietors, who are desirous of having it enclosed for their mutual benefit. A's property, upon a survey of the quantity and quality, is 394a. 3r. 34p. at 18*s.* per acre; B has 417a. 1r. 14p. at an average of 19*s.* 6*d.* per acre; and C has 714a. 3r. at a guinea an acre. Out of these an allowance of 5*s.* 6*d.* in the pound is to be made for the tithes. What quantity of land must be allotted for these tithes, at an average quality of 19*s.* 9½*d.* per acre?

LOSS AND GAIN.

Definition. Loss and Gain is a rule that discovers what is gained or lost in the buying or selling of goods; and instructs the merchant, or trader, to raise or lower the price of his goods so as to gain or lose so much *per cent. &c.*

Note. By the prime cost, or selling price of an integer, in the following propositions and rules, is meant the prime cost, or selling price, *per yard, pair, dozen, pound, cwt. gallon, tun, &c.* of any quantity of goods, or it may signify the whole value in any of the propositions, except the first, fifth, and sixth.

Proposition 1. *Given the prime cost and selling price of an integer of any quantity of goods to find the whole gain or loss.*

Rule. Calculate the value of the goods at the prime cost and selling price of an integer, by the Rule of Three, or Practice, and the difference of these values will be the gain or loss.

Prop. 2. *Given the prime cost and selling price of an integer of any quantity of goods, to find the gain or loss per cent.*

Rule. As the prime cost of an integer is to 100*l.* so is the advanced or reduced price of such integer to a fourth

number; which, if greater than 100*l.* the excess will be the gain; but, if less than 100*l.* the defect will be the loss *per cent.*

Prop. 3. *Given the prime cost of an integer, and the proposed gain or loss per cent. to find the selling price of such integer.*

Rule. As 100*l.* is to 100. with the gain added to, or the loss subtracted from, it, so is the prime cost of an integer to the required price per integer.

Prop. 4. *Given the price of an integer, with the gain or loss per cent. by such a price, to find the gain or loss at any other price.*

Rule. As the given price of an integer is to 100*l.* with the gain per cent. added to, or loss subtracted from, it, so is the proposed price to a fourth number. If this fourth number be greater than 100*l.* the excess will be the gain; but, if it be less, take it from 100*l.* and the remainder will be the loss *per cent.*

Prop. 5. *Given the price at which an integer of any quantity of goods is sold, and the gain or loss per cent. by such sale, to find the whole gain or loss.*

Rule. Find the whole value of the goods at the selling price per integer. Then, as 100*l.* with the gain *per cent.* added to, or loss subtracted from, it, is to 100*l.* so is the whole value at which the goods were sold to the whole prime cost. The difference between the whole value at which the goods were sold and the whole prime cost will give the whole gain or loss.

Prop. 6. *Given the prime cost of an integer of any quantity of goods, and the gain or loss per cent. by the whole quantity, to find the whole gain or loss.*

Rule. Find the whole value of the goods at the prime cost per integer. Then, as 100*l.* is to 100*l.* with the gain added to, or loss subtracted from, it, so is the whole value of the goods, at the price they cost, to the whole value at the gain or loss *per cent.* proposed. The difference between these values will give the whole gain or loss.

Note. More propositions and rules may be given; but, if the scholar thoroughly understand the rules already laid down, and their application, it is presumed he will not meet with any embarrassment in *Loss and Gain*, however complicated the examples may be.

Examples to Proposition 1.

(1.) Bought 119¼cwt. of sugar at 1*l*. 15*s*. per cwt. whether shall I gain or lose if I sell it by retail for 6*d*. per lb.?

1cwt. : 1*l*. 15*s*. :: 119¼cwt. : 209*l*. 11*s*. 3*d*. prime cost.
1lb. : 6*d*. :: 119¼cwt. : 335*l*. 6*s*. sold for.
Then 335*l*. 6*s*.—209*l*. 11*s*. 3*d*.=125*l*. 14*s*. 9*d*. gain.

(2.) Bought 15cwt. of cheese at 1*l*. 11*s*. 6*d*. per cwt. which I sell by retail at 4½*d*. per lb. what shall I gain or lose by so doing?

(3.) I bought 77cwt. 3qr. 14lbs. of sugar at 2*l*. 7*s*. 10*d*. per cwt. and sold it again for 6¾*d*. per lb. whether did I gain or lose, and how much?

(4.) A merchant bought 12 tuns of wine at 75*l*. 12*s*. per tun, which he sold at 7*s*. per gallon; but, by misfortune, a pipe was staved, and rendered unsaleable. Whether did the merchant gain or lose, and how much by such sale?

(5.) Bought 340 yards of cloth at 5*s*. 4*d*. a yard, and sold it again at 7*s*. 6*d*. per yard; what did I gain in the whole?

Examples to Prop. 2.

(6.) If wine be bought at 7*s*. 6*d*. per gallon, and sold for 10*s*. what is gained *per cent*. by such sale?

7*s*. 6*d*. : 100*l*. :: 10*s*. : 133*l*. 6*s*. 8*d*.
Then 133*l*. 6*s*. 8*d*.—100*l*.=33*l*. 6*s*. 8*d*. the gain per cent.
Or, 10*s*.—7*s*. 6*d*.=2*s*. 6*d*. and 2*s*. 6*d*.=⅓ of 7*s*. 6*d*. therefore 100÷3 =33*l*. 6*s*. 8*d*. answer.

(7.) A merchant has a quantity of damaged tobacco, which, including all expences, stands him in 17¼*d*. per lb. what will he lose per cent. by a sale at 13½*d*. per lb.?

(8.) Bought 27 yards of cloth for 17 guineas, and sold them again at 9s. 10*d*. per yard; what was the gain or loss per cent.?

(9.) Bought a quantity of goods for 60*l*. and sold them again for 75*l*. what was the gain per cent.?

(10.) Bought a quantity of cloth at 7*s*. 6*d*. per yard, which, upon examination, I find not so good as I expected. Now, if I sell it at 6*s*. 2¼*d*. per yard, what shall I lose per cent. by it?

Examples to Prop. 3.

(11.) Bought muslin at 4*s.* 8*d.* per yard; at what price must I sell it per yard to gain $12\frac{1}{2}$ per cent.?

100*l.* : 112*l.* 10*s.* :: 4*s.* 8*d.* : 5*s.* 3*d.* answer.

Or, 12*l.* 10*s.*$=\frac{1}{8}$ of 100*l.* and $\frac{1}{8}$ of 4*s.* 8*d.*=7*d.* Hence 4*s.* 8*d.*+7*d.* =5*s.* 3*d.* as before.

(12.) If I buy cloth at 11*s.* 6*d.* per yard, how must I sell it to gain 20*l.* per cent.?

(13.) A Manchester man bought a quantity of yarn at 6*s.* per bundle, which not proving so good as he expected, he sold it so as to lose 6 per cent. by it; what was the selling price?

(14.) If I buy tobacco at 12 guineas per cwt. at what rate must I sell it per cwt. to gain 15*l.* per cent.?

(15.) Bought a quantity of cloth at 7*s.* 6*d.* per yard, which, not proving so good as I expected, I have resolved to lose $17\frac{1}{2}$*l.* per cent. by it; how must I sell it per yard?

Examples to Prop. 4.

(16.) A stationer sold quills at 11*s.* per thousand, by which he cleared 60*l.* per cent. but they growing scarce, he raised them to 13*s.* 6*d.* per thousand; what was his gain per cent. by the latter price?

11*s.* : 160*l.* :: 13*s.* 6*d.* : 196*l.* 7*s.* $3\frac{3}{11}$*d.*

Then 196*l.* 7*s.* $3\frac{3}{11}$*d.*—100*l.*=96*l.* 7*s.* $3\frac{3}{11}$*d.* answer.

(17.) If, when I sell cloth at 8*s.* 9*d.* per yard, I gain 12*l.* per cent. what will be the gain per cent. when it is sold for 10*s.* 6*d.* per yard?

(18.) A woollen-draper in London had a quantity of black cloth by him, and, being afraid of its being damaged, he sold it at 15*s.* per yard, and, by so doing, lost 14*l.* per cent. but a general mourning coming unexpectedly, he was enabled to advance his cloth to a guinea per yard; what did he gain or lose per cent. by the latter sale?

(19.) If a plumber gain 12*l.* 10*s.* per cent. when lead is sold at 20*l.* 9*s.* 6*d.* a fother, what would he gain or lose per cent. when it is sold only at 17*l.* 1*s.* 3*d.* the fother?

Examples to Prop. 5.

(20.) A merchant sold 5t. 3hhds. 53½gall. of wine at 6*s*. 8*d*. per gallon, and by so doing gained 6½*l*. per cent. What was the prime cost of his wine, and what did he gain in the whole?

1 gall. : 6*s*. 8*d*. :: 5t. 3hhd. 52½g. : 500l. 16s. 8d. sold for.

Again, 106l. 10s. : 100l. :: 500l. 16s. 8d. : 470l. 5s. $3\frac{181\frac{1}{2}}{213}d$. prime cost.

Then 500l. 16s. 8d.—470l. 5s. $3\frac{181\frac{1}{2}}{213}d$.=30l. 11s. $4\frac{32}{213}d$. whole gain.

(21.) A merchant sold 15cwt. 3qr. 18lb. of sugar at 7½*d*. per lb. and his profit per cent. was 25*l*. what did he gain in the whole?

(22.) If I sell 500 deals at 15*d*. a piece, and 9*l*. per cent. loss, what do I lose in the whole quantity?

(23.) A had 15 pipes of Malaga wine, which he parted with to B at 4⅓*l*. per cent. profit, who sold them to C for 38*l*. 11*s*. 6*d*. advantage; C made them over to D for 500*l*. 16*s*. 8*d*. and cleared thereby 6½ per cent. what did this wine cost A per gallon?

Examples to Prop. 6.

(24.) Bought 60 reams of paper at 15*s*. per ream, by the sale of which I lost 4*l*. per cent. what did I lose in the whole?

1r. : 15s. :: 60r. : 45l. prime cost.
100 : 96 :: 45l. : 43l. 4s. selling price.
Then 45*l*.—43*l*. 4*s*.=1*l*. 16*s*. whole loss.

(25.) Sold 7 pieces of cloth, each containing 35½ yards, on account of damage, at a loss of 10*l*. per cent, what did I lose in the whole, the prime cost being 15*s*. per yard?

(26.) Bought 475 yards of cloth at 10*s*. 6*d*. per yard, by which I gained 30*l*. per cent. what did I gain in the whole?

CLASS II. *Promiscuous Examples.*

(27.) Bought 127hhds. of sugar, each containing 4½ cwt. at 3*l*. 0*s*. 8*d*. per cwt. how must I sell the sugar per lb. to gain 50 guineas by the whole?

(28.) A merchant bought 1400 casks of tallow, at 2*l*. 5*s*. per cask, and sold one half of it at 2*l*. 15*s*. per

cask; but the rest being worse than he expected, he is willing to sell it at such a price per cask, that he may exactly make his purchase-money of the whole. At what rate must he sell it?

(29.) A merchant bought 100 yards of velvet for 112*l.* at what rate must he sell it per yard to gain as much by the whole quantity as four yards are sold for?

(30.) Sold a quantity of Virginia snakeroot for 20*l.* and by so doing lost 20*l.* per cent. whereas I ought to have gained as much per cent. as the snakeroot cost. Quere my loss in point of trade?

(31.) A tea-dealer purchased 120lb. of tea; $\frac{2}{5}$ of which he sold at 10*s.* 6*d.* per lb. but the rest, being damaged, he sold it at a loss of 3*l.* 12*s.* after which he found he had neither gained nor lost. What did the tea cost him per lb. and what was the damaged tea sold for?

(32.) My factor at Leghorn returned me 800 barrels of anchovies, each weighing 14lb. neat, worth 12$\frac{1}{2}$*d.* per lb. in lieu of 7490lb. of Virginia tobacco; by which consignment I find that I have gained 17*l.* per cent. Pray what was the prime cost of a lb. of my tobacco to the factor?

(33.) A merchant sent goods to *Boulogne* to the value of 3475*l.* 15*s.* by the sale of which he gained 40*l.* sterling per cent.—The value of the goods he sent over and the gain were returned in commodities, by the sale of which in England he lost 15*l.* per cent. What was his gain at the last?

(34.) Sold a piece of cloth, containing 5000 ells Flemish, for 4250 guineas, and gained upon every yard $\frac{1}{8}$ of the prime cost of an English ell. What did the whole piece stand me in?

BARTER.

Definition. When merchants or tradesmen exchange one commodity for another, it is called *Bartering*; and, by the rule of proportion, the price and quantity of the goods so exchanged are determined, so that neither party may sustain a loss by such traffic.

Proposition 1. *Given the price of an integer of any quantity of goods, to find the corresponding quantity of any other sort of goods, at any given price per integer.*

Rule. Find the value of that commodity, whereof the quantity is given, by the Rule of Three, or Practice. Then, as the price of an integer of the *required* quantity of goods is to *that* integer, so is the value of the *given* quantity, found before, to the required quantity.

Note. Several questions that fall under this proposition may be solved, in the shortest manner, by the second rule of Compound Proportion, or by the Rule of Three Inverse.

Prop. 2. Given the price of an integer of any quantity of goods, to find the quantity of any other kind of goods, (at any given price per integer,) when part of the value is paid in money, or any other kind of merchandise.

Rule. Find the whole value of that commodity, whereof the quantity is given, by the Rule of Three, or Practice; from which subtract the sum of money to be paid down, or the value of the given quantity of goods in exchange. Then, as the price of an integer, of the *required* quantity of goods, is to *that* integer, so is the remaining value to be accounted for, to the required quantity.

Note. Several other propositions and rules in Barter are omitted as superfluous, the questions which they are intended to solve being of no real use; such as to find in what proportion one person ought to advance his goods to another who has raised *his* goods above their real value, &c. For, in the real exchange of goods, when both parties have mentioned their ready-money prices, if one person's goods are advanced to a bartering price, the other person's must be advanced in the same proportion, and consequently the balance between them will remain exactly the same as if the ready-money price *only* had been used.

Examples to Proposition 1.

(1.) A and B barter; A has $3\frac{1}{2}$lb. of pepper at $13\frac{1}{2}$*d.* per lb., B has ginger at $15\frac{1}{4}$*d.* per lb. How much ginger must B give for A's pepper?

$\frac{1}{2}$	1s.	$1\frac{1}{2}$d.	value of 1lb. of A's pepper.
		3	
	3	$4\frac{1}{2}$	value of 3lb.
		$6\frac{3}{4}$	ditto of $\frac{1}{2}$lb.
	3	$11\frac{1}{4}$	ditto of $3\frac{1}{2}$lb.

Then, $15\frac{1}{4}$*d.* : 1lb. :: 3s. $11\frac{1}{4}$d. : 3lb. $1\frac{35}{61}$ oz. answer.

Or,

$$\begin{array}{rcl} & \textit{equal} & \\ \text{If } 3\tfrac{1}{2}\text{lb. at } 13\tfrac{1}{2}\text{d.} & \times & 1 \text{ value in pence.} \\ {*}\text{lb. at } 15\tfrac{1}{4}\text{d.} & & 1 \text{ (the same) value.} \\ & \textit{equal} & \end{array}$$

$13\frac{1}{2}\times3\frac{1}{2}=47\frac{1}{4}$ dividend, which divided by $15\frac{1}{4}$, the divisor, gives $3\frac{6}{61}$lb.$=$3lb. $1\frac{35}{61}$ oz. as before.

Or,

$13\frac{1}{2}$d. : $3\frac{1}{2}$lb. :: $15\frac{1}{4}$d.*

Here it is evident that B must give A a less number of lbs. of ginger than he receives pepper, because the ginger is worth more per lb. Consequently,

$\overline{13\frac{1}{2}\times3\frac{1}{2}}\div15\frac{1}{4}=3\frac{6}{61}$lb. or 3lb. $1\frac{35}{61}$oz. as above.

(2.) A would exchange 400 gallons of Jamaica rum, worth 7*s.* 9*d.* per gallon, with B for London porter, at 9*d.* a gallon; how many gallons of porter must A receive of B in exchange for his rum?

(3.) A hop-factor, A, exchanged 5cwt. 1qr. 10lb. of hops, at 2*s.* $4\frac{1}{4}$*d.* per lb. for wheat at 5*s.* 9*d.* per bushel, with a farmer B; what quantity of wheat did B give A for his hops?

(4.) How many yards of cloth, at 18*s.* 6*d.* per yard, must I give for 5000 yards of baize, at $13\frac{1}{2}$*d.* per yard?

(5.) A delivered 6 hogsheads of brandy, at 6*s.* 8*d.* per gallon, to B for 252 yards of cloth; what ought the cloth to be worth per yard?

(6.) A has 288 ells of cloth, worth 1*s.* 3*d.* per ell, which he would barter with B for cheese at 19*s.* per cwt. what weight of cheese ought B to give for the cloth?

(7.) A and B bartered; A had 14cwt. 3qrs. of sugar, worth 1*l.* 17*s.* per cwt. which he bartered for wine worth 3*s.* 9*d.* per gallon; how much wine did A receive?

(8.) A chandler and a butcher trade as follows: the butcher has 3cwt. 2qr. 16lb. of tallow at 1*l.* 17*s.* 4*d.* per cwt. and the chandler rates his candles at 5*s.* 2*d.* per dozen. How many lbs. of candles must the chandler give the butcher for his tallow?

Examples to Prop. 2.

(9.) A and B barter as follows: A has 41 cwt. of hops at 30*s.* per cwt. for which B gives him 20*l.* in ready mo-

ney, and the rest in sugar at 6*d.* per lb. What quantity of sugar must B give A?

10*s.*	$\frac{1}{2}$	41*l.*	
		20 10	6d. : 1lb. :: 41l. 10s. : 14cwt. 3qr. 8lb. answer.

61 10 value of A's hops.
20 0 paid down.

41 10 to account for.

(10.) A and B barter; A has 750 yards of canvas, worth 10*d.* per yard, for which B gives him 475 yards of serge at $11\frac{1}{2}$*d.* per yard, and the balance in cotton at 3*s.* per yard; how many yards of cotton must A receive?

$\frac{1}{2}$d.	$\frac{1}{24}$	475	10d.	$\frac{1}{24}$	750	
		19 $9\frac{1}{2}$			£31 5	value of A's canvas.
		20)455 $2\frac{1}{2}$			22 15 $2\frac{1}{2}$	value of B's serge.
		£22 15 $2\frac{1}{2}$			£8 9 $9\frac{1}{2}$	to account for.

3s. : 1yd. :: 8l. 9s. $9\frac{1}{2}$d. : 56yds. $2\frac{7}{18}$qrs. answer.

(11.) A has 700 gallons of rum at 4*s.* 6*d.* per gallon, for which B gives him 27 guineas in money, and the rest in cotton at $11\frac{1}{2}$*d.* per lb.; how much cotton must A receive?

(12.) A has 57qrs. 6bush. of corn, worth 1*l.* 11*s.* 6*d.* per quarter, for which B will give 14cwt. 3qr. 18lb. of sugar at 4*l.* 14*s.* per cwt. and the balance in raisins at 7*d.* per lb. Should these persons barter, what quantity of raisins ought B to give A?

(13.) A has 27 cwt. of cheese, worth 1*l.* 11*s.* 4*d.* per cwt., and B has 25 pieces of cloth, worth 1*l.* 19*s.* $10\frac{1}{2}$*d.* per piece; should these persons barter together, to whom will the balance, if any, be due?

CLASS II.

(14.) A gave B 120 yards of Kersey, $3\frac{1}{2}$ yards of which cost 15*s.* 9*d.* for stockings at 7*s.* per pair, and hats at 6*s.* 6*d.* each; B gave A as many hats as pairs of stockings; how many of each did he give?

(15.) Two merchants have various kinds of goods to barter: A has 735 yards of India silk, worth 8*s.* 6*d.* per yard, 532 canes worth 3*s.* each, and 16 pieces of muslin worth 4*l.* each; B has scarlet cloth worth 1*l.* per yard, glass manufacture at 1*s.* 8*d.* per lb. and a finer kind at 2*s.* 4*d.* per lb. How many yards of cloth and pounds of each sort of glass must B give A, admitting that he gives as many pounds of each sort of glass as he gives yards of cloth?

(16.) A merchant, A, of *London*, sent 8752 yards of cloth, worth 1*l.* 11*s.* 6*d.* per yard, to B in *Jamaica*; and desired him to return him $\frac{1}{4}$ of the value in sugar at 1*l.* 15*s.* 6*d.* per cwt. $\frac{1}{3}$ of the value in pepper at 7*l.* 3*s.* 9*d.* per cwt. and the rest in rum at 5*s.* 6*d.* per gallon. Each merchant ran the risk, and paid the charges of the commodity he sent over; pray what quantity of sugar, pepper, and rum, did A receive.

(17.) A and B barter; A has 24 puncheons of rum, worth 4*s.* 9*d.* per gallon; for which B gives him 150 guineas in cash, and 714 yards of cloth. What ought B's cloth to be worth per yard?

(18.) A bartered tobacco, worth 3*s.* 4*d.* per lb. at 3*s.* 9*d.* per lb. with B for tea at 6*s.* 3*d.* per lb. When A sold the tea, he found himself a gainer of 17*l.* 6*s.* 8*d.* per cent. and in the whole 8*l.* 10*s.* 8*d.* What did A sell the tea for per lb. and what quantity of tobacco and tea were bartered?

EXCHANGE.

Definition 1. By *Exchange* is meant the bartering, or exchanging, the money of one place for that of another, by means of an instrument in writing, called a *Bill of Exchange;* and consists in finding what quantity of the money of one city or country will be equal to any given sum of another, according to a given *course* of exchange.

2. *The Course of Exchange* is the value agreed upon by merchants, or their factors; and is continually fluctuating above or below the *Par of Exchange*, according as the demand for bills is greater or less.

3. The *Par of Exchange** is that quantity of the money of one country which is intrinsically equal to a certain quantity of the money of another, whether *real* or *imaginary*.

4. The *real money* of every empire, kingdom, state, &c. signifies one piece, or more, of any kind of metal, coined by the authority of that empire, kingdom, state, &c. and current at a certain value by virtue of such authority.

5. The *imaginary money* is chiefly used in keeping accounts, and includes all the denominations made use of to express any sum of money, though no *coin* of that name may pass current, in the state, as the *pound* sterling, &c.

6. The *Agio* denotes the difference in foreign countries between the *current*, or *cash money*, and the *exchange*, or *bank-money*, the latter being compounded of a finer, or purer, metal than the former.

Note. When current, or cash-money, is taken in payment, the merchants have an allowance of so much per cent. according to what exchange-money is worth more than the current-money.

7. *Bank-notes*, in the business of exchange, are such as are obtained from foreign bankers for money lodged in their bank.—These are called *bank-money*.

8. *Usance* is a certain space of time allowed, by one country to another, for the payment of bills of exchange. It varies according to the custom of countries, and frequently in proportion to the distance of places from each other.—Bills are either payable at *sight*, or so many days after sight; at *usance*, double *usance*, or half *usance*.

The usance to England, from France, Holland, and Germany, is *one* month's date; from Spain and Portugal, *two* months' date: from Italy *three* months' date.

9. The *days of grace* are a certain number of days allowed for the payment of bills of exchange, after the expiration of the term specified in such bills, and are variable in different countries. In some countries no days of grace are allowed. The usual days of grace, in England, are *three*.

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* It is not easy to *fix* the true par of exchange, on account of the fluctuation in the comparative value of gold and silver, and the alteration made in the value of the coins of different countries by edicts, laws, &c. The par is best ascertained from the custom and speculation of merchants at particular times, which may be termed the *political par*. See Tables XX. and XXI. following.

Quotations are the lists of the courses of exchange, which are transmitted from one country to another for the use of merchants. These quotations which extend to all places in the commercial world, may be obtained at the Royal Exchange. *Lloyd's* list shews the quotation at London.

Though the quotations are continually fluctuating, the deviation from the *variable prices* exhibited in the following tables is seldom very great.

Writers on Exchange are very numerous; the principal are, *Kruse* of Hamburgh, *Corbaux* of France, and *Dubost* of London. The *Hamburgh Contorist,* by *Kruse,* is the most celebrated; an English translation of this valuable work has lately been published by *Dr. Kelly,* under the title of the *Universal Cambist.*

The following Tables and Quotations have been carefully compared with the tables, &c. in the works mentioned above. In this edition, the different species of money which are not used in exchange, have been *omitted,* and the quotations have been *added;* these are the only alterations made in the tables.

THE NECESSARY TABLES OF EXCHANGE.

TABLE I. DENMARK.

At *Copenhagen,* &c. the lowest piece of money is a *Skilling.* value $\frac{3}{10}$*d.* sterling.

Exchanges are computed in Rix-dollars, Marcs, and Skillings Danish, and sometimes in Rix-dollars, Marcs, and Sols Lubs of Hamburgh.

16 Skillings		=	1 Marc
6 Marcs Danish	= 3 Marcs Lub	=	1 Rix-dollar Lub
2 Skillings Danish		=	1 Sol Lub

QUOTATION.

Copenhagen gives to	*variable.*		*certain.*
Amsterdam........	143 Rix-dollars............	for	100 Rix-dollars
Hamburgh........	149 Rix-dollars............		100 Rix-dollars
London............	6 Rix-dollars 30 Skillings ..		£1 sterling
Paris...............	25 Skillings Danish............		1 Franc

TABLE II. SWEDEN.

At *Stockholm*, &c. the lowest piece of money is a *Runstic*, value $\frac{7}{38}d.$ sterling.

Exchanges are generally computed in Rix-dollars, Skillings, and Fennings.

8 Runstics	=	1 Copper Marc
4 Copper Marcs	=	1 Copper Dollar
12 Copper Marcs	=	1 Silver Dollar
3 Silver Dollars	=	1 Rix-dollar
12 Fennings	=	1 Skilling
48 Skillings	=	1 Rix-dollar

QUOTATION.

Stockholm gives to	*variable.*	*certain*
Amsterdam	44 Skillings	for 1 Rix-dollar
Copenhagen	36 Skillings	1 Rix-dollar
Dantzic	$9\frac{1}{2}$ Skillings	1 Florin
Hamburgh	47 Skillings	1 Rix-dollar
Leghorn	40 Skillings	1 Pezzo of 8 Rials
London	$4\frac{1}{4}$ Rix-dollars	1 Pound Sterling
Paris	24 Skillings	1 Ecu of 3 Livres
Spain	42 Skillings	1 Ducat of Exchange

TABLE III. RUSSIA.

At *Petersburg*, &c. the lowest piece of money is a *Polusca*, value $\frac{27}{400}d.$ sterling.

Exchanges are generally computed in Roubles and Copecs.

4 Poluscas	=	1 Copec
10 Copecs	=	1 Grivener
100 Copecs or 10 Griveners	=	1 Rouble

QUOTATION.

Petersburg receives from	*variable.*	*certain.*
Amsterdam	25 Stivers	for 1 Rouble
London	28 Pence sterling	1 Rouble
Paris	270 Centimes	1 Rouble
Vienna	125 Creutzers	1 Rouble
Gives to		
Constantinople	50 Copecs	1 Piastre

TABLE IV. POLAND AND PRUSSIA.

At *Dantzic*, &c. the lowest piece of money is a *Fenning*, value $\frac{7}{216}d.$ sterling.

EXCHANGES are generally computed in Florins, Groshen, and Fennings.

18 Fennings	=	1 Groshen
30 Groshen	=	1 Florin
3 Florins	=	1 *Rix dollar

QUOTATION.

Dantzic gives to	*variable.*	*certain.*
Amsterdam	370 Groshen	for £1 Flemish
Francfort	105 Groshen	1 Rix-dollar
Hamburgh	169 Groshen	1 Rix-dollar specie
Leipzig	125 Rix-dollars	100 Rix-dollars
London	24 Florins	£1 sterling

TABLE V. HAMBURGH AND ALTONA.

At *Hamburgh* the lowest piece of money is a *Fenning*, value $\frac{1}{16}$*d.* sterling.

EXCHANGES are computed in Marcs, Sols Lub, and Fennings; or in Pounds, Shillings, and Pence; also in Rix-dollars, Marcs, &c.

12 Fennings	=	1 Shilling Lubec
16 Shillings Lub, or Sols Lub	=	1 Marc
3 Marcs	=	1 Rix-dollar
6 Marcs	=	1 Danish Ducat

ALSO,

6 Fennings	=	1 Grot, or Penny Flemish
2 Pence or Grots Flemish	=	1 Sol Lub
12 Pence Flemish, or 6 Sols Lub	=	1 Shilling Flemish, or Sol Gros
20 Shill. Flemish, or 120 Sols Lub	=	1 Pound Flemish
7½ Marcs	=	1 Pound Flemish

QUOTATION.

Hamburgh gives to	*variable.*	*certain.*
Basil	25 Sols Lub	for 1 Ecu of 3 Livres
France	26 Sols Lub	3 Francs
Genoa	80 Grots Flemish	1 Pezzo of 5¾ Lire
Leghorn	86 Grots Flemish	1 Pezzo of 8 Rials
London	33 Shill. 7 Grots	1 Pound Sterling
Portugal	43 Grots	1 Old Crusade of 400 Reis.
Receives from		
Amsterdam	33 Stivers	2 Marcs
Breslau	139 Rix-dollars	100 Rix-dollars
Copenhagen	149 Rix-dollars	100 Rix-dollars
Venice	82 Soldi Picooli	1 Marc
Vienna	310 Florins	100 Rix-dollars

TABLE VI. FRANCFORT ON THE MAIN, MANHEIM, &C.

At *Francfort* the lowest piece of money is a *Fenning*, value $\frac{1}{30}d.$ sterling.

EXCHANGES are computed in Florins and Creutzers; or in Rix-dollars and Creutzers; also in Florins and Batzen.

4 Fennings	= 1 Creutzer
4 Creutzers	= 1 Batzen
60 Creutzers, or 15 Batzen	= 1 Florin
90 Creutzers, or 1½ Florin	= 1 Rix-dollar

QUOTATION.

Francfort gives to		*variable.*		*certain.*
Amsterdam	140	Rix-dollars	for 100	Rix-dollars current
Augsburg	101	Rix-dollars	100	Rix-dollars bank
Basil	101	Rix-dollars	100	New Ecu
Bremen	108	Rix-dollars	100	Rix-dollars
France	79	Rix-dollars	300	Livres
Hamburgh	150	Rix-dollars	100	Rix-dollars bank
Leipzig	100½	Rix-dollars	100	Rix-dollars
Vienna	60	Florins	100	Florins

TABLE VII. VIENNA AND AUGSBERG.

At *Vienna, Augsburg*, &c. the lowest piece of money is a *Fenning*, value $\frac{1}{30}d.$ sterling.

EXCHANGES are computed in Florins, Creutzers, and Fennings; or in Rix-dollars and Creutzers.

4 Fennings	= 1 Creutzer
60 Creutzers	= 1 Florin
90 Creutzers, or 1½ Florin	= 1 Rix-dollar of Account

At Augsburg 100 Florins of Exchange are equal to 127 Florins current.

QUOTATION.

Vienna gives to		*variable.*		*certain.*
Amsterdam	286	Rix-dollars	for 100	Rix dollars current
Augsburg	202½	Rix-dollars	100	Rix-dollars current
Constantinople	112	Florins	1	Piastre
Hamburgh	300	Rix-dollars	100	Rix-dollars bank
London	19	Florins	£1	sterling
Paris	47	Creutzers	1	Franc
Venice	184	Florins	300	Lire Piccoli
Receives from				
Genoa	30	Soldi fuori banco	1	Florin
Leghorn	28	Soldi moneta buona	1	Florin
Milan	33	Soldi currenti	1	Florin

Augsburg gives to	*variable.*	*certain.*
Amsterdam	113 Rix-dollars of exch. for	100 Rix-dollars
Francfort	102 Rix-dollars current	100 Rix-dollars
Hamburgh	118 Rix-dollars current	100 Rix-dollars
Leipzig	99 Rix-dollars current	100 Rix-dollars
London	10 Florins 45 Creutzers	£1 sterling
Nuremburg	101 Florins current	100 Florins
Paris	120 Florins	300 Francs
Receives from		
Genoa	62 Soldi fuori banco	1 Florin
Leghorn	57 Soldi moneta buona	1 Florin
Vienna	128 Florins	100 Florins

TABLE VIII. AMSTERDAM, ROTTERDAM, &C.

At *Amsterdam* the lowest piece of money is a *Penning*, value $\frac{21}{320}$d. sterling.

EXCHANGES are computed in Guilders, Stivers, and Pennings; or in Pounds, Shillings, and Pence Flemish.

8 Pennings	= 1 Grot or Penny
2 Grots, or 16 Pennings	= 1 Stiver
6 Stivers	= 1 Shilling Flemish
20 Stivers	= 1 Florin or Guilder
2½ Guilders	= 1 Rix dollar
6 Guilders	= 1 Pound Flemish

QUOTATION.

Amsterdam gives to	*variable.*	*certain.*
France	54 Grots Flemish, for	2 Francs
Genoa	86 Grots Flemish	1 Pezzo of 5¾ Lire
Hamburgh	34 Stivers	2 Marcs
Leghorn	92 Grots Flemish	1 Pezzo of 8 Rials
London	34 Shill. 8 Grots Flem.	1 Pound sterling
Portugal	44 Grots Flemish	1 old Crusade of 400 Reis
Spain	99 Grots Flemish	1 Ducat of Exchange
Vienna	20 Stivers	1 Rix-dollar
Receives from		
Antwerp	104 Florins	100 Florins
Breslau	144 Rix-dollars	100 Rix-dollars
Venice	96 Soldi Piccoli	1 Florin

TABLE IX. FRANCE.

At *Paris*, &c. the lowest piece of money is a *Denier*, value $\frac{1}{24}$d. sterling.

EXCHANGES are computed in Francs and Cents; or in Livres, Sols, and Deniers Tournois.

10 Centimes	=	1 Décime
10 Décimes or 100 Cents	=	1 Franc
80 Francs	=	81 Livres
12 Deniers	=	1 Sol
20 Sols	=	1 Livre Tournois
3 Livres or 3 Francs	=	1 Ecu of Exchange
100 Sols in Francs	=	The 5-Franc Piece

Tournois is a term of the same import in France as sterling in England. The Franc, or new Livre, is $1\frac{1}{4}$ per cent better than the old Livre Tournois; the new Livre consisting of 243 Deniers, the old 240. Hence, to reduce Francs and Cents to Livres, multiply by 81, and divide by 80.

QUOTATION.

Paris gives to	*variable.*	*certain.*
Augsburg	249 Céntimes	for 1 Florin current
Basil	101 Livres	100 Livres
Geneva	162 Francs	100 Livres current
Genoa	465 Céntimes	1 Pezzo of $5\frac{3}{4}$ Lire
Hamburgh	185 Francs	100 Marcs
Leghorn	504 Céntimes	1 Pezzo of 8 Rials
London	24 Francs	£1 sterling
Naples	4 Francs 20 Cents	1 Ducat Regno
Spain	15 Francs 40 Cents	1 Doubloon of Plate
Vienna	160 Céntimes	1 Florin
Receives from		
Amsterdam	54 Grots Flemish	3 Francs
Francfort	75 Rix-dollars	300 Francs
Lisbon	460 Reis	3 Francs

TABLE X. MADRID, CADIZ, &c.

At *Madrid, Cadiz,* &c. the lowest piece of money is a *Maravedi,* value $\frac{27}{136}$d. sterling.

Exchanges are computed in Dollars or Piastres, Rials, and Maravedis of Old Plate; also in Ducats of Exchange, and in Doubloons of Plate, or Pistoles of Exchange.

34 Maravedis	=	1 Rial
8 Rials	=	1 Dollar of Plate, Pezzo, Piastre, or Piece of Eight
375 Maravedis of Plate	=	1 Ducat of Exchange
32 Rials, or 4 Dollars of Plate	=	1 Pistole of Exchange
512 Maravedis *veillon*	=	1 Piastre
2048 Maravedis *veillon*	=	1 Pistole of Exchange

Veillon is the current money of Spain. A difference is often made between the *effective* money of Spain and the government paper, as appears by the lists of the course of exchange, published in the city of London. The paper has been of late at a considerable discount. *One thousand* Spanish dollars weigh 866 ounces English.

QUOTATION.

Madrid receives from	*variable.*		*certain.*
Paris............	15 Francs 40 Céntimes	for	1 Doubloon of Plate
Cadiz gives to			
Genoa..........	121 Dollars of Plate	..	100 Pezzos of $5\frac{1}{4}$ Lire
Leghorn	130 Dollars of Plate	..	100 Pezzos of 8 Rials
Naples..........	290 Maravedis of Plate		1 Ducat Regno
Receives from			
Amsterdam	97 Grots Flemish	......	1 Ducat of Exchange
Hamburgh	90 Grots Flemish	......	1 Ducat of Exchange
Lisbon	2470 Reis	....	1 Doubloon of Plate
London	42 Pence Sterling	......	1 Dollar of Plate
Paris	78 Sols Tournois	......	1 Dollar of Plate

TABLE XI. LISBON, &c.

At *Lisbon*, &c. the lowest piece of money is a *Rei*, value $\frac{27}{400}$*d.* sterling.

EXCHANGES are completed in Reis, and likewise in Old Crusades. Bills in Portugal are paid in the currency of the country, *viz.* half cash and half paper. The paper is at a considerable discount.

1000 Reis = 1 Mille-reis.
400 Reis = 1 Old Crusade.

QUOTATION.

Lisbon gives to	*variable.*		*certain.*
Genoa................	746 Reis	.........for	1 Pezzo of $5\frac{1}{4}$ Lire
Leghorn	810 Reis	....	1 Pezzo of 8 Rials
Paris	470 Reis	....	3 Francs
Spain	2430 Reis	....	1 Doubloon of plate
Venice	66 Reis	....	1 Lira Piccoli
Vienna	360 Reis	....	1 Florin
Receives from			
Amsterdam	45 Grots Flemish	....	1 old Crusade
Hamburgh	43 Grots Flemish	....	1 old Crusade
London	66 Pence sterling	....	1 Mille-reis

TABLE XII. GENOA.

At *Genoa* the lowest price of money is a *Denari*, value $\frac{43}{1200}$*d.* sterling.

EXCHANGES are computed in Lire, Soldi, and Denari di lira; or in Pezzos, Soldi, and Denari di Pezzo; all in currency, called *fuori di banco.*

12 Denari di Lira	=	1 Soldi di Lira
20 Soldi di Lira	=	1 Lira
$5\frac{1}{4}$ Lira	=	1 Pezzo
12 Denari di Pezzo	=	1 Soldi di Pezzo
20 Soldi di Pezzo	=	1 Pezzo
4 Lire 12 Soldi	=	1 Crown of Exchange
10 Lire 14 Soldi	=	1 Gold Crown

QUOTATION.

Genoa gives to	*variable.*	*certain.*
Augsburg	62 Soldi di Lira ··for	1 Florin
Hamburgh...........	45 Soldi di Lira ····	1 Marc
Leghorn	124 Soldi di Lira ····	1 Pezzo of 8 Rials
Naples	103 Soldi di Lira ····	1 Ducat Regno
Vienna	30 Soldi di Lira ····	1 Florin
Receives from		
Amsterdam...........	85 Grots Flemish····	1 Pezzo
France	94 Sols in Francs ····	1 Pezzo
Lisbon	718 Reis.............	1 Pezzo
London	48 Pence sterling......	1 Pezzo
Palermo.............	36 Grani	1 Lira
Spain	620 Maravedis of plate	1 Gold Crown
Venice	33 Soldi Piccoli	1 Lira

TABLE XIII. LEGHORN.

At *Leghorn* the lowest piece of money is a *Denari*, value $\frac{5}{24}$*d.* sterling.

EXCHANGES are computed in Pezzos, Soldi, and Denari di Pezzo, or in Lira, Soldi, and Denari di lira moneta buona.

12 Denari di Pezzo	=	1 Soldi di Pezzo
20 Soldi di Pezzo	=	1 Pezzo of 8 Rials
12 Denari di Lira	=	1 Soldi di Lira
20 Soldi di Lira	=	1 Lira
$5\frac{3}{4}$ Lire, moneta buona	=	1 Pezzo of 8 Rials
6 Lire, moneta lunga	=	1 Pezzo of 8 Rials

QUOTATION.

Leghorn receives from	*variable.*	*certain.*
Augsburg	204 Florins current for	100 Pezzos of 8 Rials
Amsterdam ········	95 Grots Flemish....	1 Pezzo of 8 Rials
France	104 Sols in Francs····	1 Pezzo of 8 Rials
Genoa...............	125 Soldi fuori banco	1 Pezzo of 8 Rials
Hamburgh ··········	89 Grots Flemish····	1 Pezzo of 8 Rials
Lisbon	855 Reis.............	1 Pezzo of 8 Rials
London	54 Pence sterling····	1 Pezzo of 8 Rials
Naples	118 Ducats Regno ..	100 Pezzos of 8 Rials
Palermo.............	11 Tari 15 Grani····	1 Pezzo of 8 Rials
Petersburg...........	190 Roubles ········	100 Pezzos of 8 Rials

TABLE XIV. NAPLES.

At *Naples* the lowest piece of money is a *Grano*, value $\frac{4}{10}$*d.* sterling.

EXCHANGES are computed in Ducats and Grani di Regno; or in Ducats, Carlini, and Grani.

10 Grani	=	1 Carlini
10 Carlini	=	1 Ducat Regno
100 Grani	=	1 Ducat Regno

QUOTATION.

Naples gives to	*variable.*	*certain.*
Amsterdam	54 Grani	for 1 Florin
Hamburgh	45 Grani	1 Marc
Leghorn	120 Ducats Regno	100 Pezzos
Spain	85 Grani	1 Dollar of Plate
Receives from		
France	84 Sols in Francs	1 Ducat Regno
Genoa	102 Soldi Fuori Banco	1 Ducat Regno
Lisbon	670 Reis	1 Ducat Regno
London	42 Pence sterling	1 Ducat Regno
Venice	9 Lire 15 Soldi Piccoli	1 Ducat Regno

TABLE XV. VENICE.

At *Venice* the lowest piece of money is a *Denari* Piccoli, value $\frac{1}{36}$*d.* sterling, used in buying and selling merchandise.

EXCHANGES are computed in Lire, Soldi, and Denari Piccoli; and also in Ducats.

12 Denari	=	1 Soldo
20 Soldi	=	1 Lira
6 Lire 4 Soldi	=	1 Ducat of Account, or current
8 Lire	=	1 Ducat *effective*

QUOTATION.

Venice gives to	*variable.*	*certain.*
Amsterdam	4 Lire 18 Soldi Piccoli	for 1 Florin
Augsburg	4 Lire 16 Soldi Piccoli	1 Florin
Constantinople	3 Lire 6 Soldi Piccoli	1 Piastre
Genoa	38 Soldi Piccoli	1 Lira Fuori Banco
Hamburgh	4 Lire 6 Soldi Piccoli	1 Marc
Leghorn	11 Lire 18 Soldi Piccoli	1 Pezzo
London	56 Lire Piccoli	£1 sterling
Naples	9 Lire 18 Soldi Piccoli	1 Ducat Regno
Vienna	4 Lire 8 Soldi Piccoli	1 Florin

TABLE XVI. CONSTANTINOPLE.

At *Constantinople* the lowest piece of money is a *Mangar*, value $\frac{3}{20}$*d.* sterling.

EXCHANGES are computed in Piastres, Paras, and Aspers.

4 Mangars	=	1 Asper
3 Aspers	=	1 Para
40 Paras	=	1 Piastre or Dollar
120 Aspers	=	1 Piastre or Dollar

QUOTATION.

Constantinople gives to	variable.	certain.
Amsterdam	65 Paras	for 1 Florin current
Genoa	24 Paras	1 Lira Fuori Banco
Leghorn	145 Paras	1 Pezzo of 8 Rials
London	17 Piastres	£1 sterling
Paris	200 Piastres	300 Francs
Venice	360 Paras	1 Sequin of 22 Lire
Vienna	50 Paras	1 Florin
Receives from		
Hamburgh	25 Grots Flemish	1 Piastre

TABLE XVII. EAST-INDIA SETTLEMENTS AND CANTON.

1. At *Bengal*, accounts are kept in imaginary Coins, called Current Rupees, Annas, and Pice.

12 Current Pice	=	1 Current Anna
16 Current Annas	=	1 Current Rupee
100 Sicca Rupees	=	116 Current Rupees

All real specie must be reduced to this currency, before any sum can be entered into books of accounts.

2. At *Madras*, accounts are kept in Star Pagodas, Fanams, and Cash.

80 Cash	=	1 Fanam
From 42 to 46 Fanams	=	1 Star Pagoda

Cash pieces are small copper coins struck in England, and sent to Madras for general circulation. The value is marked upon each piece. The European merchants at Madras keep their accounts at 12 Fanams the Rupee, and 42 Fanams the Star Pagoda; and the Natives at 12 Fanams 60 Cash the Rupee, and 44 Fanams 50 Cash the Star Pagoda.

3. At *Bombay*, accounts are kept in Rupees, Quarters, and Reis.

100 Reis	=	1 Quarter
4 Quarters	=	1 Rupee

The Coins real and imaginary are various at Bombay, but Accounts are confined to those above specified.

4. At *Canton* there is but one piece of Coin, made of base metal called a *Cash*. It is used to pay coolies, labourers, and for small payments in Bazars.

10 Cash	=	1 Candarine
10 Candarine	=	1 Mace
10 Maces	=	1 Tale
3 Tales	=	£1 sterling

TABLE XVIII. NEW YORK, PHILADELPHIA, BALTIMORE, &c.

At *New York, Philadelphia*, &c. EXCHANGES are computed in Dollars, Dimes, and Cents; and in some places in Pounds, Shillings, and Pence.

10 Cents	= 1 Dime
10 Dimes or 100 Cents	= 1 Dollar
12 Pence currency	= 1 Shilling currency
20 Shillings currency	= 1 Pound currency

QUOTATION.

New York gives to	*variable.*	*certain.*
Amsterdam	42 Cents.............	for 1 Guilder
Bremen	78 Cents	1 Rix-dollar
Hamburgh	35 Cents	1 Marc
London	£177 currency	£100 sterling
Receives from		
Paris	5 Francs 30 Cents......	1 Dollar
Philadelphia gives to		
Amsterdam	43 Cents.............	for 1 Guilder
Hamburgh	35 Cents	1 Marc
Receives from	*s. d.*	
London	4 6 sterling at par......	1 Dollar
Baltimore gives to		
Amsterdam	40 Cents.............	for 1 Guilder
Hamburgh	33 Cents	1 Marc
London	101 Cents	4*s.* 6*d.* sterling.

TABLE XIX. LONDON.

At *London*, &c. EXCHANGES are computed in Pounds, Shillings and Pence. *See the Table*, p. 18.

QUOTATION, or *Lloyd's List, Sept.* 22, 1818.

London receives from	*variable.*	*certain.*
Amsterdam	37 *Shillings* 10 *grots Flemish*..	for £1 *sterling*
Rotterdam	11 *Florins* 10 *Stivers current*......	£1 *sterling*
Hamburgh	34 *Shillings* 10 *Grots Flemish*	£1 *sterling*
Paris	24 *Francs* 70 *Cents*	£1 *sterling*
Venice	25 *Lire Piccoli*	£1 *sterling*
Dublin	9½ per cent. viz. £109½ Irish for	£100 *sterling*
London gives to		
Genoa	47 *Pence sterling*	1 *Pesso of* 5¾ *Lire*
Leghorn	51½ *Pence sterling*	1 *Pesso of* 8 *Rials*
Naples	43 *Pence sterling*	1 *Ducat Regno*
Lisbon..........	58¼ *Pence sterling*	1 *Mille-Reis*
Madrid	38¼ *Pence sterling*	1 *Dollar of Plate*

N. B. The words in Italics are generally omitted in *Lloyd's List*: they are inserted here by way of explanation.

Table XX.

The intrinsic *par* of Exchange between *London* and the following places in *Lloyd's List*; calculated according to the *Mint* regulations of each respective place, by comparing Gold with Gold, and Silver with Silver.

	Gold.	*Silver.*	*value.*
Amsterdam cur.	37sh. 4·9d. Flemish;	38sh. 1d. Flemish	=£1 sterling
Amsterdam Bank Agio 4 per cent.	35sh. 11·6d Flemish;	36sh. 7·5d. Flemish	=£1 sterling
Rotterdam cur.	11 Florins 4·5 Stiv.;	11 Florins 8·5 Stiv.	=£1 sterling
Hamburgh Bank	34 sh. 3·5d. Flemish;	35sh. 1d. Flemish	=£1 sterling
Paris, Old Coins	25 Liv. 9 sol. 11 den.;	25 Liv. 1 sol. 9 den.	=£1 sterling
Paris, New Coins	25 Liv. 10 sol. 6 den.;	25 Liv. 0 Sol. 9½ den.	=£1 sterling
Paris	25 Francs 21 Cents;	24 Francs 73 Cents.	=£1 sterling
Genoa	45·52 Pence sterling;	46 Pence sterling	= 1 Pezzo of 5¾ Lire
Leghorn	49·09 Pence sterl.;	46·67 Pence sterl.	= 1 Pezzo of 8 Rials
Naples	42·57 Pence sterl.;	43·5 Pence sterl.	=1 Duc. Regno
Lisbon	67·4 Pence sterl.;	69·4 Pence sterl.	=1 Mille-Reis
Madrid & Cadiz	37·3 Pence sterl;	39·22 Pence sterl.	=1 Dol. of Plate
Venice	46·28 Lire Piccoli;	47·5 Lire Piccoli	=£1 sterling

Table XXI.

The intrinsic *par* of Exchange between *London* and the following places in *Lloyd's List*; calculated from assays lately made both in London and Paris, by comparing Gold with Gold, and Silver with Silver.

	Gold.	*Silver.*	*value.*
Amsterdam cur.	37sh. 4d. Flemish;	38sh. 7½d. Flemish	=£1 sterling
Amsterdam Bank Agio 4 per cent.	35sh. 10·8d Flemish;	37sh. 1·7d. Flemish	=£1 sterling
Rotterdam cur.	11 Florins 4 Stivers;	11 Florins 14 Stiv.	=£1 sterling
Hamburgh Bank	34sh. 2·4d. Flemish;	35sh. 1d. Flemish	=£1 sterling
Paris, Old Coins	25 Liv. 9 sol. 9 den.;	25 Liv. 9 sol. 9 den.	=£1 sterling
Paris, New Coins	25 Liv 11 sol. 6·2 den;	25 Liv. 3 sol. 7½ den.	=£1 sterling
Paris	25 Francs 26 Cents;	24 Francs 87 Cents.	=£1 sterling
Genoa	45·52 Pence sterl.;	45·82 Pence sterl.	=1 Pez. 5¾ Lire
Leghorn	49·05 Pence sterl.;	46·57 Pence sterl.	= 1 Pezzo of 8 Rials
Naples	42·. Pence sterl.;	41·25 Pence sterl.	=1 Duc. Regno
Lisbon	66·5 Pence sterl.;	68·4 Pence sterl.	=1 Mille-Reis
Madrid & Cadiz	36·05 Pence sterl.;	39 Pence sterl.	=1 Dol. of Plate
Venice	46·38 Lire Piccoli;	48·9 Lire Piccoli	=£1 sterling

From the two preceding tables it appears that the par in gold generally varies from that in silver, and in some places the difference is considerable; but the assays do not differ essentially from the Mint regulations. The *commercial par* is the comparative value of the Coins of different countries, according to their weight, fineness, and the market price of the metals of which they are composed. If a sum of money in the currency of any state will buy a pound of bullion in the market of that state, and also purchase a bill for a sum of English currency, which, currency or bill, would buy a pound of bullion of the same standard in the English market, a complete commercial par of exchange is established between the two countries.

RULES FOR CALCULATING WHAT QUANTITY OF THE MONEY OF ONE COUNTRY WILL BE EQUAL TO A GIVEN QUANTITY OF THE MONEY OF ANOTHER, ACCORDING TO A GIVEN COURSE OF EXCHANGE.

CLASS I.

Places which give the uncertain price of exchange for the pound sterling.

These are HAMBURGH, AMSTERDAM, SWEDEN, DANTZIC, VIENNA, VENICE, FRANCE, &c.

Proposition I. To reduce the currency of any state into bank or exchange money, and the contrary.

Rule. As 100, with the agio added to it, is to 100, so is any given sum current to its value in bank-money.

And, As 100 is to 100 with the agio added to it, so is any given sum of bank-money to its value current.

Note. The exchange is always supposed to be made in bank money, and, therefore, the currency of any state or kingdom, which uses this denomination of money, must always be reduced to bank-money before exchange can be made.

Prop. II. Given the course of exchange between Great Britain and any foreign country, city, &c. which exchanges for the pound sterling, to change any given quantity of sterling money into the money of that country, &c.

Rule. As £1. sterling is to the given course of exchange, so is the given sum, in sterling money, to its corresponding value in foreign money.

Note. Whenever the first term of a stating is 1, as in this proposition, the work may be performed by *Practice.*

Prop. III. Given the course of exchange to or from any foreign country, city, &c. which exchanges with Great Britain for the pound sterling, to change any given quantity of such foreign money into sterling money.

Rule. As the course of exchange is to £1. sterling, so is the given sum, in foreign money, to its corresponding value in sterling money.

CLASS II.

Places which give the certain species of their money for an uncertain number of pence sterling.

These are RUSSIA, SPAIN, PORTUGAL, ITALY, &c. and almost every other place in the world, with which exchanges are made, except those already mentioned in Class I. and those which belong to Class III.

Prop. IV. Given the course of exchange between Great Britain and any foreign country, city, &c. which exchanges for any number of pence sterling, to change any quantity of sterling money into the money of that country, &c.

Rule. As the number of pence sterling, contained in the course of exchange, is to the integer of foreign money, so is the given sum, in sterling money, to its corresponding value in foreign money.

Prop. V. Given the course of exchange between Great Britain and any foreign country, city, &c. which exchanges for any number of pence sterling, to change any quantity of such foreign money into sterling.

Rule. As the integer of foreign money is to the number of pence contained in the course of exchange, so is the given quantity of foreign money to its corresponding value in sterling money.

CLASS III.

Places which exchange with GREAT BRITAIN *at an advanced rate per cent.*

These are the ISLES of MAN and IRELAND, the WEST-INDIA ISLANDS, and Part of NORTH AMERICA.

Prop. VI. Given the course of exchange between Great Britain and any place which gives a variable sum of money, more than 100*l. for* 100*l. sterling, to change any quantity of sterling money into the currency of that place.*

Rule. As 100*l.* sterling is to 100*l.* with the course of exchange per cent. added to it, so is the given sterling money to the currency required.

Note. In this and the following rule, by the course of exchange must be understood the excess of the currency above 100l. Thus, if 100l. sterling be worth 110l. currency, the exchange is at 10 per cent.

This excess, were it authorised by custom, might be called the agio.

Prop. VII. Given the course of exchange between Great Britain and any place which gives a variable sum of money, more than 100*l. for* 100*l. sterling, to change any quantity of the currency of that place into sterling money.*

Rule. As 100*l.* with the course of exchange per cent. added to it, is to 100*l.* so is the given currency to the sterling required.

OF THE GAIN OR LOSS PER CENT BY THE RISING OR FALLING OF THE COURSE OF EXCHANGE.

Prop. VIII. To determine the gain or loss per cent. by the different courses of exchange with places that exchange by the pound sterling, or with places that exchange for a variable number of pence sterling.

Rule. If the gain or loss per cent. be considered with respect to the par of exchange, say, As the par of exchange is to 100*l.* so is the given *course* of exchange to a fourth number; which, if greater than 100*l.* the excess will be the gain; but, if less than 100*l.* the defect will be the loss per cent.—But, if the gain or loss per cent. be considered with respect to any other *course* of exchange, say, As the given *course* of exchange is to 100*l.* so is the proposed *course* of exchange to a fourth number; which, if greater than 100*l.* the excess will be the gain; but, if less than 100*l.* the defect will be the loss per cent.

See *Prop.* 2. in *Loss and Gain.*

Examples to Proposition I. (*Tables* VIII. *and* XV.)

(1.) A merchant at *Amsterdam* is possessed of 3750 guilders 10 stivers currency, which he wishes to turn into bank money, the agio at $4\frac{3}{8}$ per cent., what will be the value in guilders bank?

$104\frac{3}{8}$: 100 :: 3750g. 10s. : 3593g. 5s. $13\frac{149}{167}$pen. answer.

(2.) If the agio between the current and bank-money of *Holland* be $4\frac{3}{8}$ per cent., how many guilders current will be equal to 3593 guilders 5 stivers $13\frac{149}{167}$ pennings-bank?

100 : $104\frac{3}{8}$ :: 3593g. 5s. $13\frac{149}{167}$p. : 3750g. 10s. answer.

(3.) Change 577 guilders 14 stivers current money into florins bank, agio $5\frac{1}{4}$ per cent.

(4.) Change 765 guilders 9 stivers bank into current, agio $5\frac{5}{8}$ per cent.

(5.) In 7570 guilders 15 stivers current, how many rix-dollars bank, agio $4\frac{7}{8}$ per cent.?

(6.) If the agio between the current and bank-money of Holland be 25 per cent., how many pounds Flemish bank will be equal to 797l. Flemish current?

(7.) The agio of Venice is 20 per cent., how much current money of Venice will be equal to 790 ducats bank.

Examples to Prop. II. (*Table* V.)

(8.) In 127*l.* 3*s.* 4*d.* sterling, how many *Hamburgh* marcs, &c. exchange at 32 shillings 4 grots Flemish per £. sterling?

£1 : 32*s.* 4gr. :: £127 3s. 4d. : $49340\frac{2}{3}$ grots Flemish=24670 shil. lub. 4 fen.=1541 marcs 14 sols lub. 4 fen.

(9.) How many *Hamburgh* marcs are contained in 4451*l.* 15*s.* sterling, exchange at $34\frac{1}{16}$ shillings Flemish per £. sterling?

(10.) In 475*l.* 18*s.* sterling, how many marcs, &c. exchange at 36*s.* 6*d.* Flemish per £. sterling?

(11.) In 749*l.* 14*s.* sterling, how many marcs bank, exchange at 35 shill. 1 grot Flemish per £. sterling?

(12.) In 754*l.* 18*s.* 9*d.* sterling, how many rix-dollars current, exchange at 34 shill. $9\frac{1}{2}$ grots Flemish per £. sterling, agio $18\frac{1}{4}$ per cent.?*

(*Table* VIII.)

(13.) If I pay 757*l.* 18*s.* 7*d.* in *London*, what must I draw my bill for on *Amsterdam*, exchange at 1*l.* 15*s.* 9*d.* Flemish per £. sterling?

(14.) If I pay in *London* 754*l.* 11*s.* 9*d.* sterling, how many guilders, &c. may I draw for at *Amsterdam*, exchange at 34 shill. $4\frac{1}{2}$ grots per £. sterling?

* The agio is never less than 18 per cent. and varies from 18 to 23 and 24 per cent.

(15.) In 479*l.* 14*s.* sterling, how many rix-dollars current, agio $4\frac{5}{8}$, and exchange at 34*s.* $7\frac{1}{2}$*d.* per £. sterling?

(*Table* II.)

(16.) In 547*l.* 19*s.* 10*d*, sterling, how many copper dollars of *Sweden*, exchange at $47\frac{1}{2}$ copper dollars per £. sterling.

(17.) In 3749*l.* 14*s.* $10\frac{1}{2}$*d.* how many dollars, &c., exchange at 48 copper dollars per £. sterling.

Examples to Prop. III. (*Table* V.)

(18.) Reduce 1541 marcs, 14 sols lub. 4 fen. bank-money of *Hamburgh* into sterling, exchange at $32\frac{1}{2}$ sols gros, or shillings Flemish, per £. sterling?

$32\frac{1}{2}$ sols gr. : 1*l.* :: 1541m. 14s. l. 4f. : 127*l.* 3*s.* 4*d.* answer.

(19.) In 1788 rix-dollars 21 sols lub. how many pounds sterling, exchange at $34\frac{1}{2}$ sols gros per £. sterling?

(20.) In 747 rix-dol. 2 marcs, 14 sols lub. how many £. sterling, exchange at 32s. 6d. Flemish per £. sterling?

(21.) In 743 rix dollars 4 sols gros, agio $18\frac{5}{8}$ per cent. exchange at 33*s.* 9*d.* Flemish per £. sterling, how many £. sterling?

(22.) In 1749 marcs 13 sols lub. current, agio $22\frac{3}{8}$ per cent. and 948 marcs 2 sols gros, agio $20\frac{3}{8}$ per cent. exchange at 35*s.* 8*d.* Flemish per £. sterling, how many £. sterling?

(*Table* VIII.)

(23.) Remitted from *Amsterdam* to *London* a bill of 1747*l.* 14*s.* 7*d.* Flemish, how many pounds sterling is the sum, exchange at 34*s.* 7*d.* Flemish per £. sterling?

(24.) What must I draw for at *London* if I pay at *Rotterdam* 7495 guild. 14 stiv. current, agio $5\frac{3}{8}$ per cent. exchange at 34 shill. 4 grots per £. sterling?

(25.) A merchant remits a bill of exchange from *Antwerp* to *England*, when the course is 34*s.* 3*d.* Required the value of 774*l.* 18*s.* Flemish at that rate in *London*?

(*Table* II.)

(26.) In 7123 copper dollars, 14 runstics, how many pounds sterling, exchange at $48\frac{1}{2}$ copper dollars per £. sterling?

(27.) In 5749 silver doll. 1 copper doll. 2 copper marks, 3 runstics, how many £. sterling?—Exchange at 49 copper dollars per £. sterling?

Examples to Prop. IV. (*Table* I.)

(28.) In 747*l.* 18*s.* 10*d.* sterling, how many rix-dollars of *Denmark*, exchange at 47*d.* sterling per rix-dollar?

47*d.* : 1 rix-dol. :: 747*l.* 18*s.* 10*d.* : 3819 rix-dol. 1 marc. $10\frac{26}{47}$ skillings.

(29.) In 749*l.* 16*s.* sterling, how many rix-dollars, &c. exchange at $49\frac{1}{2}$*d.* sterling per rix-dollar?

(*Table* III.)

(30.) In 7574*l.* 19*s.* sterling, how many *Russian* roubles, &c., exchange at 4*s.* 7*d.* sterling per rouble?

(31.) In 574*l.* 18*s.* sterling, &c. how many roubles, exchange at 4*s.* $9\frac{1}{2}$*d.* per rouble?

Examples to Prop. V. (*Table* I.)

(32.) In 3819 rix-dollars, 1 marc, $10\frac{26}{47}$ skillings of *Denmark*, how much sterling money, exchange at 47*d.* sterling per rix-dollar?

1 rix-dol. : 47*d.* :: 3819 rix-dol. 1 marc $10\frac{26}{47}$ skill : £747 18 10

(33.) In 9751 rix-dol. 4 m. 3 skil. how much sterling, exchange at $48\frac{3}{4}$*d.* sterling per rix-dollar?

(*Table* III.)

(34.) In 7454 roub. 4 griv. 6 cop. how many £. sterling, exchange at 4*s.* 9*d.* per rouble?

(35.) In 7479 roubles, how much sterling, exchange at 4*s.* $7\frac{1}{2}$*d.* per rouble?

CLASS II. (*Tables* III. *and* VIII.)

In the following examples, the rules belonging to the propositions hitherto made use of, are to be used occasionally.

(36.) In 4759 roub. 44 cop., exchange at 124 copecs per rix-dollar current at *Amsterdam*, agio $3\frac{1}{2}$ per cent., how much sterling money?—the exchange between *Amsterdam* and *London* being 34*s*. 6*d*. Flemish per £. sterling.

(37.) Remitted from *London* to *Petersburg*, by the way of *Amsterdam*, 495*l*. 17*s*. 6*d*. sterling, the exchange between *London* and *Amsterdam* being 34*s*. 8*d*. per £. sterling, and between *Amsterdam* and *Petersburg* 52 stivers per rouble; what is the value of this remittance in roubles, &c.?

(38.) Received from *Archangel*, per bill of exchange, 7437 roub. 5 griv. 24 cop. exchange at 121 copecs per rix-dollar current of *Amsterdam*, agio $3\frac{1}{8}$ per cent., and 34*s*. 7*d*. Flemish per £. sterling; what is the value of this bill?

(*Tables* IV. *and* VIII.)

(39.) In 7947 florins of *Dantzic*, exchange at 270 groshen per £. Flemish, and 33*s*. 5*d*. Flemish per £. sterling, how much sterling money?

(40.) In 749*l*. 17*s*. 6*d*. sterling, how many rix-dollars, &c., exchange at 274 groshen per £. Flemish, and 34*s*. 8*d*. per £. sterling?

(41.) In 4795 flor. 24 groshen, how many pounds sterling, exchange at 273 groshen per £. Flemish current, agio $3\frac{1}{8}$ per hundred guilders, and 33*s*. 7*d*. Flemish per £. sterling?

(*Table* IX.)

(42.) In 636 livres Tournois, 3 sols, $9\frac{1}{2}$ deniers, how many £. sterling, exchange at 23 francs 96 cents* per £. sterling?

(43.) Bought wine of a merchant at *Bordeaux* to the amount of 57475 livres 6 sols; for what sterling money must the merchant draw his bill, exchange at 24 livres 14 sols per £. sterling?

* London formerly exchanged with Paris by giving an uncertain number of shillings and pence for the ecu of 3 livres. The livre is still retained in Lloyd's List; but the French generally reckon in francs and cents.

(44.) A bill of 759*l.* 18*s.* 9*d.* is remitted to *Paris* by a merchant in *London;* what is the value in francs and cents, exchange at 23 francs 45 cents per £. sterling?

(45.) A gentleman (on his travels) received at *Paris* 3749 crowns, 2 livres, 10 sols, for a bill of exchange, the value whereof in *England* was 483*l.* 14*s.* 3*d.* What was the course of exchange between *England* and *France?* that is, how many francs and cents were given for £1 sterling?

(*Table* X.)

(46.) In 749*l.* 18*s.* sterling, how many piastres, or pieces of eight, at *Madrid*, exchange at $45\frac{3}{8}d.$ sterling per piastre?

(47.) In 1347 piastres, 2 rials, 24 maravedies, of *Madrid*, how much sterling, exchange at $47\frac{1}{2}d.$ per piastre?

(48.) In 9749 rials of plate, how many £. sterling, exchange at $43\frac{1}{8}d.$ per piastre?

(49.) Bought raisins of a merchant at *Malaga* to the amount of 7549 rials Veillon; for what sterling money must the merchant draw his bill, exchange at $41\frac{1}{4}d.$ per piastre?

(*Table* XI.)

(50.) In 7434 crusades 347 reis, how many £. sterling exchange at 65*d.* per mille-reis?

(51.) A merchant at *Lisbon* remits to *London* 4756 mille-reis 290 reis, exchange at $64\frac{1}{2}d.$ per mille-reis; how much sterling must be paid in *London* for this remittance?

(52.) If a bill of 1788*l.* 17*s.* sterling be drawn upon *London*, what is the value at *Oporto* in mille-reis, exchange at $66\frac{1}{2}d.$ per mille-reis?

(53.) If 2000 mille-reis were paid at *Lisbon* for a bill upon *London* of 666*l.* 13*s.* 4*d.*, what was the course of exchange?

(*Tables* XII. *and* XIII.)

(54.) How much sterling money may a person receive in *London*, if he pays in *Genoa* 947 pezzos, exchange at $53\frac{1}{4}d.$ per pezzo?

(55.) *London* is indebted to *Genoa* 1749*l.* 17*s.* 6*d.*, for how many pezzos may *Genoa* draw on *London*, the exchange at 47¾*d.* per pezzo?

(56.) In 747*l.* 16*s.* 4*d.* sterling, how many pezzos of *Leghorn*, exchange at 46⅛*d.* per pezzo?

(57.) *London* is indebted to *Leghorn* 7430 pezzos, or piastres, 9 soldi, 3 denari; what sterling money stands as an equivalent in the *London* merchant's books, the exchange being 48⅜*d.* per piastre?

(58.) A bill of 574*l.* 15*s.* is remitted to *Leghorn*, to be paid in *piastres* of 6 lires each, exchange at 54*d.* per piastre; how many will be received?

Examples to Prop. VI.

(59.) *London* remits to *Ireland* 574*l.* 15*s.* sterling, how much currency of *Ireland* must be received, exchange at 7*l.* 10*s.* per cent.?

100l. : 107l. 10s. :: 574l. 15s. : 617. 17s. 1½d. answer.

(60.) The value of 694*l.* 18*s.* 6*d.* sterling is required in *Irish* currency, exchange at £5¼ per cent.?

(61.) *London* receives a bill of exchange from *North Carolina* for 917*l.* 18*s.* sterling; for how much currency was *London* indebted, exchange at 76 per cent.?

Examples to Prop. VII.

(62.) *Dublin* draws upon *London* for 879*l.* 6*s.* 6¼*d.* *Irish*, exchange at 11⅜ per cent. How much must *London* pay *Dublin* to discharge the bill?

111⅜. : 100l. :: 879l. 6s. 6¼d. : 787l. $15\frac{5}{2673}$sh.

(63.) What must be paid in *London* for a remittance of 6747*l.* 14*s.* *Irish*, exchange at 11½ per cent.?

(64.) *Jamaica* remits to *London* 475*l.* 14*s.* 10*d.* currency, what sterling money must be received for it, exchange being at £135 currency for £100 sterling?

CLASS II. *exercising the 6th and 7th Propositions.*

(65.) A merchant in *London* consigns to his factor in *Jamaica* goods amounting to 734*l.* 14*s.* 9*d.* sterling, which are sold for 900*l.* currency; what sterling ought the factor to remit, after deducting 5 per cent. for his commission and charges; and whether does the merchant gain or lose, and how much; the exchange being at 25 per cent.?

(66.) My factor at *Barbadoes* bought goods for me to the amount of 7150*l.* 14*s.* currency; what is the value in sterling money, allowing the factor $2\frac{1}{2}$ per cent. for commission, the exchange being at 35 per cent?

(67.) A merchant at *Boston* stands indebted to his correspondent in *London* 7549*l.* 18*s.* 4*d.* currency; what sterling sum stands as an equivalent in the *London* merchant's books, exchange at 57 per cent.?

(68.) Sold sugar in *London* for my employer in *Jamaica* to the amount of 1757*l.* sterling; what currency ought I to remit, after deducting $2\frac{1}{2}$ per cent. for commission, the exchange between *London* and *Jamaica* being £157 currency for £100 sterling?

Examples to Prop. VIII.

(69.) *London* draws upon *Holland* for a sum of money when the exchange is at 35*s.* 6*d.* Flemish per £. sterling, and afterwards draws again when the exchange is at 34*s.* 6*d.* What does *London* lose or gain per cent. by this negociation when compared with the former?

35s. 6d. : 100l. :: 34s. 6d. : 97l. 3s. $7\frac{67}{71}$d.
Then 100l.—97l. 3s. $7\frac{67}{71}$d.=2l. 16s. $4\frac{6}{71}$d. loss per cent.

(70.) *London* draws upon *Amsterdam* for a sum of money when the exchange is at 34*s.* 6*d.* Flemish per £. sterling, and afterwards draws again when the exchange is at 35*s.* 6*d.* How much does *London* gain or lose per cent. by this transaction, when compared with the former?

34s. 6d. : 100l. :: 35s. 6d. : 102l. 17s. $11\frac{135}{207}$d.
Then 102l. 17s. $11\frac{135}{207}$d.—100=2l. 17s. $11\frac{135}{207}$d. gain per cent.

(71.) If the par of exchange between *London* and *Amsterdam* be *$37\frac{1}{9}s$. Flemish per £. sterling, what does *London* gain or lose per cent. by drawing bills upon *Holland* at 33*s*. 4*d*. Flemish per £. sterling.

(72.) Suppose *London* exchanges with *Holland* when the course of exchange is at 35*s*. 6*d*. per £. sterling, what will be the gain or loss per cent. to *London*, admitting the *par* of exchange to be 33*s*. 4*d*. per £. sterling?

(73.) A bill of exchange was drawn upon *Amsterdam* when the course of exchange was 34*s*. 3*d*. Flemish per £. sterling; and, some time after, another was drawn, when the course of exchange was 33*s*. 6*d*. Flemish per £. sterling; what was gained or lost per cent. by this negociation when compared with the former?

CLASS II.

(74.) If the *par* of exchange between *London* and *Portugal* be 5*s*. $7\frac{1}{2}d$. sterling per mille-reis, what is gained or lost per cent. in *London*, when the course of exchange is 5*s*. $2\frac{1}{2}d$. per mille-reis?

(75.) Suppose *London* exchanges with *Portugal* for a mille-reis at 5*s*. 6*d*. sterling, and afterwards at 5*s*. $1\frac{1}{2}d$.—What is gained or lost per cent. by the latter negociation, when compared with the former?

(76.) Suppose the course of exchange between *London* and *Madrid* to be $41\frac{3}{4}d$. sterling per piastre, at which time a bill of exchange is drawn by *London*; what would have been the gain or loss per cent. to *London*, had the bill been drawn when the exchange was at $53\frac{1}{2}d$. sterling per piastre, by comparing the latter negociation with the former?

ARBITRATION OF EXCHANGE.

Definition. *By Arbitration*, or the comparison of exchange, is to be understood a method of remitting to, or drawing upon, foreign places in such a manner as shall be most advantageous to the merchant.

* The table, given at p. 387 of the Negociator's Magazine, is calculated upon this principle.

I. *Simple Arbitration.*

Definition. When the exchanges among three places *only* are concerned, it is called *Simple* Arbitration, and the *arbitrated* price is such a rate of exchange between two of the places as shall be in proportion with the rates assigned between each of them and a third.

Note. All questions in simple arbitration may be solved with a little consideration by one or more statings in the direct or inverse rule of three.—If a gain or loss per cent. is mentioned, after you have found the proportional gain or loss by the rule of three, the gain or loss per cent. by a variation of exchange, may be found by Proposition 8 preceding, if it has no regard to time. But, if time, commission, brokerage, &c. are considered, the several allowances to be made for these purposes must be calculated by the rules of interest, commission, brokerage, &c. previous to the operation for the gain or loss per cent.

II. *Compound Arbitration of Exchange*, called by Merchants, *The Chain Rule of Three.*

Definition. *Compound Arbitration* has respect to the exchanges of four or more countries, or cities, and its utility consists in discovering the best and most advantageous method of negociating exchanges with different places.

Proposition. To determine whether a direct or circular exchange will be preferable, having the course of exchange between several places given.

Rule. Distinguish the several courses of exchange into *antecedents* and *consequents:* place the *antecedents* in one column, and the *consequents* in another, to the right-hand of the *antecedents,* in such a manner that the first *consequent* may be of the same name and denomination as the second *antecedent,* and the second *consequent* as the third *antecedent,* &c. through the whole. Then multiply all the *antecedents* together for a divisor, and all the *consequents* together for a dividend; the quotient produced from this divisor and dividend will be the value of the sum required. Then calculate the value of the sum by the direct exchange or by any other circular exchange; and by comparing these

values together, may be seen which will be the most advantageous.

Note 1. By this rule the weights, measures, &c. of different countries may be compared. If an allowance for commission, &c. is to be made from place to place, the most certain method will be to find the value of the *sum*, at each place, by the rule of three, and deduct the commission therefrom as you proceed.

2. The work may sometimes be shortened by subtracting the sum of the logarithms of the antecedents from the sum of the logarithms of the consequents. The remainder will be the logarithm of the answer.

Examples in Simple Arbitration.

(77.) When *London* exchanges with *Paris* at £1 sterling for 23 francs 45 cents, and with *Amsterdam* at 36*s*. 4*d*. Flemish per £. sterling; what ought the course of exchange to be between *Paris* and *Amsterdam*, that a merchant in *London* may remit a sum of money to *Amsterdam* by way of *Paris*, instead of remitting immediately from *London* thither, without loss; the exchange between *Paris* and *Amsterdam* being 3 francs for a certain number of Flemish pence?

23fr. 45c. : 36s. 4d. Flemish :: 3fr. : 55 pence 6 grots.

Therefore the course of exchange between Paris and Amsterdam ought to be at 3 francs for 55 pence 6 grots Flemish.

(78.) If *Amsterdam* exchanges with *London* at 33*s*. 7*d*. per £. sterling, and with *Lisbon* at 51½*d*. Flemish for the crusade of 400 reis, how ought the exchange to go between *London* and *Lisbon*?

(79.) A merchant of *Amsterdam* orders his factor at *London* to remit to his correspondent at *Paris* at £1 sterling for 23 francs *, and to draw upon *Rotterdam* for the value at 37*s*. Flemish per £. sterling; but, when the order came to hand, the exchange was on *Paris* at 24 francs per £. sterling. At what rate of exchange ought the factor

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* *France* formerly exchanged with *England* by giving 1 crown for a variable number of pence English: now the French give 23 or 24 francs, and a variable number of cents for 1*l*. sterling. See the QUOTATION to Table IX.

to draw upon *Rotterdam* to execute his orders without loss to his employer.

(80.) A factor in *London* is ordered to remit to *Venice* at 50*d.* per ducat, and to draw for the value upon *Madrid* at 42*d.* per dollar; but, on receipt of the order, bills upon *Venice* were at 53$\frac{1}{2}$*d.* At what rate must he draw upon *Spain* to compensate this loss?

CLASS II.

(81.) A merchant at *London* is desirous of transferring a sum of money to *Amsterdam* in the most advantageous manner, either directly to *Amsterdam*, or through *Paris*, at a time when the course of exchange between *London* and *Amsterdam* is 34*s.* 5*d.* per £. sterling, and between *London* and *Paris* 31$\frac{1}{4}$*d.* sterling per crown?—by advice he finds the course of exchange between *Paris* and *Amsterdam* to be 52*d.* Flemish per crown, upon which he remits directly to *Amsterdam*, and draws for the value upon *Paris*. What does he gain per cent. by these means; and what would he have lost per cent. had he remitted the money to *Amsterdam* by way of *Paris*, and then drawn upon *Amsterdam* for the value, supposing he had received no advice of the course of exchange between *Paris* and *Amsterdam*?

(82.) A Spanish merchant ordered his factor in *London* to remit the value of 900 ducats to *Venice*, at 50$\frac{1}{4}$*d.* per ducat, and to draw upon him at *Madrid* for the value at 41*d.* per piastre. When the order arrived, the exchange at *Venice* was 51*d.* per ducat, and at *Spain* at 42$\frac{1}{4}$*d.* per piastre; whether did the merchant gain or lose by this negociation?

(83.) A merchant in *London* remitted to *Amsterdam* 500*l.* sterling, at the rate of 18*d.* sterling per guilder; his correspondent at *Amsterdam* was to remit the same, by order, to *Bordeaux*, at 3 guilders per crown, rebating $\frac{1}{3}$ per cent. for his commission; but, when he received this order, the exchange between *Amsterdam* and *Bordeaux* was at 3$\frac{1}{2}$ guilders per crown. The merchant at *London*, not apprized of this, drew upon *Bordeaux* at 55*d.* sterling per crown; whether did he gain or lose, and how much per cent.?

(84.) A merchant at *Amsterdam* was indebted to another at *Paris* a bill of 3000 florins current, agio 4 per cent., and exchange at $90\frac{1}{4}d.$ per ecu of 60 *sols Tournois;* but, when this bill became negociable, the exchange was down at $89\frac{1}{2}d.$ per crown, and the agio advanced to 5 per cent. Did the *Paris* merchant gain or lose by this turn of affairs?

Examples in Compound Arbitration.

(85.) Sold goods to a house in *Amsterdam* to the amount of £824 Flemish, which my correspondent advises me he will remit; but, as the exchange on *Amsterdam* was so low as 34*s*. 4*d*. per £. sterling, I have desired him to remit it to *France* at 48*d*. Flemish per crown; thence he orders it to be remitted to *Vienna*, at 100 crowns for 60 ducats, thence to *Hamburgh*, at 100*d*. Flemish per ducat; thence to *Lisbon*, at 50*d*. Flemish per crusade of 400 reis; and lastly, from *Lisbon* to *England* at 5*s*. 8*d*. sterling per mille-reis. Whether shall I gain or lose by the circular exchange?

By the circular Exchange.

Antecedents.		Consequents.
48d. Flemish	=	1 crown.
100. crowns	=	60 ducats.
1 ducat	=	100d. Flemish.
50d. Flemish	=	400 reis or 1 crusade.
1 mille-reis, or 1000 reis	=	68d. sterling.
How many pounds sterling	=	824*l*. Flemish.

$$\frac{1\times\cancel{60}\times\cancel{100}\times\overset{2}{\cancel{400}}\times68\times\overset{103}{\cancel{824}}}{\underset{8}{\cancel{48}}\times\cancel{100}\times1\times\cancel{50}\times\underset{5}{\cancel{1000}}}=\frac{103\times68\times2}{5\times5}=560l.6s\,4\tfrac{4}{5}d$$

By the direct Exchange.

34*s*. 4d. Flemish : £1 sterling :: £824 Flem. : £480 sterling. Then 560l. 6s. $4\frac{4}{5}$d.—£480=80l. 6s. $4\frac{4}{5}$d. advantage by the circular exchange.

(86.) A banker in *Paris* remits to his factor at *Amsterdam*, 22641 francs 75 cents, first to *London* at 24 francs per £. sterling; thence to *Rome*, at 65*d*. per stampt crown; thence to *Venice*, at 100 stampt crowns for 142 ducats bank; thence to *Leghorn*, at 105 ducats bank for 100 piastres; and from *Leghorn* to *Amsterdam*, at 87*d*.

Flemish per piastre. How many guilders bank will be received at *Amsterdam*, and what will the banker gain, supposing the direct exchange between *Paris* and *Amsterdam* to be 51 grots Flemish for 3 francs?

(87.) A merchant in *London* is desirous to remit £759 sterling to *Genoa*. He can remit by way of *Paris*, at 56*d*. per ecu; thence to *Venice*, at 100 crowns for 60 ducats bank; thence to *Rome*, at 140 ducats bank for 100 stampt crowns; and from *Rome* to *Genoa*, at 115 stampt crowns for 125 pezzos.—He can likewise remit by way of *Amsterdam*, at 33*s*. Flemish per £. sterling: thence to *Frankfort*, at 2 rix-dollars for 16*s*. Flemish; thence to *Venice*, at 12 ducats for 11 rix-dollars; thence to *Rome*, &c. as above. How many pezzos by each of these methods will the merchant have for his money, and which method will be the more advantageous?

CLASS II.

(88.) A merchant in *London* has credit at *Leghorn* for 7547 piastres, whence he receives advice that a remittance can be made at 52*d*. per piastre. The merchant upon this orders them to be remitted to *Venice*, at 95 piastres for 100 ducats bank; thence to *Cadiz*, at 321 maravedis per ducat; thence to *Lisbon*, at 631 reis per piastre; thence to *Amsterdam*, at 50*d*. Flemish per crusade; thence to *Paris*, at 56*d*. Flemish per ecu; and lastly from *Paris* to *London*, at 31½*d*. per crown. What ought to be the arbitrated price between *London* and *Leghorn*; whether will the merchant gain or lose, and how much per cent. by the circular exchange?

(89.) I have ordered my factor at *Amsterdam* to remit 1757*l*. 15*s*. Flemish (the exchange between *London* and *Amsterdam* being 34*s*. 7*d*. Flemish per £. sterling) to *France* at 54*d*. Flemish per ecu; thence to *Venice* at 100 crowns for 56 ducats bank; thence to *Hamburgh*, at 100*d*. Flemish per ducat; thence to *Portugal*, at 45*d*. Flemish per crusade; and thence to *London*, at 63*d*. per mille-reis. How much sterling money ought I to receive, allowing my factor ½ per cent. for commission at each place; and whether will be the more advantageous—the circular or the direct exchange?

(90.) If 100lb. weight of *England* make 88lb. at *Rouen*, 78lb. at *Rouen* 94lb. at *Lyons*, 69lb. at *Lyons* 53lb. at *Geneva*, 72lb. at *Geneva* 100lb. at *Marseilles*, 121lb. at *Marseilles* 100lb. at *Hamburgh*, 103lb. at *Hamburgh* 101lb. at *Paris*.—What is the difference between the weight of a pound at *London* and *Paris?*

INVOLUTION.

Definition 1. *When any given number* is multiplied by itself and that product by the same number, and so on to any assigned number of products, the process is called *Involution*, or the involving a number to any assigned power.

2. The given number is called the root, or first power; the first power multiplied by itself gives the second power, or square; the second power multiplied by the first, gives the third power, or cube; the third power multiplied by the first, gives the fourth power, or biquadrate, &c.

3. *The number denoting the power* is called the *index*, or *exponent*, of that power. Thus, if a number is to be involved to the fourth power, then 4 is the index of the power.

4. *Powers are generally denoted* by writing the exponent over the first. Thus the square of 205 is written $\overline{205}|^2$, the cube $\overline{205}|^3$; also the fourth power of $705 \times 9{\cdot}15$ may be expressed thus, $\overline{705 \times 9{\cdot}15}|^4$, &c.

Note I. A *general* rule for the practice of Involution is evidently contained in the *2d definition*. A fraction may be involved to any power by a continual multiplication of its terms in a similar manner.

Proposition. To find the power of any number, above the cube, without finding all the intermediate powers.

Rule. Find, by the second *definition*, two or more such powers of the given number as that the sum of their indices may make the index of the power required. Then multiply these powers continually together, and the last product will be the power required.

2. If any number end with 5 or 6, all the powers of that number will end with 5 or 6.

3. The *sum* of any two numbers, differing by an unit, is equal to the *difference* of the *squares* of those numbers.

4. The *sum* of any two numbers, multiplied by their *difference*, is equal to the *difference* of the *squares* of the same numbers.

5. The *sum* of any two numbers will measure the *sum* of their cubes; and the *difference* of any two numbers will measure the difference of their cubes.

6. The product of two square numbers is a square number, and of two cubes a cube number, and so on for higher powers; likewise every power of a square number is a square number, and every power of a cube-number a cube-number, &c.

A TABLE OF POWERS.

Root or first power.	Square or second power.	Cube or third power.	Biquadrate, or square squared, or 4th power.	Sursolid or the 5th power.	Cube squared, or the square cubed, or the 6th power.	Second sursolid, or the 7th power.	Biquadrate squared, or the eighth power.	Cube cubed, or the ninth power.
In. 1	In. 2	Ind. 3	Ind. 4	Index. 5	Index. 6	Index. 7	Index. 8	Index. 9
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

Examples.

(1.) Involve 1·05 to the 9th power.

1·05 × 1·05 × 1·05 × 1·05 × 1·05 × 1·05 × 1·05 × 1·05 × 1·05 = 1·551328215978515625, the power required.

Or thus, by the rule, page 199.

1 + 1 + 1	=	3,	index of the power.
1·05 × 1·05 × 1·05	=	1·157625	power.
3 + 3	=	6,	index of the power.
1·157625 × 1·157625	=	1·340095640625	power.
6 + 3	=	9,	index of the power.
1·340095640625 × 1·157625	=	1·551328215978515625	power.

(2.) Square 1754.
(3.) Square 549.
(4.) Cube 3·1416.
(5.) Cube ·7854.
(6.) Involve 57·5 to the 4th power.
(7.) Involve 1·732 to the 5th power.
(8.) What is the 9th power of 735?
(9.) Involve 365 to the 6th power.

EVOLUTION.

Definition 1. *The method of finding* the first power, or root, by having the *second*, third, &c. power given, is called *Evolution*, or the extraction of roots, and is exactly the reverse of Involution. Though, in Involution, there is no number whereof we cannot find the exact power, yet, in Evolution, there are many numbers of which we cannot find the precise root.

2. *The roots which are perfectly accurate* are called *rational roots*, and those roots, which are continually approximating nearer to the truth, yet never arrive at it, are called *surd-roots*.

3. *Roots are sometimes denoted* by writing the character $\surd$ before the power, with the index of the root in it; or by putting the index of the root above the power in the form of a fraction. Thus the square root of 21 may be expressed by $\surd 21$, or $\overline{21}|^{\frac{1}{2}}$, and the cube-root of 24+7 by $\sqrt[3]{24+7}$, or $\overline{24+7}|^{\frac{1}{3}}$, &c.

Note. There is no such thing, according to our present notation of numbers, as the exact square-root of 2, 3, 5, 6, 7, 8, 10, &c. nor the exact cube-root of 2, 3, 4, 5, 6, 7, 9, &c. Hence if the root of any number is not composed of some of the natural series, 1, 2, 3, 4, 5, 6, 7, &c. *ad infinitum*, it is a *surd*.

SQUARE ROOT.

Proposition 1. *To extract the square-root of any whole number, or a pure or mixed decimal.*

Rule. If there be decimals in the given number, make them to consist of *two*, *four*, or *six*, &c. places, by annexing ciphers to the right-hand; then, separate the whole into periods of *two* figures each, beginning at the right-hand, and the left-hand period will consist of *one* or *two* figures, according as the number of figures in the whole number is odd or even.

2. Find a square number equal to, or the next less than, the left-hand period, and put the root thereof in the quotient; subtract this square from the left-hand period, and to the remainder bring down the next period for a dividend.

3. Double the quotient for a divisor, then consider what figure must be annexed to the right hand thereof, so that if the result be multiplied by that figure, the product may be equal to, or the nearest less number than, the dividend, and it will be the second figure in the root. Then bring down the next period, double the figures in the quotient for a divisor, and proceed in all respects as above till you have finished the operation.

For the proof. Square the root found, and to that product add the remainder, if any; and that sum will be the same as the number given to be extracted.

Squares	1 .	4 .	9 .	16 .	25 .	36 .	49 .	64 .	81
Roots	1 .	2 .	3 .	4 .	5 .	6 .	7 .	8 .	9

Hence we may observe, that, if any number end with 2, 3, 7, or 8, the square root of that number can never be exactly found.

Prop. 2. *To extract the square-root of a vulgar fraction.*

Rule I. Multiply the numerator by the denominator, and extract the square-root of the product. The nu-

merator of the given fraction, written above this root, or the denominator written below it, will express the root of any fraction when reduced to its lowest terms.

If the product of the numerator by the denominator does not extract even, annex ciphers to the right hand thereof, and continue the root as far as is necessary, which divide by the denominator of the fraction to obtain its true root.

Thus $\sqrt{\frac{N}{D}}=\frac{\sqrt{N}}{\sqrt{D}}=\frac{N}{\sqrt{N\times D}}=\frac{\sqrt{N\times D}}{D}$, whether $\frac{N}{D}$ represent a proper or an improper fraction. The product of the numerator by the denominator will always extract even, when the fraction is not a surd. Vide note 6, p. 200.

Rule II. 1. Reduce the given fraction to its lowest terms. Then extract the square root of the numerator for a new numerator, and the square-root of the denominator for a new denominator.

2. If the fraction will not extract even, reduce it to a decimal, and then extract the square-root.

3. When the number to be extracted is a mixed fraction, reduce the fractional part to a decimal, and annex it to the whole number, then extract the square-root.

Prop. 3. *To find a mean proportional between two given numbers.*

Rule. Multiply the two given numbers together, and extract the square-root of their product.

Prop. 4. *To find the side of a square equal in area to any given superficies.*

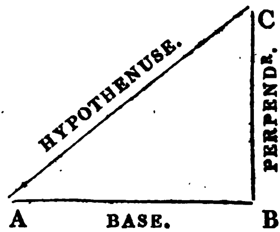
Rule. Extract the square-root of the number expressing the superficies of the given figure; and it will be the side of a square of equal area, and of the same measure as the given figure; viz. yards, or feet, &c. according as the given superficies consists of yards, or feet, &c.

Prop. 5. *Given the area or surface of a circle to find the diameter.*

Rule. Divide the area by ·7854, and extract the square root of the quotient.

Prop. 6. *The base and perpendicular of a right angled triangle being given, to find the hypothenuse.*

Rule. To the square of the base add the square of the perpendicular, the square-root of the sum will give the hypothenuse.



Prop. 7. *Given the hypothenuse, or longest side of a right angled triangle, and either of the other sides, to find the third side.*

Rule. Multiply the sum of the two given sides by their difference, and extract the square-root of the product.

Examples to Proposition 1.

(1.) Extract the square root of 1340095640625.

```
1|34|00|95|64|06|25(1157625, the root.
1

21)34
   21

225)1300
    1125

2307)17595
     16149

23146)144664
      138876

231522)578806
       463044

2315245)11476225
        11476225
```

(2.) Extract the square root of 5678·243.

```
56|78|·24|30, &c.(75·35 root.
49

145)778

1503)5324

15065)81530

      6205 rem.
```

Note. After you have found the root to five or six figures, two or three more may be found by plain or contracted division.

(3.) What is the square root of 393129?

(4.) Extract the square root of 3272869681.
(5.) Extract the square root of 152415787501905 21.
(6.) Required the square root of 57132.
(7.) What is the square root of 75·347?
(8.) Required the square root of 1788·57′.
(9.) What is the square root of ·4325?
(10.) Required the square root of 5·3′.

Examples to Prop. 2.

(11.) What is the square root of $\frac{2025}{2116}$?

2025 × 2116 = 4284900, the square root of which is 2070; then $\frac{2025}{2070}=\frac{45}{46}$, or $\frac{2070}{2116}=\frac{45}{46}$, the root required.

(12.) Extract the square root of $\frac{7}{9}$.

7 × 9 = 63, the square root of which is 7·9372532332; this root divided by 9, the denominator, gives ·8819171036, &c. for the square root of $\frac{7}{9}$.

(13.) What is the square root of $\frac{448}{567}$?
(14.) Required the square root of $\frac{275}{539}$.
(15.) Required the square root of $\frac{45}{54}$.
(16.) What is the square root of $15\frac{5}{8}$?
(17.) Required the square root of $20\frac{4}{25}$.

Examples to Prop. 3.

(18.) Find a mean proportional between 3 and 27.

$\sqrt{3\times27}=\sqrt{81}=9$. *Answer.*
For 3 : 9 :: 9 : 27.

(19.) Of three numbers in geometrical progression, the first is 18, and the third 32, what is the middle one?

(20.) In a pair of scales, a body weighed 90lb. in one scale, and only 40lb. in the other scale: required the true weight, and the ratio of the lengths of the two arms of the balance on each side of the point of suspension?

Examples to Prop. 4.

(21.) The area of a fish-pond is 9 acres, 2 roods, 15 perches; how many yards are contained in the side of a square of equal superficies?

a. r. p.
9. 2. 15=1535 square perches.
$5\frac{1}{2}\times5\frac{1}{2}=30\frac{1}{4}$ square yards in 1 perch.
$\sqrt{1535\times30\frac{1}{4}}=\sqrt{46433\cdot75}=215\cdot485$ yards, nearly. *Answer.*

(22.) An army of 56169 men is to be formed into a square, how many men will the front contain?

(23.) If the area of a circular piece of ground be 231·2575 acres, how many yards will the side of a square be that will contain the same number of acres?

Examples to Prop. 5.

(24.) A circular fish-pond is to be dug in a garden that shall take up just an acre, what must be the length of the cord which describes the circle?

An acre=4840 square yards.

Then $\sqrt{4840\div\cdot7854}=\sqrt{6162\cdot465}=78\cdot514$ yards the diameter of the circle, hence the length of the cord, or the radius,=39·257 yards.

(25.) The area of one end of a circular piece of timber is 4356·6 square inches, what is the diameter?

(26.) In a field adjoining my house I wish to plant four acres of wood in the form of a circle, and to have a gravel walk round it of six feet wide; what must the lengths of the cords be which describe each of the circles?

Examples to Prop. 6.

(27.) The base of a right-angled triangle is 24 feet, the perpendicular 18 feet; what is the length of the hypothenuse?

$24\times24=576$ the square of the base.
$18\times18=324$ the square of the perpendicular.

Sum 900, the square-root of which is 30 the hypothenuse.

(28.) The wall of a fort standing on the brink of a river is 42·426 feet high, the breadth of the river is 23 yards; what length must a cord be to reach from the top of the fort across the river?

(29.) Two ships sail from the same port, the one due

east 50 miles, the other due south 84 miles; how far are they asunder?

Examples to Prop 7.

(30.) The hypothenuse of a right-angled triangle is 30, and the base 24; what is the length of the perpendicular?

$30+24=54$ sum of the *given* sides.
$30-24=\ 6$ difference of the *given* sides.

$\sqrt{54\times6}=\sqrt{324}=18$. *Answer.*

(31.) A line 27 yards long will reach from the top of a fort on the opposite bank of a river to the water edge on this side of the river; what is the height of the fort, the river being 24 yards across?

(32.) A ladder of 100 feet in length was placed against a building of 100 feet high, in such a manner that the top of it reached the top of the building within 6 inches; what was the distance of the foot of the ladder from the base of the edifice?

CLASS II.

(33.) A gentleman hired a number of labourers, at a shilling per day each, to dig a fish-pond. When they had finished their work, their wages amounted together to 120*l.* 1*s.* What were the wages of one man, each man worked as many days as there were men in company?

(34.) A number of men, drinking porter in *London*, spent, at a reckoning, half a crown and a farthing; when they came to pay the landlord, they found that each man had as many farthings to pay as there were men in company. Pray how many men were there?

(35.) The wall of a town, which is surrounded by a moat 24 feet wide, is 18 feet high; what length must a ladder be made to reach from the outer edge of the moat to the top of the wall?

(36.) A ladder, 50 feet long, will reach to a window 30 feet from the ground on one side of the street; and, without moving the foot, will reach a window 40 feet high on the other side. The breadth of the street is required.

(37.) The longer diameter of an ellipsis is 81 inches,

and the shorter diameter 64 inches; what is the diameter of a circle of equal superficies?

(38.) If a ladder 50 feet in length will exactly reach the coping of a house when the foot is 10 feet from the upright of the building, how long must a ladder be to reach the bottom of the second-floor window, which is 17·9897 feet from the coping, the foot of this ladder standing 6 feet from the upright of the building; and what is the height of the wall of the house?

(39.) A society collected among themselves, for charitable purposes, the sum of 30*l.* 9*s.* 2¼*d.*, each member contributed as many farthings as there were members in the whole society. What did each contribute?

(40.) A line of 380 feet will reach from the top of a precipice that stands close by the side of a brook, to the opposite bank; the precipice is 128 feet high, how broad is the brook?

(41.) There are five numbers in geometrical progression, the first is 5, and the fifth is 1280, what are all the rest?

(42.) An irregular piece of ground, consisting of 420 acres, 3 roods, 14 perches, is to be exchanged for a square piece of the same surface; what will be the length of one of its sides? This square is likewise to be divided into 40 equal squares, what will be the extent of a side of each?

(43.) There are three towers, A, B, and C, standing in a direct line, the heights whereof are 64, 90·249, and 50, feet respectively. The distance between the top of the tower A and that of B is 97 feet; and the distance between the bottom of the tower B and that of C is 76 feet. By these data it is required to find the distances the tops and bottoms of the towers are from each other?

(44.) A gentleman has a garden in the form of an equilateral triangle, the sides whereof are each 50 feet: at each corner of the garden stands a tower;—the height of A is 30 feet, that of B 34 feet, and that of C 28 feet. At what distance from the bottom of each of these towers must a ladder be placed that it may just reach the top of each tower, and what will be the length of the ladder, the ground of the garden being horizontal?

CUBE ROOT.

Prop. 1. *To extract the cube root of any number.*

RULE I.

1. If there be decimals in the given number, make them to consist of *three, six,* or *nine,* &c. places, by annexing ciphers to the right-hand; then, separate the whole into periods of *three* figures each, beginning at the right-hand. The left-hand period may consist of *one, two,* or *three* figures.

2. Find the nearest less cube to the left-hand period, and subtract it therefrom; put the root in the quotient, and bring down the figures in the next period for a *dividend.*

3. Find a divisor by multiplying the square of the quotient by 300, seek how often it is contained in the dividend, and put the answer in the quotient.

4. Multiply the *last* figure in the quotient by the preceding figure (or figures,) and that product by 30; add the result, together with the square of the *last* quotient figure, to the divisor; this sum, when multiplied by the *last* quotient figure, will give the *subtrahend.*

5. Take the subtrahend from the dividend, and bring down the next period for a new dividend. Then find a divisor as above, and repeat the operation.

For the proof. Cube the root found, and to the product add the remainder, if any, and that sum will be the same as the number given to be extracted.

Cubes	1	8	27	64	125	216	343	512	729
Roots	1	2	3	4	5	6	7	8	9

Note. The above rule is the same in principle as those usually given by other authors; but, when the number to be extracted is large, or has not a *rational root,* and is required to be extracted to several figures, the operation by this rule is very tedious: the following rules will be found preferable in those cases.

RULE II.

1. Find the root to three places of figures, by Rule I., and call it the assumed root. Then,

2. As the sum of the *given number* and *double the cube* of the assumed root, is to the sum of *double the given number* and *the cube* of the assumed root, so is the assumed root to the root required, nearly.

3. For greater exactness, call the root last found the assumed root, and repeat the operation.

Or, Let N represent the number to be extracted, r the nearest root, to be found by repeated trials.

Then will $\frac{1}{2}r + \sqrt{\frac{N-r^3}{3r} + \frac{1}{4}r^2} = \frac{1}{2}r + \frac{1}{2}\sqrt{\frac{4N-r^3}{3r}} =$ $\frac{1}{2}r + \frac{1}{2}\sqrt{\frac{4N}{r} - \frac{r^2}{3}} = \frac{1}{2}r + \sqrt{\frac{N}{3r} - \frac{1}{12}r^2}$, be the root as above; being the same as the *irrational* formula of Dr. Halley. The last theorem is the same as *Birks'* rule.

Or, $\frac{2N+r^3}{N+2r^3} \times r =$ the cube root of N; that is, $N+2r^3 : 2N+r^3 :: r : \sqrt[3]{N}$; being the same as the rational formula of Dr. Halley.

As these algebraical theorems or rules, are all exactly the same in principle, the learner may use that which he conceives to be most convenient. But in the application of any one of them, the operation will, in general, be shorter if you find the root by Rule I. to three places of figures, instead of finding it by repeated trials.

Prop. 2. *To extract the cube root of a vulgar fraction.*

Rule. Reduce the fraction to its lowest terms, then extract the cube root of the numerator for a new numerator, and the cube root of the denominator for a new denominator; but, if the terms will not extract even, multiply the numerator by the square of the denominator, and the cube root of the product, divided by the denominator will give the root required.—Or reduce the fraction to a decimal, and then extract the root.

Here $\sqrt[3]{\frac{N}{D}} = \frac{\sqrt[3]{N}}{\sqrt[3]{D}} = \frac{N}{\sqrt[3]{N \times D^2}} = \frac{\sqrt[3]{N \times D^2}}{D}$ as in the square root.

Examples to Prop. 1, *Rule* 1.

(1.) Extract the cube root of 48627·125.

48|627.|125(36·5 *root.*

$3\times3\times3=27$

$\overline{3}|^2\times300=2700$ *divisor.* | 21627 *dividend.*

$6\times3\times30=\ 540$

$\overline{6}|^2=\ 36$

$3276\times6\ =\ 19656$ *subtrahend.*

$\overline{36}|^2\times300=388800$ *divisor.* | 1971125 *dividend.*

$5\times36\times30=\ 5400$

$\overline{5}|^2=\ 25$

$394225\times5\ =\ 1971125$ *subtrahend.*

....

(2.) Required the cube root of 122615327232.
(3.) Required the cube root of 41421736.
(4.) Extract the cube root of 705·919947284.
(5.) Required the cube root of 17·54.
(6.) What is the cube root of 254358061056000 ?
(7.) The cube root of ·57345 is required.
(8.) Extract the cube root of 75·3857.
(9.) What is the cube root of ·7854 ?
(10.) Required the cube root of 517·375475.
(11.) Extract the cube root of 20874107909304.
(12.) Extract the cube root of 1551328·215978515625.

Examples to Rule 2.

(13.) Extract the cube root of 98003·449 to 6 places of decimals.

98|003·|449|(4·61 *assumed root.*

$4\times4\times4=64$

$\overline{4}|^2\times300=4800$ *divisor.* | 34003 *dividend.*

$6\times4\times30=\ 720$

$\overline{6}|^2=\ 36$

$5536\times6\ =\ 33336$ *subtrahend.*

$\overline{46}|^2\times300=634800$ *divisor.* | 667449 *dividend.*

$1\times46\times30=\ 1380$

$\overline{1}|^2=\ 1$

$636181\times1\ =\ 636181$ *subtrahend.*

31268000 *divisor, &c.*

Secondly.

The cube of 46·1=97972·181, the cube of the assumed root.

97972·181	98003·44
2	2
195944·362	196006·898
98003·449	97972·181

293947·811 : 293979·979 :: 46·1 : 46·104937, &c. the root sought. ——By cubing 46·104937, and repeating the latter part of the operation, the root may be obtained, truly, to nearly 20 figures.

(14.) What is the cube root of 7154·10916753 ?

(15.) Extract the cube root of 8302348000000· to four places of decimals.

(16.) Extract the cube root of 2 to eleven places of decimals.

(17.) Extract the cube root of ·0001357 to ten places of decimals.

(18.) Extract the cube root of 13·6′ to nine places of decimals.

(19.) Extract the cube root of 92398647506217 to four places of decimals.

Examples to Prop. 2.

(20.) Extract the cube root of $\frac{243}{576}$.

First $\frac{243}{576}=\frac{27}{64}$; the cube root of 27 is 3, and that of 64 is 4, therefore the cube root of $\frac{27}{64}$ is $\frac{3}{4}$.

(21.) Extract the cube root of $\frac{2}{3}$.

2×3 squared=18, the cube root of which (according to the preceding examples) is 2·620741, &c. This, divided by 3, gives ·87358, &c. for the cube root of $\frac{2}{3}$; or $\frac{2}{3}$=·66666, &c. the cube root of which is ·87358, &c. as before.

(22.) What is the cube root of $\frac{175}{476}$?

(23.) Required the cube root of $\frac{1912}{3078}$.

(24.) What is the cube root of $\frac{81}{375}$?

(25.) Required the cube root of $\frac{5}{9}$.

(26.) What is the cube root of $\frac{23}{24}$?

CLASS II.

The six following questions depend upon the 33rd proposition of the XIth book of *Euclid*, and the 12th and 18th propositions of the XIIth book; or, the 9th, 19th, and 21st propositions of the Xth book of *Keith's Geometry*; where it is demonstrated, *that all solid bodies are in proportion to each other as the cubes of their similar sides, diameters, lines, &c.*

(27.) If the diameter of a globe be 1 inch, its solidity will be ·5236 inch; what will be the solidity of a globe of 15 inches diameter?

(28.) The solid content of a block of marble is 31185 inches; what will be the side of a cubical piece of equal solidity?

(29.) A malster agreed with a carpenter to make him a cubical bin, to hold 60 quarters of barley; what will be the internal length of one of its sides, 2150·42 cubic inches being a *Winchester* bushel?

(30.) If a stone, 20 inches long, 15 inches broad, and 8 inches thick, weigh 217lb., what will be the length, breadth, and thickness, of a similar stone that weighs 9000lb.?

(31.) Admit the length of a ship's keel to be 125 feet, the breadth of the mid-ship beam 25 feet, and the depth of the hold 15 feet; required the dimensions of two other ships, of a similar construction, the one to carry 3 times, the other $\frac{1}{2}$, the burthen of that given above?

(32.) A sugar-loaf, in the form of a cone, the perpendicular height whereof is 20 inches, is to be divided into 3 equal parts; what will be the perpendicular height of each part?

(33.) It is required to find *two* mean proportionals between 4 and 108; or, which is the same thing, there are four numbers in geometrical progression, the first term is 4 and the last 108, what are the two middle terms?

(34.) There are seven numbers in geometrical progression, the *first* is 9, and the *seventh* 36864, what are all the intermediate terms; or, which is the same thing, find *five* mean proportionals between 9 and 36864.

TO EXTRACT ANY ROOT OF A POWER.

Rule I. Point the root into periods as the question requires. Find the nearest root to the first period, and subtract its power therefrom; to the remainder bring down the first figure in the next period for a *dividend*. Involve the root to the next lower power than the given one, and multiply it by the index of the given power for a *divisor*, the quotient is the next figure in the root. Then involve the whole root as before, and subtract. Repeat the operation till all the figures are brought down.

1. For a fraction $\sqrt[n]{\frac{N}{D}}=\frac{\sqrt[n]{N}}{\sqrt[n]{D}}=\frac{N}{\sqrt[n]{N^{n-1}\times D}}=\frac{\sqrt[n]{N\times D^{n-1}}}{D}$

universally where N=the numerator, D=the denominator, and n the index of the root.

2. When the index of the power to be extracted is a composite number, the work may be performed more concisely than by this general rule. Indeed, rules of this kind will never be made use of, except by those who have not acquired such a knowledge of the mathematics as will enable them to make use of better methods. Thus, the square root of the square root=the biquadrate, or fourth root, for $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$. The cube root of the square root, or the square root of the cube root =the sixth root, for $\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$. The square root of the fourth root= the eighth root, for $\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$, or $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$. The cube root of the cube root=the ninth root, for $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$, &c.

Rule II. If N be any given power whatever, whose root is sought, n the index of the power, r the nearest rational root; or r^n the nearest rational power to N, whether greater or less. Then will

$$\frac{\overline{n+1}\times N;\ +\overline{n-1}\times r^n}{\overline{n-1}\times N;\ +\overline{n+1}\times r^n}\times r=\text{the root sought.}$$

Examples.

(1.) Extract the 5th root of 307682821106715625.

By Rule I.

307|86282|11067|15625|(3145 *root.*

$\overline{3}|^5=3\times3\times3\times3\times3=243$ subtrahend.

$\overline{3}|^4\times5=405$) 648 first dividend.

$\overline{31}|^5=28629151$ subtrahend.

$\overline{31}|^4\times5=4617605$)21391311 second dividend.

$\overline{314}|^5=3052447761824$ subtrahend.

$\overline{314}|^4\times5=48605856080$)243804492431 third dividend.

$\overline{3145}|^5=307682821106715625$ subtrahend.

By Rule II.

Here the nearest root to the first period is 3, hence $r=3000$, and $r^n=\overline{3000}|^5$, $N=3076828211$, &c.; $n+1=6$, and $n-1=4$, therefore

$$\frac{6\times3076828211,\ \&c.+4\times\overline{3000}|^5}{4\times3076828211,\ \&c.+6\times\overline{3000}|^5}\times3000=3144,$$ the root nearly, and by taking $r=3144$, and repeating the operation, the root will be had.

(2.) Extract the square root of 2.
(3.) Required the cube, or third, root of 5.
(4.) What is the fourth root of 1728?
(5.) Required the 5th root of 57·54.
(6.) Required the 6th root of 3·1416.
(7.) Required the 7th root of 547·5.
(8.) What is the 8th root of 547·5?
(9.) Required the 9th root of 1·5513282159785l5625.
(10.) Required the 365th root of 1·05.
(11.) Required the 40th root of 1·04.
(12.) The amount of £1. for 40 years at compound interest is £4·8010206, what is the rate per cent.?

DUODECIMALS.

Definition. *Duodecimals* are so called because every superior place is 12 times its next inferior in that scale of notation. This way of conceiving an unit to be divided is chiefly in use among *artificers* who generally take the linear dimensions of their work in *feet*, *inches*, and parts.

Note, 12 Inches′	= 1 Foot,	12 Thirds‴	1 Second″,
12 Seconds″	= 1 Inch,	12 Fourths iv	1 Third‴, &c.

Different works are computed by different measures, viz. glazing, &c. by the foot; painting, plastering, paving, &c. by the yard; flooring, roofing, tiling, &c. by the square of 100 feet; bricklayer's work, &c. by the rod of $16\frac{1}{2}$ feet, the square of which is $272\frac{1}{4}$ feet. Bricklayers always value their work at the rate of $1\frac{1}{2}$ brick thick, therefore the content of the wall, &c. must be multiplied by the number of $\frac{1}{2}$ bricks it is in thickness, and then be divided by 3, before the value of the work is estimated. Several other observations, equally useful, might here be inserted, but this part rather belongs to *mensuration* than *arithmetic*. See *Keith's Mensuration.*

A general rule for multiplying duodecimally, or squaring the dimensions of artificers' work.

Under the multiplicand write the corresponding denominations of the multiplier. Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier; write each result under its respective term, observing to carry an unit for every 12, from each lower denomination to its next superior. In the same manner multiply all the multiplicand by the inches in the multiplier, and write the result of each term one place removed to the right-hand of those in the multiplicand. Work in a similar manner with the seconds in the multiplier, setting the result of each term removed two places to the right-hand of those in the multiplicand.—Proceed in like manner with the rest of the denominations, and their sum will give the answer required.

Note. This may be performed by the rule of practice; thus, after you have multiplied by the feet, take aliquot parts of the multiplicand, with the inches, &c. Or the inches, &c. may be reduced to the fraction of a foot, by Prop. 9, Vulgar Fractions, and then multiplied together. Or, turn the inches, &c. into the decimal of a foot by Prop. 2, Rule 2, in Reduction of Decimals, and then multiply them together by some of the rules in Multiplication of Decimals. By reducing the inches, &c. into decimals of a superior name, it will often happen, that these decimals will be infinite; and hence the scholar may have a good opportunity of examining the truth and ceertainty of the rules I have laid down for managing recurring, or infinite, decimals; for, though the multiplier and multiplicand may be infinite in a decimal scale, yet they will be finite in a fractional or duodecimal one.

Examples.

(1.) Multiply 4ft. 6in. 5 parts by 9ft. 4in. 7 parts.

	Ft.	In.	Pts.			
	4	6	5			
	9	4	7			
	40	9	9			prod. by 9 feet.
	1	6	1	8		do. by 4 in.
		2	7	8	11	do. by 7pts.
Prod.	42	6	6′	4″	11‴	

By Practice.

		Ft.	In.	Pts.		
4 in.	$\frac{1}{3}$	4	6	5		
				9		
		40	9	9		
6	$\frac{1}{2}$	1	6	1	8″	
1′	$\frac{1}{6}$		2	3	2	6‴
			4	6	5	
		42	6	6	4	11

Note. The same answer may be *exactly* found either by fractions or decimals.

(2.) Mult. 7ft. 5in. by 4ft. 7in.
(3.) Mult. 9ft. 6in. by 8ft. 7in.
(4.) Mult. 3ft. 11in. by 9ft. 10in.
(5.) Mult. 25ft. 6in. by 34ft. 9in.
(6.) Mult. 15ft. 7in. by 5ft. 11in.
(7.) Mult. 207ft. 9in. by 7ft. 10in.
(8.) Mult. 77ft. 3in. 6pts. by 54ft. 4in. 7pts.
(9.) Mult. 15ft. 3in. 6pts. 5″ by itself.
(10.) Mult. 10ft. 4in. 5pts. by 7ft. 8in. 9pts.
(11.) Mult. 25ft. 11in. 6pts. 8″ 7‴ by itself.

CLASS II.

(12.) If a window be 7ft. 3in. high, and 3ft. 5in. broad, how many square feet of glazing are contained therein?

(13.) There is a house with three tiers of windows, 7 in a tier; the height of the first tier is 6ft. 11in., of the second 5ft. 4in., and of the third 4ft. 3in., the breadth of each window is 3ft. 6in. What will the glazing come to at 14½*d.* per foot?

(14.) What will the paving a court-yard come to at 3*s.* 4*d.* per yard, the length being 24ft. 5in., and breadth 12ft. 7in.?

(15.) What will be the expence of paving a rectangular court-yard, its length being 62ft. 7in., and breadth 44ft. 5in., and in which there is laid a foot path the whole length of it, and 5½ feet broad, with broad stones at 3*s.* per yard, the rest being paved with pebbles at half a crown a yard?

(16.) If the national debt be £500,000,000, how long a foot path, of a yard wide, would this sum pave if reduced to guineas?—a guinea being one inch in diameter.

(17.) What will be the expence of plastering a ceiling at 11½*d.* per yard, supposing the length 22ft. 7in., and breadth 13ft. 11in.?

(18.) A gentleman had a room painted at 8½*d.* per square yard, the measure whereof is as follows: the height 11ft. 7in., the compass 74ft. 10in., the door 7ft. 6in. by 3ft. 9in.; 5 window-shutters, each 6ft. 8in. by 3ft. 4in., the breaks in the windows 14in. deep and 8ft. high, the chimney 6ft. 9in. by 5ft., the shutters and doors being coloured on both sides; what will the whole come to?

(19.) If a house measure 57ft. 7in. in length, and 31ft. 5in. in breadth, and if the roof be of a true pitch, what will it cost roofing at half a guinea per square?

(20.) How many square rods are there in a wall 63½ feet long, 14ft. 11in. high, and 2½ bricks in thickness?

(21.) Admit the end-wall of a house to be 28ft. 10in. in breadth, and the height of the roof from the ground 55ft. 8in., the gable (or triangular part above the side walls) to rise 42 courses of bricks, reckoning 4 courses to a foot; and that 20 feet high be 2½ bricks thick, 20 feet more 2 bricks thick, and the remaining 15ft. 8in. 1½ brick thick. What will the work come to at 5*l.* 16*s.* per rod, the gable being 1 brick in thickness?

END OF THE FIRST PART.

THE

COMPLETE PRACTICAL

ARITHMETICIAN.

PART II.

ALLIGATION.

DEFINITION. When different sorts of wine, corn, spices, metals, &c., or any number of simples, of different qualities are required to be mixed together, the method of proportioning such a mixture is called Alligation, from the quantities being generally linked, or joined, together by lines.

Note 1. The first proposition and rule are usually called Alligation medial; the second Alligation alternate, and is the reverse of Alligation medial; the third Alligation partial; and the fourth Alligation total.

Proposition 1. *Given the particular quantities mixed, and their respective rates, or prices to find the mean rate, or price, of the compound.*

Rule. Multiply the quantities of the mixture by the respective rates, or prices, reduced to one denomination, and divide the sum of the products by the sum of the quantities, the quotient will be the mean rate, or price.

The method of proof. Find the whole value of the mixture at the mean price, and if it be the same with the total value of the several ingredients, at their respective prices, the work is right.

Prop. 2. Given the rates, or prices, of several ingredients to find the quantities thereof, so that the mixture may be sold at a given rate or price.

Rule I. Reduce the particular rates to the same denomination as the mean rate; write them orderly under each other, beginning with the greatest, and place the mean rate to the left-hand of them.—Then connect the simple rates together, so that each rate *less* than the mean may be coupled with *one* greater, or with *each* greater; and each rate *greater* than the mean with *one* less, or with *each* less.

2. Take the difference between each simple rate and the mean rate, and place it *alternately;* that is, against the rate with which it is linked.—Then, if only one difference stand against any rate, it will be the quantity belonging to that rate; but, if there be several, their sum will be the quantity.

Questions under this and the following rules may be proved by the rule to the first proposition.

Note. Questions that fall under this and the following propositions are called by *algebraists* indeterminate or unlimited problems, because they will admit of an infinite number of different answers; for finding which, *algebra* furnishes us with an universal rule. But the rule given above is limited, in its immediate effect, to the different answers obtained by the various methods of linking the simples; viz. just as many different answers may be obtained as there are different ways of linking together a greater and less rate than the mean.—Though the preceding rule is in some measure limited, yet an infinite number of answers may be deduced from it; for, after the rates are coupled, and their several differences taken, instead of any or every couple of such differences, we may take any *equimultiples* thereof, and place them *alternately;* and these or other quantities proportional to them, will be the quantities required.

Prop. 3. Given the rates, or prices, of several ingredients, the quantity of one, and the mean rate, to find the several quantities of the rest in proportion to that given.

Rule. Take the difference between each rate and the mean, as before.—Then, as the difference standing against the price of the given quantity is to *that* quantity, so are the other several differences to their respective quantities.

Prop. 4. Given the rates, or prices, of several ingredients, the mean rate, and the whole quantity of the mixture, to find the particular quantities of each sort.

Rule. Take the difference between each rate and the mean, as before.—Then, as the sum of these differences is to the whole quantity of the mixture, so is each particular difference to its respective quantity.

Examples to Prop. I.

(1.) A vintner would mix 2 gallons of wine, at 14*s.* per gallon, with 1 gallon at 12*s.*, two gallons at 9*s.* and 4 gallons at 8*s.* per gallon. What will be the worth of a gallon of this mixture?

2	gallons multiplied	by 14s. gives	28 product.
1	———	by 12s. ——	12
2	———	by 9s. ——	18
4	———	by 8s. ——	32
9			90

9 Sum of the products 90, this divided by 9, (the sum of the quantities) gives 10s. the value, or *mean rate* of a gallon, answer.

(2.) A grocer would mix 4 cwt. of sugar of 2*l.* 18*s.* per cwt., 7cwt. 2qr. at 2*l.* 13*s.* per cwt., 5cwt. 1qr. at 1l. 19*s.* per cwt., and 3cwt. 3qr. at 1*l.* 14*s.* per cwt. together; what is the worth of a cwt. of this mixture?

(3.) A tobacconist mixed 50lb. of tobacco, at 11½*d.* per lb. with 40lb. at 14*d.* per lb. 27lb. at 2*s.* 6*d.* per lb., and 87lb. at 3*s.* per lb. What was the worth of 1lb. of this mixture?

(4.) A farmer mixed 2qr. 4bush. of corn, worth 2*l.* per quarter; 4qr. 4bush. of an inferior kind, worth 1*l.* 4*s.* per quarter; and 5qr. of a third kind, worth only 16*s.* per quarter; required the value of a quarter of this mixture?

Examples to Prop. 2.

(5.) A vintner would mix four sorts of wine, of different prices, together, viz. at 14*s.* 12*s.* 9*s.* and 8*s.* per gallon; what quantity of each sort must he put into the compound, that he may be enabled to sell it at 10*s.* per gallon?

Answer.

10s.	14s.	2 gal. at 14s.
	12s.	1 — at 12
	9s.	2 — at 9
	8s.	4 — at 8

Or thus.

10s.	14s.	1 gal. at 14s.
	12s.	2 — at 12
	9s.	4 — at 9
	8s.	1 — at 8

Otherwise.

	s.	
10s.	14	2+1=3 gal. at 14s.
	12	2 =2 — at 12
	9	4 =4 — at 9
	8	4+2=6 — at 8

Or,

0s.	14	2+1=3 gal. at 14s.
	12	1+2=3 — at 12
	9	3+4=6 — at 9
	8	4+2=6 — at 8, &c, &c.

(6.) A grocer wishes to mix sugar at 4*d*., 6*d*., and 10*d*. per lb., so that he may sell the mixture at 8*d*. per lb. What quantity of each may he take?

(7.) A goldsmith would mix gold of 23 carats * fine with gold of 20 carats, some of 18, some of 17, and some of 14 carats fine; how much of each sort must he melt together to form a composition of 19 carats fine?

(8.) A provider for the army, desirous of mixing wheat at 4*s*. per bushel, with rye at 3*s*. per bushel, barley at 2*s*. per bushel, peas at 1*s*. 4*d*. per bushel, and oats at 1*s*. per bushel, wishes to be informed how to proportion the mixture that it may be worth 1*s*. 8*d*. per bushel?

Note. By reducing the several rates into pence, 24 answers, in whole numbers, may be obtained to this question by the different methods of linking the simples *only*.

Examples to Prop. 3.

(9.) A merchant proposes to mix four sorts of wine together, *viz*. 2 gallons of one sort, at 14*s*. per gallon, with others at 12*s*., 9*s*., and 8*s*., per gallon; how many gallons of each sort must he take to make a composition worth 10*s*. per gallon?

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* A carat is an imaginary weight, which expresses the degrees of goodness or fineness of gold. The whole mass is conceived to be divided into 24 equal parts, called carats, and the purity of the mass is expressed by the number of carats of pure gold it contains. Thus, gold of 23 carats fine, which is the standard for French gold, is compounded of $\frac{23}{24}$ of pure gold and $\frac{1}{24}$ of some other metal, called alloy. Gold of 22 carats, is that which is composed of $\frac{22}{24}$ of pure gold, and $\frac{2}{24}$ of silver or copper, or that which, in refining, loses two parts in 24 of its weight. This is the standard for English gold, and here the carat is divided into 4 grains.

	s.	diff.		
16s.	14	2 =2.	2	
	12	1+2=3.	2 diff. : 2 gal. :: 3 diff : 3 gal.	ans.
	9	2 =2.	2 diff. : 2 gal. :: 2 diff. : 2 gal.	
	8	4+2=6.	2 diff. : 2 gal. :: 6 diff. : 6 gal.	

Note. Different answers may be obtained by linking the quantities differently.

(10.) A distiller would mix 80 gallons of brandy, at 12*s.* per gallon, with another sort at 7*s.*, and a third at 4*s.* per gallon; what quantity of each sort must he take to make a composition worth 8*s.* per gallon?

(11.) A grocer would mix teas at 12*s.*, 8*s.*, and 6*s.* per lb. with 28lb. at 4*s.* 6*d.* per lb. What quantity of each must he take to make a composition worth 7*s.* per lb.?

(12.) A person is desirous of mixing corn at 4, 3, and 2 shillings per bushel, with 24 bushels of an inferior kind, worth 1*s.* 6*d.* per bushel; how many bushels of each must he take that he may afford to sell the mixture at 3*s.* 4*d.* per bushel?

Examples to Prop. 4.

(13.) A merchant proposes to mix four sorts of wine; the best at 14*s.* per gallon, the second at 12*s.*, the third at 9*s.* and the fourth at 8*s.* per gallon. How many of each will make a mixture of 12 gallons worth 10*s.* per gallon?

	s.	diff.		
10s.	14	1 =1	As 12d. : 12 gal. :: 1d. : 1 gal.	Answer.
	12	1+2=3	12d. : 12 — :: 3d. : 3 —	
	9	4+2=6	12d. : 12 — :: 6d. : 6 —	
	8	2 =2	12d. : 12 — :: 2d. : 2 —	

Sum of the differences 12

Note. Other answers may be obtained by linking the simples differently.—In order to give the scholar a clearer idea of this subject, I have given the *same* example to each of the propositions.

(14.) A grocer would mix four sorts of sugar, *viz.* at 2*s.*, 1*s.* 8*d.*, 1*s.*, and 8*d.* per lb. What quantity of each must he take to make a composition of 72lb. at 1*s.* 4*d.* per lb.?

(15.) It is required to mix brandy at 8*s.* 7*s.* and 1*s.* per gallon, with water at 0*s.* per gallon, so that a composition of 16 gallons thereof may be worth 5*s.* per gallon?

(16.) How much gold, of 8, 9, and 24, carats fine, must be mixed together, to make a composition of 64oz. of 14 carats fine?

POSITION.

Definition. The Rule of Position, or trial and error, is so called because we suppose some uncertain number, or numbers; and, by reasoning from them according to the nature of the question, and paying proper attention to the error, or errors, obtain a true answer.

SINGLE POSITION.

Definition. By Single Position, or a single supposition, are solved those questions wherein the results are proportional to their suppositions.

RULE.

Suppose some convenient number, and proceed with it according to the nature of the question; then, if the result be either too much or too little, say, as the false number resulting is to the true number given, so is the whole, or any part, of the supposed number to the whole, or corresponding part, of the required number.

Examples.

(1.) A drover being asked how many sheep he had got, replied, If, sir, you add $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$, of the number together, the sum will be 18. How many had he?

Suppose he had 12

Then $\frac{1}{3}$ of 12 = 4
$\frac{1}{4}$ of do. = 3
$\frac{1}{6}$ of do. = 2

The sum is 9, but should be 12.

Hence, as 9 : 18 :: 12 : 24 sheep answer.

(2.) Three persons are to pay a reckoning of 20*s.*; A is to pay $\frac{1}{2}$, B$\frac{1}{3}$, and C$\frac{1}{4}$; how much must each person pay of the reckoning?

(3.) A can do a piece of work in 7 days, B can do the same in 5, and C in 6. Set them all at work together, in what time will they finish it?

(4.) One-fifth part of an army were killed in battle, $\frac{1}{6}$ part were taken prisoners, and $\frac{1}{10}$ part died by sickness; if 4000 men were left, how many men did the army at first consist of?

(5.) I have a cistern which has three cocks, D, E, and F. Now, if D be opened by itself, when the cistern is full, it will empty it in 9 hours; if E be opened by itself, it will empty the cistern in 11 hours; and, if F be opened by itself, it will empty the cistern in 13 hours. In what time will they empty the cistern if I set them all open together?

(6.) A person delivered to another a sum of money, to receive interest for the same at 4 per cent. per annum, (simple interest.) At the end of 3 years he received for principal and interest 176*l.* 8*s.* What was the sum lent?

DOUBLE POSITION.

Definition. By Double Position, or two suppositions, are solved those questions wherein the errors are proportional to the difference between the true number, and each supposition.

RULE.

Suppose any two convenient numbers, and proceed with them according to the nature of the question, marking the errors (with + or —) according as they exceed or fall short of the truth.

Then,

Multiply the first supposition by the second error, and the second supposition by the first error, and divide the sum of the products by the sum of the errors, if they are differently marked; or the difference of the products by the difference of the errors, if they are marked alike, and the quotient will be the number sought.

Or, II.

Multiply the difference between the two supposed numbers by the less error, and divide the product by the sum of the errors, if they are differently marked; or by the difference if they are marked alike; and the quotient will be a correction of the number belonging to the *less* error, and must be added to it, if *that* error be less than the truth, or subtracted, if it be greater.

Note 1. Mr. *Ward*, Mr. *Malcolm*, and several other writers, have omitted the rule of Position, because all questions that can be solved by it are more readily solved by a simple equation in *algebra*. Though this observation be true, yet Position has its use; for it may frequently be applied to the solution of adfected and exponential equations in *algebra*, better than any other method, (particularly the second rule,) for, by repeating the process, the answer will continually approximate to the true number within any assigned degree of exactness. For this reason it is of essential service in the more abstruse parts of the mathematics; for, in many difficult problems, there is hardly any other way to obtain a solution.

2. *In any enquiry, where it is possible to prove the truth of the answer, when discovered.* Make two suppositions as near the truth as you are able to guess, and from them deduce an answer, as directed in the *second* rule; then take the nearer of the two suppositions; and, the result above gained, as two other suppositions; and, in like manner, deduce another answer: proceed thus, till you have obtained an answer sufficiently exact.

Examples.

(1.) What number is that, which, being multiplied by 3, the product increased by 4, and that sum divided by 8, the quotient may be 32?

Suppose 12	Again, suppose 108
×3	×3
36	324
+4	+4
8)40	8)328
Quotient 5	Quotient 41
should be 32	should be 32
Error—27	Error+9

By Rule I.

		its error.
1st supposition	12	—27
2d supposition	108	+ 9

27 its error 12

2916   108
108

$27+9=36$)3024(84 answer.
144

By Rule 2.

$\frac{\overline{108-12\times 9}}{27+9}=24$, correction of the number (108) belonging to the less error. Hence $108-24=84$, as before.

(2.) A man has 2 excellent horses; and a single-horse chaise and furniture worth 150*l.* Now, if the first horse be put in the chaise, his value with the furniture, &c. will be three times that of the second horse without it; but, if the second horse be put in the chaise, his value will be double that of the first. What are the horses worth?

(3.) A person being asked the time of the day, replied, the day is now 16 hours long, and the sun rises at 4 o'clock. Now, if you add $\frac{1}{5}$ of the hours that have passed since the sun rose to $\frac{1}{4}$ of those which must elapse before the sun sets, you will have the *exact* time of the day.

(4.) A person received 11 crowns and 7 dollars for a debt of 4*l.* 10*s.* 10*d.*, and at another time received 4 crowns and 3 dollars for a debt of 1*l.* 15*s.* What was the value of a crown and of a dollar in English money?

(5.) A person distributed in charity 2*d.* a piece among several poor children, and had 4*d.* left. He would have given them 3*d.* a piece, but wanted 10*d.* to be able to do it. What was the number of children?

Examples exercising Note 2.

(6.) If $g\times\overline{4372-g}|^{6}=4\times\overline{4732-4}|^{6}$ what is g?
Answer, 2916.

(7.) If $r^3+12r=300$, what is r? *Answer*, 6·096, &c.

(8.) Given $x^x=100$ to find the value of x.

Note. By the 2d note, and a table of logarithms the most intricate adfected equations in algebra may be solved.

ARITHMETICAL PROGRESSION.

Definition. When a series of numbers increase, or decrease, by an equal excess or difference, those numbers are said to be in *arithmetical progression;* such as,

2, 4, 6, 8, 10, &c.; or 15, 14, 13, 12, 11, &c.

and the numbers which form such series are called the terms of the progression. The first and last terms are usually called the extremes.

Note 1. If *three* numbers be in arithmetical progression, the sum of the extremes will be equal to double the mean; and the product of the extremes, increased by the square of the common difference, will be equal to the square of the mean. Thus, if 5. 7. 9. be in arithmetical progression, then will $5+9=7\times2$, and $\overline{9\times5}+\overline{2\times2}=7\times7$.

2. If *four* numbers be in arithmetical progression, the sum of the two extremes will be equal to the sum of the means.

Thus, if 2. 5. 8. 11. be in arithmetical progression,
Then will $2+11=5+8$.

3. If a series of numbers, (consisting of any number of terms) be in arithmetical progression, the sum of the extremes will always be equal to the sum of any two means equidistant from the extremes; or to double the mean, if the terms be odd.

Thus, if 3. 5. 7. 9. 11. 13. &c. be in arithmetical progression,
Then will $3+13=5+11=7+9$.
Or, if 1. 4. 7. 10. 13. &c. be in arithmetical progression, then will $1+13=4+10=7\times2$.

4. The difference between the extremes is equal to the product of the common difference by the number of terms less one.

Thus, if 3. 5. 7. 9. &c. be in arithmetical progression,
Then will $9-3=2\times\overline{4-1}$.

5. The number of terms, where the terms are odd, is equal to the sum of the terms, divided by the mean.

Thus, $3+5+7+9+11=35\div7=5$.

6. The sum of the terms is equal to the number of terms multiplied by the mean term.

Thus $3+5+7+9+11=5\times7$.

7. If out of any series of numbers in arithmetical progression there be taken any series of *equidistant* terms, *that* series will also be in arithmetical progression.

Thus, if 2. 4. 6. 8. 10. 12. 14, &c. be in arithmetical progression,
Then will 4. 8. 12. &c. be in arith. prog.

8. In any series of numbers in arithmetical progression, the common excess or difference is as often repeated as there are terms in the progression, wanting one; viz. every term except the first is continually increased or diminished by the common excess or difference.

9. The rules for an ascending or descending series are the same; for, a descending series becomes an ascending one by beginning at the least term.

Proposition 1. *Given the least term, the greatest term, and the number of terms, of an arithmetical progression, to find the sum of the terms.*

Rule. To the least term add the greatest, multiply the sum by half the number of terms, and the product will be the sum of the terms.

Prop. 2. *Given the least term, the greatest term, and the number of terms, to find the common excess, or difference.*

Rule. Divide the difference between the greatest and the least term by the number of terms less unity, and the quotient will be the common excess, or difference.

Prop. 3. *Given the least term, the greatest term, and the common excess, or difference, to find the number of terms.*

Rule. Divide the difference between the *greatest* and the *least* term, by the common excess, or difference, the quotient, increased by an unit, will give the number of terms.

Prop. 4. *Given the greatest term, the number of terms, and the common excess, or difference, to find the least term.*

Rule. Multiply the common excess, or difference, by the number of terms less 1: subtract the product from the greatest term, and the remainder will be the least term.

Prop. 5. *Given the number of terms, the common excess, or difference, and the sum of the terms, to find the least term.*

Rule. Divide the sum of the terms by the number of terms; and, from the quotient, subtract half the product of the common excess, or difference, by the number of terms less 1, the remainder will be the least term.

Prop. 6. *Given the least term, the number of terms, and the common excess, or difference, to find the greatest term.*

Rule. Multiply the number of terms by the common excess, or difference, and to that product add the least term; from this sum subtract the common excess, or difference, and the remainder will be the greatest term.

Note. The following propositions and theorems contain the whole practice of arithmetical progression, (including the propositions and rules already given.)

Where $\begin{cases} l = \text{the least term.} \\ g = \text{the greatest term.} \\ n = \text{the number of terms.} \\ s = \text{the sum of the terms.} \\ d = \text{the common excess or difference.} \end{cases}$

Proposition 1. *Given l, g, and n, to find s and d.*

Theorem I. $\overline{l+g}\times\frac{n}{2}=s.$ *Theo.* II. $\frac{g-l}{n-1}=d.$

Prop. 2. *Given l, g, and s, to find n and d.*

Theo. III. $\frac{2s}{g+l}=n.$ *Theo.* IV. $\frac{\overline{g+l}\times\overline{g-l}}{2s-\overline{g+l}}=d.$

Prop. 3. *Given l, g, and d, to find n and s.*

Theo. V. $\frac{g-l}{d}+1=n.$ *Theo.* VI. $\overline{\frac{g-l}{d}+1}\times\frac{g+l}{2}=s.$

Prop. 4. *Given l, n, and s, to find g and d.*

Theo. VII. $\frac{2s}{n}-l=g.$ *Theo.* VIII. $\frac{\overline{s-ln}\times 2}{\overline{n-1}\times n}=d.$

Prop. 5. *Given l, n, and d, to find g and s.*

Theo. IX. $l+\overline{n-1}\times d=g.$ *Theo.* X. $\overline{\overline{nd-d}+2l}\times\frac{n}{2}=s.$

Prop. 6. *Given l, s, and d, to find g and n.*

Theo. XI. $\frac{1}{2}\sqrt{8ds+\overline{2l-d}^{2}}-\frac{1}{2}d.=g.$

Theo. XII. $\frac{\sqrt{8ds+\overline{2l-d}^{2}}-\overline{2l-d}.}{2d}=n.$

Prop. 7. *Given g, n, and s, to find l and d.*

Theo. XIII. $\frac{2s}{n} - g = l.$ *Theo.* XIV. $\frac{\overline{ng - s} \times 2}{\overline{n-1} \times n} = d.$

Prop. 8. *Given g, n, and d, to find l and s.*

Theo. XV. $g - \overline{n-1} \times d = l.$ *Theo.* XVI. $\overline{2g + d - nd} \times \frac{n}{2}.$

Prop. 9. *Given g, s, and d, to find l and n.*

Theo. XVII. $\frac{1}{2}\sqrt{\overline{2g+d}^2 - 8ds} + \frac{1}{2}d = l.$

Theo. XVIII. $\frac{\overline{2g+d} + \sqrt{\overline{2g+d}^2 - 8ds}}{2d} = n.$

Prop. 10. *Given n, s, and d, to find l and g.*

Theo. XIX. $\frac{s}{n} - \overline{n-1} \times \frac{1}{2}d = l.$ *Theo.* XX. $\frac{s}{n} + \overline{n-1} \times \frac{1}{2}d = g$

The application of the preceding theorems is very evident and easy. Rules might here be inserted, were they of any use, for finding the sum of *polygonal* and *figurate* numbers, constituting part of the ancient *Pythagorean* speculations about numbers, &c. Should any person wish to become acquainted with such numbers, he may consult Mr. *Malcolm's* Arithmetic, from page 396 to 441.

Examples to Proposition 1.

(1.) If the least term of a series of numbers in arithmetical progression be 4, the greatest 100, and the number of terms 17, what is the sum of the terms?

$4+100=104$, and $104 \times 17=884$, answer.

(2.) If the least term be 3, the greatest 108, and the number of terms 14, what is the sum of the terms?

(3.) How many strokes does the hammer of a clock strike in 12 hours?

(4.) If 100 stones be laid in a straight line, and exactly the space of a yard be left between one stone and another, how far must a person travel who gathers up these stones singly, returning with every one to a basket a yard distant from the first?

Examples to Prop. 2.

(5.) If the least term of a series of numbers in arithmetical progression be 4, the greatest 100, and the number of terms 17, what is the common difference between each term?

100—4=96 divisor, and 17—1=16 dividend, hence 96 divided by 16 gives 6, the common difference.

(6.) If the least term be 3, the greatest 108, and the number of terms 14, what is the common difference?

(7.) A person travelled from London to a certain place in 8 days; he travelled 2 leagues the first day, and every day he travelled farther than he did the preceding by an equal number of leagues; the last day he travelled 23 leagues: how far did he travel every day?

Examples to Prop. 3.

(8.) The least term of a series of numbers in arithmetical progression is 4, the greatest 100, and the common difference between each term is 6; what is the number of terms?

100—4=96 dividend, which divided by 6, gives 16 for the quotient; this increased by an unit, gives 17 for the number of terms.

(9.) If the least term be 3, the greatest 108, and the common difference 5, what is the number of terms?

(10.) A man, going a journey, travelled the first day 2 leagues, and the last day 23; he increased his journey every day 3 leagues; how many days did he travel?

Examples to Prop. 4.

(11.) The greatest term of a series of numbers in arithmetical progression is 100, the number of terms 17, and the common difference between each term 6; what is the least term?

$\overline{17-1}\times 6=96$; then 100—96=4, answer.

(12.) If the greatest term be 108, the number of terms 22, and the common difference 5, what is the least term?

(13.) A man in 6 days went from London to a certain place; every day's journey was greater than the preceding one by 4 miles; his last day's journey was 40 miles: what was his first?

Examples to Prop. 5.

(14.) The number of terms is 17, the common difference 6, and the sum of the terms, of a series of numbers, in arithmetical progression, is 884; what is the least term?

$884 \div 17 = 52$, and $\overline{17-1} \times 6 = 96$; then $52 - \frac{96}{2} = 4$, the least term.

(15.) If the number of terms be 22, the common difference 5, and the sum of the terms 1221, what is the least term?

(16.) A man is to receive 300*l.* at 12 payments, each succeeding payment to exceed the former by 4*l.* What will his first payment be?

Examples to Prop. 6.

(17.) If the least term of a series of numbers in arithmetical progression be 4, the number of terms 17, and the common difference 6, what is the greatest term?

$17 \times 6 = 102$, and $102 + 4 = 106$; then $106 - 6 = 100$, the greatest term.

(18.) If the least term be 3, the number of terms 22, and the common difference 5, what is the greatest term?

(19.) A man bought 100 yards of cloth; the first yard cost him 2*s.*, and each succeeding yard 1*s.* more to the last; what did the last yard stand him in?

GEOMETRICAL PROGRESSION.

Definition. When a series of numbers increase by a common multiplier, or decrease by a common divisor, those numbers are said to be in *geometrical progression*; such as 2, 4, 8, 16, &c.; or 27, 9, 3, 1, &c. The first and last terms are usually called the extremes, and the common multiplier or divisor the *ratio*.

Note 1. If *three* numbers be in geometrical progression, the product of the two extremes will be equal to the square of the mean.

Thus, if 3. 9. 27. be in geometrical progression.
Then will $3\times27=9\times9$.

2. If *four* numbers be in geometrical progression, the product of the two extremes will be equal to the product of the means.

Thus, if 2. 4. 8. 16. be in geometrical progression,
Then will $2\times16=4\times8$.

3. If a series of numbers (consisting of any number of terms) be in geometrical progression, the product of the two extremes will be equal to the product of any two means equidistant from the extremes; or to the square of the mean, if the terms be odd.

Thus, if 1. 2. 4. 8. 16. 32. &c. be in geometrical progression,
Then will $1\times32=2\times16=4\times8$.
Or, if 1. 2. 4. 8. 16. &c. be in geometrical progression,
Then will $1\times16=2\times8=4\times4$.

4. If, out of any series of numbers in geometrical progression, there be taken any series of *equidistant* terms, *that* series will likewise be in geometrical progression.

Thus, if 2. 4. 8. 16. 32. 64. &c. be in geometrical progression,
Then will 4. 16. 64. &c. be in geometrical progression.

Proposition 1. *Given the number of terms the ratio, and either of the extreme terms, of a limited geometrical series, to find the other extreme.*

Rule. Write down a few terms of a geometrical series, beginning with, and formed by, the given ratio; over which place the arithmetical series 1. 2. 3. 4. 5. &c. as *indices:* observe what figures of these indices, when added together, will give a number an unit less than that expressing the number of terms; and find the product of the terms in the geometrical series which stand under these indices. This product multiplied by the first term given in the question, or the first term divided by this product, according as the progression is increasing or decreasing, will give the term sought.

Prop. 2. *Given one extreme, the ratio, and the number of terms, of a geometrical series, to find the sum of the terms.*

Rule. Find the other extreme by *Proposition* 1. Then divide the difference between the extremes by the ratio less 1; the quotient increased by the *greater* extreme will give the sum of the terms.

Prop. 3. *In any series of numbers in geometrical progression, decreasing, ad infinitum,—given the first term and the ratio to find the sum of the series.*

Rule. Subtract the second term from the first; the square of the first term, divided by this difference, will give the sum of the series.

See the 7th note in circulating decimals, Part I. page 105.

Note. If l=the least term, g=the greatest, n=the number of terms, s=the sum of the terms, r=the ratio, log=logarithm of any letter.

Then will the following theorems exhibit all the possible cases of geometrical progression, including those already given.

Proposition 1. *Given l, g, and n, to find s and r.*

Theo. I. $$g+\frac{g-l}{\overline{\frac{g}{l}}\Big|^{\frac{1}{n-1}}-1}=s$$

Theo. II. $$\frac{g}{l}\Big|^{\frac{1}{n-1}}=r.$$

Prop. 2. *Given l, g, and s, to find n and r.*

Theo. III. $$r^{n-1}=\frac{g}{l},\ or,\ \frac{\log.\ g-\log.\ l}{\log.\ \overline{s-l}-\log.\ \overline{s-g}}+1=n.$$

Theo. IV. $$\frac{s-l}{s-g}=r,\ or\ \log.\ \overline{s-l}-\log.\ \overline{s-g}=\log.\ r.$$

Prop. 3. *Given l, g, and r, to find n and s.*

Theo. V. $$r^{n-1}=\frac{g}{l}\ or,\ \frac{\log.\ g-\log.\ l}{\log.\ r}+1=n.$$

Theo. VI. $$\frac{\overline{r\times g-l}}{r-1}=s,\ or,\ g+\frac{g-l}{r-1}=s.$$

Prop. 4. *Given l, n, and s, to find g and r.*

Theo. VII. $$g\times\overline{s-g}\Big|^{n-1}=l\times\overline{s-l}\Big|^{n-1}$$

Theo. VIII. $\frac{sr}{l}-r^{n}=\frac{s-l}{l}$. The value of g in the first equation, and the value of r in the second, must be found as directed in the 2d note in *Double Position.*

Prop. 5. *Given l, n, and r, to find g and s.*

Theo. IX. $lr^{n-1}=g$. *Theo.* X. $\frac{r^{n}-1}{r-1}\times l=s$.

Prop. 6. *Given l, s, and r, to find g and n.*

Theo. XI. $\frac{s\times\overline{r-1}+l}{r}=g$.

Theo. XII. $r^{n}=\frac{s\times\overline{r-1}}{l}+1$; *or, log.* $\frac{s\times\overline{r-1}}{l}+1\div$ *log.* $r=n$.

Prop. 7. *Given g, n, and s, to find l and r.*

Theo. XIII. $l\times\overline{s-l}^{\,n-1}=g\times\overline{s-g}^{\,n-1}$

Theo. XIV. $\frac{sr^{n-1}-r^{n}}{s-g}=\frac{g}{s-g}$

Where the value of l in the first equation, and the value of r in the second, must be found as directed in the second note in *Double Position*, being the most difficult proposition in geometrical progression.

Prop. 1. *Given g, n, and r, to find l and s.*

Theo. XV. $\frac{g}{r^{n-1}}=l$. *Theo.* XVI. $\frac{r^{n}-1}{r^{n}-r^{n-1}}\times g=s$.

Prop. 9. *Given g, s, and r, to find l and n.*

Theo. XVII. $\overline{rg+s}-rs=l$.

Theo. XVIII. $r^{n-1}=\frac{g}{rg+s-rs}$, *or,* $\frac{\log.\ g-\log.\ \overline{rg+s-rs}}{\log.\ r}+1=n$.

Prop. 10. *Given n, s, and r, to find l and g.*

Theo. XIX. $\frac{sr-s}{r^{n}-1}=l$. *Theo.* XX. $\frac{r^{n}-r^{n-1}}{r^{n}-1}\times s=g$.

The above theorems will answer for any *finite* series of numbers, either increasing or decreasing; (see note 9th Arithmetical Progression.) But, if the series decrease, *ad infinitum*, then n will be infinite, or greater than any assignable number, and $l=0$. Hence the three following theorems answer all the possible cases of an infinitely decreasing geometrical progression.

Prop. 11. *Given g and r to find s.*

Theo. XXI. $\frac{rg}{r-1}=s.$ Or proceed by Prop. 3rd, page 235.

Prop. 12. *Given r and s to find g.*

Theo. XXII. $\frac{s\times\overline{r-1}}{r}=g.$

Peop. 13. *Given g and s to find r.*

Theo. XXIII. $\frac{s}{s-g}=r.$ In the three preceding theorems, if the ratio be a fraction, then r must represent the reciprocal of that fraction. Thus if the ratio be $\frac{7}{9}$, then $r=\frac{9}{7}$, &c.

Examples to Proposition 1.

(1.) The first, or least, term of a series of numbers in geometrical progression is 3, the ratio 3, and the number of terms 14, what is the greatest, or last term?

1 . 2 . 3 . 4 . 5, &c. indices
3 . 9 . 27 . 81 . 243, &c. geometrical series.
5 + 5 + 3 = 13, an *unit* less than the number of terms
243 × 243 × 27 = 1594323
Then 1594323 × 3 = 4782969 the 14th term.

(2.) If the first, or least, term be 2, the ratio 2, and the number of terms 19, what is the last, or greatest, term?

(3.) A draper sold 20 yards of cloth; the first yard for 3*d.*, the second for 9*d.*, the third for 27*d.*, &c. in triple proportion geometrical; what did he sell the last yard for?

(4.) The first, or least, term of a geometrical series is 5, the ratio 3, and the number of terms 12; what is the last, or greatest, term?

1 . 2 . 3 . 4 . 5, &c. indices
3 . 9 . 27 . 81 . 243, &c. geometrical series
3 + 4 + 4 = 11 *one* less than the number of terms
27 × 81 × 81 = 177147
Then 177147 × 5 = 885735, *answer.*

Note. If the greatest term (885735) had been given to find the least (5), the operation would have been the same, excepting that 885735 must have been divided by 177147.

(5.) If the first, or least, term be 7, the ratio 2, and the number of terms 19, what is the last, or greatest, term?

(6.) A thrasher worked 20 days for a farmer, and received (by agreement) for the first day's work 4 barley-corns, for the second 12, for the third 36, &c., in triple proportion geometrical; what did he receive for his last day's work, admitting 7680 barley-corns to fill a pint measure, and the barley to be worth 2*s.* 6*d.* per bushel?

Examples to Prop. 2

(7.) If the first term of a series of numbers in geometrical progression be 5, the ratio 3, and number of terms 12, what is the sum of the terms?

The last or greatest term (by example 4,) is 885735.
Then 885735—5=885730 difference between the extremes.
And 3—1=2 ratio less 1. Hence 885730÷2=442865; and 442865+885735=1328600, sum of the terms.

(8.) If the first term be 4, the ratio 3, and the number of terms 7, what is the sum of the terms?

CLASS II. *exercising all the preceding propositions.*

(9.) What would a horse be sold for that has 4 shoes on, with 8 nails in each shoe, at 1 farthing for the first nail, 2 for the second, 4 for the third, &c. And, supposing another horse to be sold with only two shoes on, on the same conditions, what would be the difference in their prices?

(10.) If a servant should agree with his master to serve him 11 years, without any other reward than the produce of a wheat-corn for the first year; and, for the second year, ground sufficient to sow his first year's produce on, &c. from year to year to the end of the time: what would his wages amount to, admitting each wheat-corn to yield ten by sowing, 7680 wheat-corns to fill a pint-measure, and that he could sell his wheat at 8*s.* per bushel?

(11.) A nobleman dying, left ten sons, to whom and to his executor he bequeathed his estate as follows: to his executor he gave 1024*l.*, the youngest son was to have as much and half as much, and every son to succeed the next younger in the same ratio of $1\frac{1}{2}$; what was the eldest son's fortune, and what did the nobleman die worth?

(12.) Required the sum of 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, &c. continued 15 terms?

(13.) Required the sum of $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, $\frac{3}{10000}$, &c. carried to 12 terms.

(14.) The greater extreme of a descending series in geometrical progression is 1835008, the ratio 2, and the number of terms 19; what is the sum of the terms?

Examples to Prop. 3.

(15.) Required the sum of $\frac{3}{10}+\frac{3}{100}+\frac{3}{1000}+\frac{3}{10000}$, &c. *ad infinitum.*

$\frac{3}{10}-\frac{3}{100}=\frac{27}{100}$, difference between the first and second terms. $\frac{3}{10}\times\frac{3}{10}=\frac{9}{100}$, square of the first term. Then $\frac{9}{100}\div\frac{27}{100}=\frac{1}{3}$, answer. Hence we may infer, that if a ball were put in motion by a force, which moved it $\frac{3}{10}$ of a league, or 1584 yards, the first minute, (or any portion of time,) $\frac{3}{100}$ of a league, or $158\frac{2}{5}$ yards, the second, &c. for *ever*, it would go no farther than 1 mile! For, it is evident, that $\frac{3}{10}+\frac{3}{100}$, &c. *ad infinitum*, $=$·3333, &c. *ad infinitum*; and this is equal to $\frac{1}{3}$ precisely, by the nature of vulgar fractions and infinite decimals.

(16.) Required the sum of $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$, &c. *ad infinitum.*

(17.) Required the sum of $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}$, &c. *ad infinitum.*

(18.) If a body be put in motion by a force which moves it 10 miles the first portion of time, 9 miles in the second equal portion, and so on (in the ratio of $\frac{9}{10}$) for *ever*, how many miles will it pass over?

VARIATIONS.

Definition. By Variations are meant, the different ways any number of things may be altered, or changed, with respect to their places. These are sometimes called *Changes, Permutation, Alternation*, &c.

Proposition 1. *To find the number of changes that can be made of any given number of things, all different from each other.*

Rule. Multiply continually together the numbers 1, 2, 3, 4, 5, &c., to the number of terms; and the last product will be the answer.

Prop. 2. *Given any number of different things, to find how many changes can be made of them, by taking any given number of them at a time.*

Rule. Multiply the number of things by itself less 1, and that product by the same number less 2, &c. diminishing each succeeding multiplier, by an unit, till you have made as many products (abating one) as there are things taken at a time; the last product will be the answer.

Examples to Proposition 1.

(1.) How many changes may be rung by 8 bells?

$1\times2\times3\times4\times5\times6\times7\times8=40320$, answer.

(2.) How many changes may be rung on 9 bells?

(3.) An arithmetician asked a farmer with whom he lodged, what he should give him per annum for *board* and lodging; the farmer asked him 25*l*. The arithmetician said that was somewhat dear; however, he would give him that sum if he would find him with board and lodging so long as he could place himself and the honest farmer's family (consisting of 6 persons) in a different position at dinner. How long might he stay for 25*l*.

(4.) How many changes may be rung on 12 bells, and how long would they take in ringing once over, supposing 10 changes to be rung in a minute, and the year to consist of 365 days 6 hours?

Examples to Prop. 2.

(5.) How many changes may be rung with 4 bells out of 8?

$8\times\overline{8-1}\times\overline{8-2}\times\overline{8-3}=8\times7\times6\times5=1680$, answer.

(6.) How many changes may be rung with 7 bells out of 12?

(7.) Required the number of words that can be made with 5 letters of the 26 in the alphabet, allowing any 5 letters to make a word?

COMBINATIONS.

Definition. By *Combinations* must be understood a method of taking a less number of quantities out of a greater, as often as possible, without respect to their places, and combining them together.

Proposition. To find the combinations of a less number of things out of a greater, all different.

Rule. Take the series 1, 2, 3, 4, 5, &c. up to the less number of things, and multiply them continually together: then take a series of as many terms, decreasing by an unit, from the greater number of things, and multiply *them* continually together.—Divide the latter product by the former, and the quotient will be the answer.

Examples.

(1.) How many combinations can be made with 5 letters out of the 26 of the alphabet?

$1 \times 2 \times 3 \times 4 \times 5 = 120$ divisor.

$26 \times \overline{26-1} \times \overline{26-2} \times \overline{26-3} \times \overline{26-4} = 7893600$ dividend; and $7893600 \div 120 = 65780$, answer.

Or,

$$\frac{\overset{13}{\cancel{26}} \times \overset{5}{\cancel{25}} \times \overset{2}{\cancel{24}} \times 23 \times 22}{1 \times \cancel{2} \times \cancel{3} \times \cancel{4} \times \cancel{5}} = 13 \times 5 \times 2 \times 23 \times 22 = 65780.$$

(2.) A successful general was asked by his sovereign what reward he should confer upon him for his services; the general modestly asked only a farthing for every file of 10 men in a file which he could make with a body of 100 men; what sterling money will this amount to?

Those who wish for farther information in the *doctrine of combinations, permutations,* &c. may consult Mr. *Emerson's* Treatise on the subject.

COMPOUND INTEREST BY DECIMALS

Put p = the principal or money lent.

r = the ratio, or amount of £1 for a year, ½ year, ¼ year, &c. according as the payments are made yearly, ½ yearly, quarterly, &c.

t = the time, or *number of payments.*

a = the amount.

Rates per cent.	The amounts of £1, or values of r for		
	Yearly payments.	Half-yearly payments.	Quarterly payments.
3	1·03	1·015	1·0075
$3\frac{1}{2}$	1·035	1·0175	1·00875
4	1·04	1·02	1·01
$4\frac{1}{2}$	1·045	1·0225	1·01125
5	1·05	1·025	1·0125
$5\frac{1}{2}$	1·055	1·0275	1·01375
6	1·06	1·03	1·015

The amounts, or values of r, in the preceding table, are calculated thus:

$$100 : 100+3 :: 1 : 1{\cdot}03 = r \text{ for yearly payments.}$$
$$100 : 100+1\tfrac{1}{2} :: 1 : 1{\cdot}015 = r \text{ for } \tfrac{1}{2} \text{ yearly payments.}$$
$$100 : 100+\tfrac{3}{4} :: 1 : 1{\cdot}0075 = r \text{ for quarterly payments.}$$

This method is most commonly used.—Some writers find the value of r thus: let m = the amount of £1 for half a year, at 3 per cent. then 1·03 is undoubtedly the true amount for a year; hence, according to the principles on which the rules of compound interest are founded.

$1 : m :: m : 1{\cdot}03 \therefore m = \sqrt{1{\cdot}03} = 1{\cdot}014889$, &c. $= r$, for $\frac{1}{2}$ yearly payments.

$1 : m^2 :: m^2 : 1{\cdot}03 \therefore m = \sqrt[4]{1{\cdot}03} = 1{\cdot}007417$, &c. $= r$, for quarterly payments.

Or, if m = the amount of £1 for $\frac{1}{n}$ of a year, at R per cent. then $r = \overline{\frac{R}{100}+1}\Big|^{\frac{1}{n}}$ *universally.*

And these values of r appear to be more correct than those given above, especially in the calculation of annuities; for which reason the following table is inserted, that the reader may use which he pleases. Mr. *Ward*, in his *Clavis Usuræ*, published in 1710, makes use of this method.

Rates per cent.	The amount of £1, or values of r, for		
	Yearly payments.	Half yearly payments.	Quarterly payments.
3	1·03	1·014889	1·007417
$3\frac{1}{2}$	1·035	1·017349	1·008637
4	1·04	1·019803	1·009853
$4\frac{1}{2}$	1·045	1·022252	1·011065
5	1·05	1·024695	1·012272
$5\frac{1}{2}$	1·055	1·027132	1·013475
6	1·06	1·029563	1·014673

Proposition 1. *Given the principal, rate, and time, to find the amount or interest.*

Rule. Find the amount of £1. for the first payment, by simple interest, which involve to such a power as is denoted by the number of payments.—This power, multiplied by the principal, will give the amount; from which deduct the principal, and the remainder will be the interest.

Or, Theo. I. $p \times r^t = a$, *when p, r, and t, are given.*

Logarithmically, $\log. p. + \log. r \times t = \log. a.$

Prop. 2. *Given the amount, rate, and time, to find the principal.*

Rule. As the amount of £1., at the rate and for the time given, is to £1., so is the amount given to the principal required.

Or, Theo. II. $\frac{a}{r^t} = p$, *when a, r, and t, are given.*

Logarithmically, $\log. a - \log. r \times t = \log. p.$

Prop. 3. *Given p, a, and t, to find r.*

Theo. III. $\left.\frac{a}{p}\right|^{\frac{1}{t}} = r.$

Logarithmically, $\frac{\log. a - \log. p.}{t} = \log. r.$

Prop. 4. *Given p, a, and r, to find t.*

Theo. IV. $\frac{a}{p} = r^t.$ If t be not a whole number, it cannot be found without logarithms.

Logarithmically, $\frac{\log. a - \log. p.}{\log. r.} = t.$

Examples to Proposition I.

(1.) What will 200*l.* amount to in 6 years, at 5 per cent. per annum, compound interest, and what interest will it gain?

Here the amount of £1 for the first payment is £1·05, and 1·05 ×

$1{\cdot}05\times1{\cdot}05\times1{\cdot}05\times1{\cdot}05\times1{\cdot}05=1{\cdot}340095640625$ $(=r^t)$. This multiplied by 200, gives $268{\cdot}019128125=$ £268 0 4½ 363 $(=p\times r^t)$ the amount; from which deduct 200l. the principal, and the remainder, 68l. 0s. 4½d. 363, will be the interest.

(2.) What will 275*l.* amount to in 3 years, at 5 per cent. per annum, compound interest?

(3.) What is the compound interest of 700*l.* 15*s.* for 7 years, at 4 per cent. per annum?

(4.) What is the compound interest of 800*l.* for 9 years, at 5 per cent. per annum?

Examples to Prop 2.

(5.) What principal, put to interest for 6 years, will amount to 268*l.* 0*s.* 4½*d.*363 at 5 per cent. per annum?

First, £268 0 4½ ·363 = 268·019128125 $(=a,)$ and $1{\cdot}05\times1{\cdot}05\times1{\cdot}05\times1{\cdot}05\times1{\cdot}05\times1{\cdot}05=1{\cdot}340095640625$ $(=r^t)$ amount of 1l. for 6 years. Hence,

$1{\cdot}340095640625 : 1$l :: $268{\cdot}019128125 : 200$l. the principal required $(=\frac{a}{r^t}.)$

(6.) What principal, put to interest for 3 years, will amount to 318*l.* 6*s.* 11¼*d.* at 5 per cent. per annum?

(7.) What principal, put to interest for 4 years, at 4 per cent. per annum, will amount to 819*l.* 15*s.* 6¾*d.* ·2504832?

(8.) What principal, put to interest for 9 years, at 5 per cent. per annum, will amount to 1241*l.* 1*s.* 3.017467875*d.*

Examples to Prop. 3. *Theorem* III.

(9.) At what rate per cent. will 200*l.* amount to 268*l.* 0*s.* 4½*d.*363 in 6 years time?

First, £268 0 4½ ·363 = £268·019128125 $(=a,)$ and

$268{\cdot}019128125\div200=1{\cdot}340095640625$ $(=\frac{a}{p})$; the 6th root of

which (by the rule page 214,) is 1·05 $\left(=\overline{\frac{a}{p}}\right|^{\frac{1}{t}}.)$

Or, the square root of 1·340095640625 *is* 1·157625, *and the cube-root of* 1·157625 *is* 1·05 *as above. Hence the rate is* 5 *per cent.*

(10.) At what rate per cent. will 275*l.* amount to 318*l.* 6*s.* 11½*d.* in 3 years time?

(11.) At what rate per cent. will 700*l.* 15*s.* amount to 819*l.* 15*s.* 6¾*d.*·2504832 in 4 years?

(12.) At what rate per cent. will 800*l.* amount to 1241*l.* 1*s.* 3·017467875*d.* in 9 years?

Examples to Prop. 4. *Theo.* IV.

(13.) In what time will 200*l.* amount to 268*l.* 0*s.* 4½*d.*·363, at 5 per cent. per annum?

$$£268\ 0\ 4\tfrac{1}{2}{\cdot}363 = £268{\cdot}019128125\ (= a.)$$

$$\text{Then } 268{\cdot}019128125 \div 200 = 1{\cdot}340095640625\ \left(=\frac{a}{p}=r^t\right)$$

which, being divided by 1·05, (= *r*,) and the *quotient* by 1·05, &c. till nothing remains, the number of divisions will shew the time, 6 years.

Note. This method of finding the time, by repeated divisions, is made use of by Mr. Ward, *(see Ex. 3rd, page 255, 8th edit. of his Math. Guide,) and several writers have followed his example; but it is far from being an eligible or correct method. It may serve to prove a question, when the time happens to be whole years. The best method of solving the questions in this and the preceding proposition is by logarithms.*

(14.) In what time will 275*l.* amount to 318*l.* 6*s.* 11½*d.*, at 5 per cent. per annum?

(15.) In what time will 700*l.* 15*s.* amount to 819*l.* 15*s.* 6¾*d.*·2504832, at 4 per cent. per annum?

(16.) In what time will 800*l.* amount to 1241*l.* 1*s.* 3·017467875*d.* at 5 per cent. per annum?

EQUATION OF PAYMENTS AT COMPOUND INTEREST.

Proposition. Having several debts, due at different times, from one person, to find the TRUE *equated time for paying the whole at once, without loss either to the debtor or creditor, allowing compound interest.*

Rule 1. Find the amount of each debt from the time it becomes due to the time of the last payment, [by Prop. 1. Compound Interest,] add these amounts, together with the last payment, into one sum.

2. Find in what *time* [by Prop. 4, Compound Interest,] the sum of the debts will amount to the sum of the amounts found above:—this *time*, subtracted from the time the last payment becomes due, will give the *true* equated time.

Note. This rule, which is *Sir Samuel Moreland's*, is founded on the same manner of reasoning as the common rule, Part I. and will bring out the *same* answer, allowing simple interest instead of compound.—Were this a place for algebraical demonstrations, it might easily be shewn, that the above rule is universally true, allowing *compound interest*, whether we argue from *Burrow's*, *Kersey's*, or *Mulcolm's* principles, it being deducible from each.

Examples.

(1.) A owes to B 1000*l.*, 200*l.* of which will be due one year hence, 200*l.* two years hence, 150*l.* three years hence, 300*l.* four years hence, and 150*l.* five years hence; should these persons agree to have the whole discharged at once, what will be the *true* equated time, reckoning interest at 5 per cent. per annum?

$\overline{1\cdot05}\rvert^4 \times 200 = 243\cdot10125$	amount of the first payment.
$\overline{1\cdot05}\rvert^3 \times 200 = 231\cdot525$	ditto of the second.
$\overline{1\cdot05}\rvert^2 \times 150 = 165\cdot375$	ditto of the third.
$\overline{1\cdot05}\rvert^1 \times 300 = 315\cdot$	ditto of the fourth.
$150\cdot$	last payment.
£1105·00125	sum of the amounts.

200+200+150+300+150=£1000, sum of the debts. Now we have to find in what time £1000 will amount to £1105·00125, at 5 per cent. compound interest; *and here we must be under the necessity of making use of logarithms, since the method made use of in Ex.* 13, *page* 245, *will by no means do.*

$$\text{First, } 1105\cdot00125 = \frac{884001}{800}\text{, then } log.\ \frac{884001}{800} - log.\ 1000 =$$

0·0433628 (=*log. a*—*log. p*;) this divided by *log.* 1·05 (=*log. r*) = 0·0211893 gives 2·0464, &c. for the quotient, (= *t.*) Hence 5 years—2·0464, &c. years=2·9535, &c. the *true* equated time.

(2.) A person has 320*l.* due to him; and, at the end of 5 years, 96*l.* more will be due from the same debtor; now

both parties have agreed for the whole to be discharged at once. The *true* equated time is required, reckoning interest at 5 per cent. per annum?

(3.) There is 100*l.* payable one year hence; and 105l. payable three years hence, what is the *true* equated time, allowing compound interest at 5 per cent. per annum?

(4.) There is 100*l.* payable one year hence, 200*l.* two years hence, 300*l.* three years hence, and 500*l.* five years hence; required the *true* equated time for paying the whole at once, reckoning compound interest at 5 per cent. per annum?

ANNUITIES CERTAIN.

Definition 1. *Annuities certain* signify any interest of money, rents, or pensions, payable yearly, or from time to time, to *some* certain period, or for *ever.* They are divided into two parts, viz. annuities in *possession*, or such as are either entered upon, or are to be entered upon immediately; and annuities in *reversion*, or such as are not to be entered upon till some particular future event has happened, or till some given period of time has elapsed; and the time the purchaser holds the annuity, after he has entered upon it, is called the *reversion.*

2. *An annuity is said to be in arrears* when the debtor keeps it in his hands for any certain time after the term of payment; and the sum of all the single payments, together with the interest due upon each payment from the time of its becoming due to the time the whole is paid off, is called the amount of such annuity.

3. *When an annuity*, to be entered on immediately, or some time hence, is sold for ready money, the price which ought to be paid for it is called the present worth.

ANNUITIES AT COMPOUND INTEREST.

Let n = the annuity, pension, rent, or *payment*, whether yearly, half-yearly, or quarterly.

t = the time, or *number of payments.*

r = the ratio, or amount of £1. for a year, $\frac{1}{2}$ year, $\frac{1}{4}$ year, &c. according as the payments are made yearly, half-yearly, quarterly, &c. by either of the methods or tables given in compound interest.

a = the amount.

ANNUITIES *in Arrears, at Compound Interest.*

Proposition 1. Given the annuity, payable in whole years, or at any equal number of payments, the rate per cent., and time, to find the amount.

Rule. Make an unit the first term of a geometrical series, the amount of £1 for 1 year, $\frac{1}{2}$ year, $\frac{1}{4}$ year, &c. the *ratio*, according as the payments are made yearly, half-yearly, quarterly, &c.—Carry the series to as many terms as there are payments, and find its sum, (by Prop. 2, of Geometrical Progression), which multiply by the annuity, and the product will be the amount.

Or, *Theo.* I. $\frac{r^t-1}{r-1}\times n=a$, when n, r, and t are given.

Logarithmically. $log.\ \overline{r^t-1}+log.\ n.-log.\ \overline{r-1}=log.\ a.$

Prop. 2. *Given a, r, and t, to find n.*

Theo. II. $\frac{r-1}{r^t-1}\times a=n.$

Logarithmically. $log.\ \overline{r-1}+log.\ a-log.\ \overline{r^t-1}=log.\ n.$

Prop. 3. *Given a, r, and n, to find t.*

Theo. III. $\frac{\overline{r-1}\times a}{n}+1=r^t.$ If t be not a whole number, it cannot be found with logarithms.

Logarithmically. $\frac{log.\ \overline{\overline{ar-a+n}}-log.\ n.}{log.\ r.}=t.$

Prop. 4. *Given a, n, and t, to find r.*

Equation. $\frac{ar}{n}-r^t=\frac{a-n}{n}$. After this equation is reduced to numbers, the value of r must be found as directed in the 2d note in Double Position. If t be a mixed fraction, the value of r cannot be found without logarithms.

Examples to Proposition 1.

(1.) What is the amount of an annuity of 100*l.* to continue 5 years, at 6 per cent. per annum, compound interest?

$$1+1{\cdot}06+\overline{1{\cdot}06}\,^2+\overline{1{\cdot}06}\,^3+\overline{1{\cdot}06}\,^4=\frac{\overline{1{\cdot}06}\,^4-1}{1{\cdot}06-1}+\overline{1{\cdot}06}\,^4=$$

5·63709296; this multiplied by 100, the annuity, gives 563·709296 =563l. 14s. 2·23104d. for the amount required.

Or, by Theorem I.

$1{\cdot}06\times1{\cdot}06\times1{\cdot}06\times1{\cdot}06\times1{\cdot}06=1{\cdot}3382255776=r^t$ and $1{\cdot}3382255776-1=\cdot3382255776=r^t-1$; also, $1{\cdot}06-1=\cdot06=r-1$.

Hence $\dfrac{\cdot3382255776}{\cdot06}\times100=563{\cdot}709296$, the amount as before.

(2.) What is the amount of an annuity of 80*l.* unpaid, or in arrears, for 9 years, at 5 per cent?

(3.) Required the amount of an annuity of 560*l.* to continue 7 years, at 5 per cent. per annum?

The following Examples are performed by Logarithms.

(4.) If a pension of 356*l.* per annum, payable half-yearly, be unpaid for 9 years, what will it amount to at 6 per cent?

Here $n=178$, $r=1{\cdot}03$, (by Table I. p. 242), and $t=18$.

$log.\ r=log.\ 1{\cdot}03=0{\cdot}0128372$
$\qquad\qquad\qquad\qquad\qquad 18$

$log.\ r^t \quad = \quad 0{\cdot}2310696$, the number answering to which is $1{\cdot}702433=r^t$

$log.\ r^t-1=log^t\ \cdot702433=-1{\cdot}8466048$
$log.\ n.=log.\ 178=+2{\cdot}2504200$

$+2{\cdot}0970248$
$log.\ r-1=log.\ \cdot03=-2{\cdot}4771213$

log of the amount$=+3{\cdot}6199035$, the number answering to which is 4167·768=4167l. 15s. 4·32d. answer.

(5.) What is the amount of an annuity of 350*l.*, payable half-yearly, unpaid for 4 years, at 4½ per cent.?

(6.) If an annuity of 70*l*. payable quarterly, be unpaid for 5 years, what will it amount to at 5 per cent.?

Examples to Prop. 2. Theo. II.

(7.) What annuity, unpaid for 5 years, will amount to 563*l*. 14*s*. 2·23104*d*. at 6 per cent?

£563 14 2·23104=563·709296=a.

1·06—1=·06=r—1 dividend.

1·06×1·06×1·06×1·06×1·06=1·3382255776=r^5, and $r^5-1=$ ·3382255776, divisor.

Then $\frac{\cdot06}{\cdot338225576}\times563\cdot709296=£100=n$, the annuity.

(8.) What annuity, unpaid for 9 years, will amount to 882*l*. 2*s*. 6·04*d*. at 5 per cent.?

(9.) What annuity, in arrears for 7 years, will amount to 4559*l*. 10*s*. 5*d*. 3·7444, at 5 per cent.?

The following Examples are performed by Logarithms.

(10.) What annuity, payable half-yearly, will amount to 4167*l*. 15*s*. 4·32*d*. at 6 per cent. if unpaid for 9 years?

£4167 15 4·32=4167·768=a, by the 4th question $r^t-1=\overline{1\cdot03}|^{18}-1=\cdot702433$.

$$\begin{array}{rr} log.\ r-1=log.\ \cdot03= & -2\cdot4771213 \\ log.\ a=log.\ 4167\cdot768= & +3\cdot6199035 \\ \hline & +2\cdot0970248 \\ log.\ \cdot702433= & -1\cdot8466048 \\ \hline \end{array}$$

log. of ½ the annuity=+2·2504200, the number answering to which is 178. Hence the annuity is 356*l*.

(11.) What annuity, payable half-yearly, will amount to 1515*l*. 6*s*. 1.1*d*. if unpaid for four years, at 4½ per cent.?

(12.) What annuity, payable quarterly, will amount to 394*l*. 16*s*. 10½*d*. at 5 per cent. if unpaid for 5 years?

Examples to Prop. 3. Theo. III.

(13.) To find the time in which an annuity of 100*l*. will amount to 563*l*. 14*s*. 2·23104*d*. at 6 per cent. per annum?

£563 14 2·23104=563·709296=a.
1·06−1=·06=$r-1$.

563·709296 × ·06=33 82255776=$\overline{r-1} \times a$.

Then $\frac{33\cdot82255776}{100}+1=1\cdot3382255776=\overline{1\cdot06})$ *which number being continually divided by r=1·06 (according to Mr. Ward and others) till nothing remains, the number of divisions will be 5, the years required.*

(14.) In what time will an annuity of 80*l*., unpaid, amount to 882*l*. 2*s*. 6·04*d*. at 5 per cent.?

(15.) What time must an annuity of 560*l*. continue in arrears, at 5 per cent., to raise a stock of 4559*l*. 10*s*. 5*d*. 3·7444?

The following Examples are performed by Logarithms.

(16.) What time must an annuity of 356*l*., payable half-yearly, be continued, at 6 per cent., to raise a stock of 4167*l*. 15*s*. 4·32*d*.?

£4167 15 4·32=4167·768=a
1·03=r

4292·80104=ar
4167·768 =a

125·03304=$ar-a$
178 =n

303·03304=$\overline{ar-a}+n$

log. 303·03304=2·4814899
log. 178=2·2504200

log. r=*log*. 1·03=·0128372)0·2310699(18=t, the number of payments; hence the time is 9 years.

(17.) How long must an annuity of 350*l*., payable half-yearly, remain in arrears at 4½ per cent., to raise a stock of 1515*l*. 6*s*. 1·1*d*.?

(18.) How long must an annuity of 70*l*., payable quarterly, remain in arrears, at 5 per cent., to raise a stock of 304*l*. 16*s*. 10¼*d*.?

Examples to Prop. 4.

(19.) Required the rate per cent. compound interest,

for an annuity of 100*l.*, continuing 5 years, to raise an amount of 563*l.* 14*s.* 2·23104*d.*?

£563 14 2·23104=563·709296=q.

$$\text{Then } \frac{563{\cdot}709296r}{100} - r^5 = \frac{563{\cdot}709296-100}{100}$$

Or, $5{\cdot}63709296r - r^5 = 5{\cdot}63709296 - 1$. Now, by (note 2, page 226) Double Position, I find $r=1{\cdot}06$; hence the rate is 6 per cent.

(20.) An annuity of 80*l.* in arrears for 9 years, amounted to 882*l.* 2*s.* 6 04*d.*, what was the rate per cent.?

PRESENT WORTH OF ANNUITIES IN ARREARS,

AT COMPOUND INTEREST.

Proposition 1. *To find the present worth of an annuity at compound interest, the time of its continuance, and the rate per cent. being given.*

Rule. Find the amount of the annuity, (by Prop. I. page 249,) supposing it in arrears, till the last payment is due. Then find the present worth of that amount, (by Prop. 2. Compound Interest,) and it will be the answer.

Or,

Find the present worth of each payment by itself, discounting from the time it falls due, (by Prop. 2. Compound Interest,) the sum of these present worths will be the present worth of the whole.

Or, *Theo.* I. $\dfrac{1-\frac{1}{r^t}}{r-1} \times n = p$; the present worth when n, r, and t, are given.

Logarithmically, $\log. \overline{1-\dfrac{1}{r^t}} + \log. n - \log. \overline{r-1} = \log. p.$

Prop. 2. *Given* p, r, *and* t, *to find* n.

Theo. II. $\dfrac{r-1}{1-\frac{1}{r^t}} \times p = n.$

Logarithmically, $\log. \overline{r-1} + \log. p. - \log. \overline{1-\dfrac{1}{r^t}} = \log. n.$

Prop. 3. *Given p, r, and n, to find t.*

Theo. III. $\frac{n}{n+p-pr}=r^{t}$. If t be not a whole number, it cannot be found without logarithms.

Logarithmically, $\frac{\log.\ n-\log.\ \overline{n+p-pr}}{\log\ r}=t.$

Prop. 4. *Given p, n, and t, to find r.*

Equation, $\frac{\overline{n+p}}{p}\times r^{t}-r^{t+1}=\frac{n}{p}$. After this equation is reduced to numbers, the value of r must be found (as directed in the second note, page 226,) by Double Position. If t, be not a whole number, the value of r cannot be found without logarithms.

Examples to Proposition 1.

(1.) Required the present worth of an annuity of 100*l.* to be continued 5 years, allowing 6 per cent. compound interest.

The amount of 100*l.* for 5 years is 563·709296 (See *Example* I. *page* 249,) and 1·06 × 1·06 × 1·06 × 1·06 × 1·06 = 1·3382255776, the amount of 1*l.* for 5 years.

As 1·3382255776 : 1*l.* :: 563·709296 : 421·236378 = 421*l.* 4*s.* 8·731*d.* the present worth.

Or thus,

Ratio = 1·06	)100 (94·339622, present worth of 100*l.* for 1 year.
$\overline{1{\cdot}06}\vert^{2}$ = 1·1236	)100 (88·999644, ditto for the 2d year.
$\overline{1{\cdot}06}\vert^{3}$ = 1·191016	)100 (83·961928, ditto for the 3d year.
$\overline{1{\cdot}06}\vert^{4}$ = 1·26247696	)100 (79·209366, ditto for the 4th year.
$\overline{1{\cdot}06}\vert^{5}$ = 1·3382255776	)100 (74·725817, ditto for the 5th year.

The sum is the present worth, 421·236378 = £421 4 8·731, as above.

(2.) Required the present worth of an annuity of 80*l.* to continue 9 years, at 5 per cent. per annum, compound interest.

(3.) Required the present worth of an annuity of 560*l.* to continue 7 years, at 5 per cent. per annum compound interest.

The following Examples are performed by Logarithms.

(4.) What is the present worth of an annuity of 356*l*., payable half-yearly, to continue 9 years, allowing 6 per cent. compound interest to the purchaser?

Here $n = \frac{356}{2} = 178$, $r = 103$, (*by Table* I. *page* 242,) and $t = 18$.

$log.\ r = log.\ 1 \cdot 03 = 0 \cdot 0128372$
$\quad 18$

$log.\ r^t = 0 \cdot 02310696$; this subtracted from the *log.* of $1 = 0$, gives $-1 \cdot 7689304$ for the *log.* of $\frac{1}{r^t}$; the number answering to which is $\cdot 5873946$; then $1 - \cdot 5873946 = \cdot 4126053 = 1 - \frac{1}{r^t}$

$$log.\ 1 - \frac{1}{r^t} = log.\ \cdot 4126053 = -1 \cdot 6155348$$
$$log.\ n = log.\ 178 = +2\ 2504200$$
$$+1 \cdot 8659548$$
$$log.\ r - 1 = log.\ \cdot 03 = -2 \cdot 4771213$$

log. of the present worth $+3 \cdot 388335$ the number answering to which is $2448 \cdot 1253 = £2448\ 2\ 6 \cdot 072$ answer.

(5.) What is the present worth of an annuity of 350*l*., payable half-yearly, to continue 4 years, allowing 4 per cent. interest?

(6.) What is the present worth of an annuity of 70*l*, payable quarterly, to continue 5 years, allowing 5 per cent. per annum, compound interest?

Examples to Prop. 2. *Theo.* II.

(7.) What annuity, to continue 5 years, at 6 per cent. will be worth 421*l*. 4*s*. 8·731*d*.?

First, $£421\ 4s.\ 8 \cdot 731 = £421 \cdot 23637916 = p$. and $421 \cdot 23637916 \times \cdot 06 = 25 \cdot 27418275 = \overline{r - 1} \times p$, dividend.
$1 \cdot 06 \times 1 \cdot 06 \times 1 \cdot 06 \times 1 \cdot 06 \times 1 \cdot 06 = 1 \cdot 3382255776 = r^t$.
$1 - 1 \div 1\ 3382255776 = 1 \cdot - \cdot 7472581725 = \cdot 2527418275$, divisor.
Then $25 \cdot 27418275 \div \cdot 2527418275 = £100 = n$, the annuity required.

(8.) What annuity, to continue 9 years, will be worth 568*l*. 12*s*. 6·177*d*., allowing the purchaser 5 per cent. compound interest for ready money?

(9.) What annuity, to continue 7 years, will 2842*l*. 7*s*. 9·0183*d*. ready money, purchase, allowing 5 per cent. per annum compound interest?

The following Examples are performed by Logarithms.

(10.) What annuity, (payable half-yearly,) to continue 9 years, will 2448*l*. 2*s*. 6·072*d*. purchase, allowing compound interest at 6 per cent.?

Here $t=18$, $r=1·03$, and £2448 2 6·072 = 2448·1253 = p.

By example 4, page 254, $1-\frac{1}{r^t}=·4126053$

$$\begin{aligned} log.\ r-1 = log.\ ·03 &= -2·4771213 \\ log.\ p = log.\ 2448·1253 &= +3·3888336 \\ \hline &+1·8659549 \\ log.\ 1-\frac{1}{r^t} = log.\ ·4126053 &= -1·6155348 \\ \hline \end{aligned}$$

log. of $\frac{1}{2}$ the annuity $=+2·2504201$, the number answering to which is 178; hence the annuity is 356*l*.

(11.) What annuity, (payable half-yearly,) to continue 4 years, will 1268*l*. 5·08*s*. ready money, purchase, allowing compound interest at $4\frac{1}{2}$ per cent.?

(12.) What annuity, (payable quarterly,) will 307*l*. 19*s*. 9·6*d*. ready money, purchase, allowing the purchaser 5 per cent. per annum compound interest, for his money?

Examples to Prop. 3. *Theo.* III.

(13.) In what time will an annuity of 100*l*. be worth 421*l*. 4*s*. 8·731*d*. present money, if it continues in arrears; *or, which is the same thing, if it be received annually,* allowing 6 per cent.?

$$\begin{aligned} £421\ 4\ 8·731 = 421·23687916' &= p. \\ 100 &= n \\ 421·23687916' &= p \\ \hline 521·23687916' &= n+p \\ 421·23687916' \times 1·06 = 446·511091916' &= pr \\ \hline 74·72578725 &= n+p-pr. \end{aligned}$$

$100 \div 74·72578725 = 1·338226+ = \overline{1·06}|^t$; *whence the value of t may be found by repeated divisions,* or rather by logarithms.

(14.) In what time will an annuity of 80l. be worth [illegible]l. 12s. 9·177d., allowing the purchaser 5 per cent. per annum for present payment?

(15.) How long may a person enjoy an annuity of 560l. if he pays 2042l. 7s. 9·0138d. ready money, and is allowed 5 per cent. interest?

The following Examples are performed by Logarithms.

(16.) How long must an annuity of 356l., (payable half-yearly,) continue, at 6 per cent., to be worth 2448l. 2s. 6·072d. ready money?

First, £2448 2 6·072 = 2448·1253 = p.

$$
\begin{array}{rl}
178\cdot & = n \\
2448\cdot1253 & = p \\
\hline
2626\cdot1253 & = n+p. \\
2448\cdot1253 \times 1\cdot03 = 2521\cdot369059 & = pr \\
\hline
104\cdot55624 & = n+p-pr
\end{array}
$$

$log.\ n = log.\ 178 = 2\cdot2504200$

$log.\ \overline{n+p-pr} = log.\ 104\cdot55624 = 2\cdot0193508$

$log.\ r = log.\ 1\cdot03 = 0\cdot0128372)\ 0\cdot2310692\ ($18 payments: hence the term is 9 years.

(17.) How long must an annuity of 350l. (payable half-yearly) continue to be worth 1268l. 5·08s. ready money, allowing the purchaser $4\frac{1}{2}$ per cent. for his money?

(18.) How long must an annuity of 70l. (payable quarterly) continue to be worth 307l. 19s. 9·6d. ready money, allowing 5 per cent. per annum interest?

Examples to Prop. 4.

(19.) An annuity of 100l. to continue for 5 years, was purchased for 421l. 4s. 8·731d., what rate per cent. was the purchaser allowed for his ready money?

£421 4 8·731 = 421·23637416′ = p.

$$\text{Then, } \frac{100+421\cdot23637416'}{421\cdot23637416'} \times r^{5} - r^{5+1} = \frac{100}{421\cdot23637416'}.$$

Or, $521\cdot23637416' r^{5} - 421\cdot23637416' r^{6} = 100$.

By note 2, Double Position, I find $r = 1\cdot06$, very near.

(20.) An annuity of 80*l.* to continue 9 years, was sold for 568*l.* 12*s.* 6·177*d.* ready money; what rate per cent. was the purchaser allowed.

PRESENT WORTH *of Annuities in Reversion, at Compound Interest.*

Here t=the *reversion*, or the time the purchaser holds the annuity. T=the time which must elapse before he enters upon it; r, n, &c. as before.

Proposition 1. *To find the present worth of an annuity in reversion, at Compound Interest.*

Rule. Find the present worth of the annuity (for the time of its continuance) as though it were to be entered on immediately, (by Prop. 1, page 253,) then find what principal put to interest, at the same rate per cent. for the time between the purchase and commencement of the annuity, will amount to *that* present worth, (by Prop. 2, page 248,) and it will be the answer.

Or *Theo.* I. $\dfrac{r^t-1}{\overline{r-1}\times r^{T+t}}\times n=p$, the present worth, when T, t, r, and n, are given.

Logarithmically, *log.* $\overline{r^t-1}$ + *log.* n — *log.* $r\times\overline{T+t}$ + *log.* $\overline{r-1}$ = *log.* p.

Prop. 2. *Given T, t, r, and p, to find n.*

Theo. II. $\dfrac{\overline{r-1}\times r^{T+t}}{r^t-1}\times p=n$.

Logarithmically, *log.* $r\times\overline{T+t}$ + *log.* $\overline{r-1}$ + *log.* p — *log.* $\overline{r^t-1}$ = *log.* n.

Examples to Proposition 1.

(1.) The *reversion* of a lease of 175*l.* per annum, to continue 11 years, which commences 9 years hence, is to be sold; what is it worth, allowing the purchaser 6 per cent. per annum for his ready money?

$1 + 1\cdot06 + \overline{1\cdot06}|^2 + \overline{1\cdot06}|^3 + \overline{1\cdot06}|^4 + \overline{1\cdot06}|^5 + \overline{1\cdot06}|^6 + \overline{1\cdot06}|^7 + \overline{1\cdot06}|^8 + \overline{1\cdot06}|^9 + \overline{1\cdot06}|^{10} = \dfrac{\overline{1\cdot06}|^{10} - 1}{1\cdot06 - 1} + \overline{1\cdot06}|^{10} = 14\cdot97164264,$

this, multiplied by 175, gives £2620·037462, amount of the annuity.

$\overline{1\cdot06}|^{11} = 1\cdot898298558$: 1l. :: 2620·037462 : £1380·203052, present worth of the annuity, supposing it were to be entered upon immediately. Again,

$\overline{1\cdot06}|^{9} = 1\cdot689478959$: 1l. :: 1380·203052 : £816·240065= £816 18 $2\frac{1}{2}$·4624, the present worth of the *reversion*.

Or thus by the Theorem.

Here $t=11$, $r=1\cdot06$, $n=175$, and $T=9$.

$\overline{1\cdot06}|^{11} = 1\cdot898298558 = r^t$.

$1\cdot898298558 - 1 = \cdot898298558 = r - 1$, and $\cdot898298558 \times 175 = 157\cdot20224765 = \overline{r^t - 1} \times n$, dividend.

$r^{T+t} = \overline{1\cdot06}|^{9+11} = \overline{1\cdot06}|^{20} = 3\cdot207135472$; this multiplied by ·06, gives $\cdot1924281283 = \overline{r - 1} \times r^{T+t}$ divisor.

Hence 157·20224765 ÷ ·1924281283 = 816·940065 = £816 18s. $9\frac{1}{2}$·4624d. as before.

(2.) What is the present worth of the *reversion* of a lease of 50*l.* per annum, to continue for 5 years, but not to commence till the end of 3 years, allowing 5 per cent. for present payment?

(3.) What ought a person to pay in ready money for the reversion of 1000*l.* a year, to continue 20 years, on a lease which cannot commence till the expiration of 5 years, allowing the purchaser compound interest at 5 per cent.?

Examples to Prop. 2. *Theo.* II.

(4.) What annuity, or yearly rent, to be entered upon 9 years hence, and thence to continue 11 years, may be bought for 816*l.* 18*s.* $9\frac{1}{2}$·4624*d.* ready money, allowing the purchaser 6 per cent.?

£816 18 $9\frac{1}{2}$·4624=816·94005=p, $1\cdot06 - 1 = \cdot06 = r - 1$.

$r^{T+t} = \overline{1\cdot06}|^{9+11} = \overline{1\cdot06}|^{20} = 3\cdot207135472.$

$\cdot06 \times 3\cdot207135472 \times 816\cdot940065 = 157\cdot2022476575 = \overline{r-1} \times r^{T+t} \times p$, dividend. $\overline{1\cdot06}|^{11} - 1 = \cdot8982985583 = r^t - 1$, divisor, Hence $157\cdot2022476575 \div \cdot8982985583 = $ £175, the annuity required.

(5.) What annuity, or yearly rent, to be entered upon 3 years hence, and then to continue 5 years, may be bought for 187*l*. 0*s*. 0¾·4176*d*. ready money, at 5 per cent.?

(6.) The reversion of a lease, to be entered on 5 years hence, and thence to continue 20 years, was sold for 9764*l*. 9*s*. 4¼·088*d*., allowing the purchaser 5 per cent., what ought the yearly rent to be?

PURCHASING FREEHOLD ESTATES, OR PERPETUAL ANNUITIES TO BE ENTERED ON IMMEDIATELY.

Proposition 1. *Given the annual rent of any perpetual annuity, or freehold estate, to find the value thereof, allowing the purchaser any assigned rate per cent. for his money.*

Rule. Divide the rent by the ratio less 1, and the quotient will be the present worth of the estate.

Or, *Theo.* I. $\frac{n}{r-1}=p$, when n and r are given. If the rents are to be paid either ½ yearly or quarterly, as is generally the case, then the ratio, or r, must represent the amount of £1 for that time, and the annuity, or n, must be divided by 2, 4, &c. to represent the ½, ¼, &c. rent—Here we may observe, that though there be no such thing as a *limited time* considered in the purchase of perpetual annuities, yet a due regard ought to be had to the *times* the annuities or rents are *paid*; for, it is evident the less the intervals between the payments of the rents are, the purchase is more valuable, and *vice versa*.

Prop. 2. *When any sum of money is proposed to be laid out in a perpetual annuity, or freehold estate, to find what annual rent that sum will purchase at any given rate per cent.*

Rule. Multiply the proposed sum to be laid out by the ratio less 1, and the product will be the yearly rent.

Theo. II. $p\times\overline{r-1}=n$, when p and r are given.

Prop. 3. *The annual rent of any perpetual annuity, or freehold estate, and the sum paid down for it, being given, to find what rate of interest per cent. is paid to the purchaser.*

Rule. Divide the annual rent by the sum that is paid for the purchase, the quotient, increased by an unit, will be the ratio, whence the rate per cent. may be found.

Theo. III. $\frac{n}{p}+1=r$, when p and n are given.

Examples to Prop. 1.

(1.) An estate brings in 25*l.* yearly rent; required the present worth thereof, allowing the purchaser 4 per cent. compound interest for his money.

First, $1{\cdot}04-1={\cdot}04$, the ratio less 1.
Then $25 \div {\cdot}04=$ £625, the present worth required.

(2.) Suppose a freehold estate of 250*l.* yearly rent is to be sold; what is it worth, allowing the buyer 6 per cent. compound interest for his money?

(3.) What is the present worth of a freehold estate of 250*l.* per annum, the rent payable half yearly *, allowing the purchaser 4 per cent. for his money?

(4.) What is the present worth of a perpetual annuity of 2000*l.* payable quarterly, (viz. 500*l.* per quarter), allowing the buyer 4½ per cent. compound interest for his money?

Examples to Prop. 2.

(5.) I propose to lay out 625*l.* in the purchase of a perpetual annuity, and to make 4 per cent. compound interest for my money; what ought the annuity to be?

$1{\cdot}04-1={\cdot}04$, the ratio less 1.
Then, ${\cdot}04\times625=$ £25, the annuity or annual rent required.

(6.) A freehold estate was bought for 4166*l.* 13*s.* 4*d.*; what ought the yearly rent to be, allowing the buyer 6 per cent. compound interest for ready money?

(7.) A person is desirous of laying out 1760*l.* in the purchase of a freehold estate, so as to get 4½ per cent. compound interest for his money; what must be the annual income of such an estate?

* It may not be improper to observe in this place, that, if the ratio be taken according to Table I. p. 242, it will make no difference whether the rents are payable yearly, half-yearly, or quarterly, but, if it be taken according to Table II. page 242, the difference, *in this example*, will be 61*l.* 17*s.* 1¼*d.*: this shews, that the second method or table is more accurate than the first; for it is certainly more advantageous to receive the rents half-yearly than yearly.

Examples to Prop. 3.

(8.) Suppose 625*l.* to be paid for a freehold estate which yields 25*l.* per annum, what rate of interest has the purchaser for his money?

625)25.00(·04
1·

1·04 the ratio; hence the rate per cent. is 4*l.*

(9.) Suppose a freehold estate of 250*l.* per annum, costs 4166*l.* 13*s.* 4*d.*, what rate of interest per cent. is allowed to the purchaser?

(10.) A freehold estate of 80*l.* a year rent was sold for 1200*l.*, what was the rate per cent. (compound interest) allowed the purchaser for the ready money which he paid for the estate?

THE BUYING AND SELLING FREEHOLD ESTATES TO BE ENTERED ON IMMEDIATELY, ACCORDING TO A NUMBER OF YEARS' RENT, OR INCOME, FOR THE PURCHASE-MONEY.

Proposition 1. *The purchase-money, or present worth, of a freehold estate being given, to find at what rent it must be let to clear itself in a given time.*

Rule. Divide the present worth by the proposed time, and the quotient will be the annual rent.

Prop. 2. *Given the purchase, or present worth, of a freehold estate, and the annual rent it lets for, to find in what time it will clear itself, or bring in the purchase-money.*

Rule. Divide the present worth by the annual rent, and the quotient will be the time required.

Prop. 3. *Given the annual rent of a freehold estate, and the time in which it will clear itself, to find the purchase, or present worth, of such an estate.*

Rule. Multiply the rent by the time.

Prop. 4. *Given the time in which a freehold estate brings in the purchase-money, or clears itself, to find what rate per cent. the purchaser has for his money.*

Rule. Divide the time more 1 by the time, and the quotient will be the ratio, whence the rate may be found.

Prop. 5. *When a person proposes to lay out any sum of money in a freehold estate, so that he may make a certain rate per cent. of the money laid out, to find in what time the estate will clear itself.*

Rule. Divide an unit by the ratio less 1, and the quotient will be the time.

Examples to Proposition 1.

(1.) A freehold estate was purchased for 625*l.* At what rent must it be let that it may bring in the purchase-money in 15 years?

$625 \div 15 = 41\frac{2}{3} =$ 41*l.* 13*s.* 4*d.*

(2.) A freehold estate was purchased for 1200*l.* At what rent must it be let to clear itself in 20 years?

Examples to Prop. 2.

(3.) I purchased a freehold estate for 4500*l.* which I let at 250*l.* per annum; in what time will it clear itself?

$4500 \div 250 = 18$ years, answer.

(4.) I purchased a freehold estate for 625*l.*, which brings me in 25*l.* per annum; in what time will it clear itself?

Examples to Prop. 3.

(5.) If a freehold estate, which lets for 40*l.* per ann. will clear itself in 20 years, what is its present worth?

$40 \times 20 =$ £800, answer.

(6.) If a freehold estate, which lets for 25*l.* per ann. will clear itself in 25 years, what was it bought for?

Examples to Prop. 4.

(7.) If a freehold estate be sold for 20 years purchase, what rate per cent. compound interest is the purchaser allowed for his money?

$\frac{20+1}{20} = \frac{21}{20} = 1{\cdot}05$, the ratio. Hence the rate is 5 per cent.

(8.) If a freehold estate be sold for $22\frac{2}{3}$ years purchase, what rate per cent. is the buyer allowed for his money?

Examples to Prop. 5.

(9.) Suppose I want to lay out 1200*l.* in a freehold estate, and to have 5 per cent. allowed me for my money; in what time will the estate bring in the purchase-money, or clear itself?

$1 \div \cdot 05 = 20$ years, answer.

(10.) I wish to lay out 625*l.* in a freehold estate, and to have 4 per cent. allowed me for my money; in what time will the estate clear itself?

PURCHASING FREEHOLD ESTATES OR PERPETUAL ANNUITIES IN REVERSION.

Here n = the annuity or yearly rent; T = the time before the annuity commences; p = the present worth; r = the ratio, &c.

Proposition 1. *The yearly rent of a freehold estate, and the rate per cent. being known, to find the present worth of the reversion of such an estate.*

Rule. Find the present worth for the *reversion* (by Prop. 1, page 259.)

Then, by (Prop. 2, page 243) find what principal will amount to the full value of the estate for the time before it commences, and it will be the present worth required.

Theo. I. $\dfrac{n}{\overline{r-1} \times r^{T}} = p$, when n, r, and T, are given.

Logarithmically. $Log.\ n - log.\ r \times T + log.\ \overline{r-1} = log.\ p.$

Prop. 2. *The sum given for the reversion of a freehold estate being known, to find the yearly income, allowing the purchaser so much per cent. for his money.*

Rule. Find the amount of the purchase-money, to the time when the *reversion* begins, (by Prop. 1, page 243.) Then find the yearly income which that amount will purchase, (by Prop. 2, page 259,) and it will be the answer.

Theo. II. $\overline{r-1} \times r^{T} \times p = n.$

Logarithmically. $Log.\ \overline{r-1} + log.\ r \times T + log.\ p = log.\ n.$

Examples to Proposition 1.

(1.) The reversion of a freehold estate of 500*l*. per annum, to commence 5 years hence, is to be sold; what is it worth in ready money, allowing the purchaser 4 per cent. for his money?

500 ÷ ·04 = 12500*l*. value of the estate, if entered on immediately.

1·04 × 1·04 × 1·04 × 1·04 × 1·04 = 1·2166529024, amount of 1*l*. for 5 years.

1·2166529024 : 1*l*. :: 12500 : 10274·088834 = £10274 1 9¼·281, present worth of the reversion.

Or thus by Theorem I.

Here $n = 500$, $r = 1·04$, and $T = 5$.

1·04 × 1·04 × 1·04 × 1·04 × 1·04 = 1·2166529024 = r^T, and

1·2166529024 × ·04 = ·048666116096 = $\overline{r-1} \times r^T$.

Hence 500 ÷ ·048666116096 = 10274·088834 = £10274 1 9¼·281, as before.

(2.) If a freehold estate of 60*l*. 10*s*. per annum, to commence 10 years hence, is to be sold; what is it worth, allowing the purchaser 5 per cent. for present payment?

(3.) A freehold estate of 290*l*. per annum, to commence 4 years hence, is to be sold; what is it worth, allowing the purchaser 4 per cent.?

Examples to Prop. 2.

(4.) A freehold estate, to commence 5 years hence, is sold for 10274*l*. 1*s*. 9¼*d*·281, allowing the purchaser 4 per cent. for his money; what is the yearly rent?

First, £10274 1 9¼·281 = 10274·088834.

1·04 × 1·04 × 1·04 × 1·04 × 1·04 = 1·2166529024.

1·2166529024 × 10274·088834 = 12500 (nearly) the amount of the purchase-money to the time the reversion begins.

Then, 12500 × ·04 = £500, the yearly rent.

By Theo. II.

·04 × $\overline{1·04}|^5$ × 10274·088834 = $\overline{r-1} \times r^T \times p$ = ·04 × 1·216652904 × 10274·088834 = £500, (nearly) the annuity required.

(5.) If a freehold estate, to commence 10 years hence, is sold for 742*l*. 16*s*. 8¼*d*·8, allowing the purchaser 5 per cent.; what is the yearly rent?

(6.) If a freehold estate which commences 4 years hence, be sold for 6197*l.* 6*s.* 5½*d.*, allowing the purchaser 4 per cent, for his money, what ought the yearly rent to be?

Note. The preceding rules and examples include all kinds of annuities which do not depend upon *chance.*

SIMPLE INTEREST BY DECIMALS.

Put $p=$ the principal, or sum put to interest.
$r=$ the ratio, being the rate per cent. divided by 100.
$t=$ the time, or years, the money is at interest.
$i=$ the interest for the time t.
$a=$ the amount.

Then, at 2½ per cent.	$r=\cdot025$	At 4 per cent.	$r=\cdot04$
— 3 —	$r=\cdot03$	— 4½ —	$r=\cdot045$
— 3½ —	$r=\cdot035$	— 5 —	$r=\cdot05$, &c.

And		Decimals.
1 day	$=\frac{1}{365}$ of a year	$=\cdot002739726$, &c.
1 week	$=\frac{7}{365}$ of a year	$=\cdot019178082$, &c.
1 month	$=\frac{1}{12}$ of a year	$=\cdot08\dot{3}$
1 quarter	$=\frac{1}{4}$ of a year	$=\cdot25$
1 half	$=\frac{1}{2}$ of a year	$=\cdot5$

Hence the decimal parts of a year, for any number of days, weeks, months, &c. may be readily found.

Proposition* 1. *Given the principal, time, and rate per cent., to find the interest or the amount.

Rule. Multiply the principal, time, and ratio, together, the last product will be the *interest;* to which add the principal to find the amount.

Theorem, $ptr=i$, and $ptr+p=a$. When p, t, and r, are given.

Prop.* 2. *Given the amount, (or the interest,) time, and rate, to find the principal.

Rule. Multiply the time by the ratio, and add an unit to the product; by this sum divide the *amount,* and the quotient will be the principal.—*Or,* divide the *interest* by the product of the time and ratio, and the quotient will be the principal.

Theo. $\frac{a}{tr+1}=\frac{i}{tr}=p$. When a, (or i,) t, and r, are given.

Prop. 3. *Given the principal, time, and amount, (or the interest,) to find the rate per cent.*

Rule. Divide the difference between the amount and the principal (*viz.* the interest) by the product of the principal and time, and the quotient will be the ratio, which multiply by 100 to obtain the rate per cent.

Theo. $\frac{a-p}{pt}=\frac{i}{pt}=r$, when p, t, and a, (or i,) are given.

Prop. 4. *Given the principal, rate, and amount, (or interest,) to find the time.*

Rule. Divide the difference between the amount and the principal (*viz.* the interest) by the product of the principal and ratio, and the quotient will be the time.

Theo. $\frac{a-p}{pr}=\frac{i}{pr}=t$. When p, r, and a, (or i,) are given.

Examples to Proposition 1.

(1.) What will 567*l.* 10*s.* amount to in 9 years, at 4 per cent. per annum?

$$\begin{array}{rl}
£567{\cdot}5 & = p \\
9 & = t \\
\hline
5107{\cdot}5 & = pt \\
{\cdot}04 & = r \\
\hline
204{\cdot}300 & = ptr = i \\
567{\cdot}5 & = p \\
\hline
£771.800 & = ptr+p = a. \\
20 & \\
\hline
s.\ 16\ 000 & \\
\hline
\end{array}$$

Answer 771l. 16s.

(2.) What is the amount of 235*l.* at simple interest, for $3\frac{3}{4}$ years, at 5 per cent. per annum?

(3.) What is the interest of 550*l.* for 5 years at $3\frac{1}{2}$ per cent. per annum?

(4.) What is the interest of 715*l.* 15*s.* for 240 days, at 5 per cent. per annum?

(5.) What will 510*l.* amount to in 5 years 120 days, at 5 per cent. per annum?

Examples to Prop. 2

(6.) What principal, in 9 years, will amount to 771*l.* 16*s.* at 4 per cent. per annum?

$9 = t$
$\cdot 04 = r$ Now a=771l. 16s.=£771·8, dividend.
Hence 771·8÷1·36=£567·5=567l. 10s. answer.
$\cdot 36 = tr$
$1\cdot$
$1\cdot 36 = tr + 1$ divisor.

(7.) What principal, put to interest for 9 years, will gain 204*l.* 6*s.* interest, at 4 per cent. per annum?

$9 = t$ Now i=£204 6s =£204·3, dividend.
$\cdot 04 = r$ Hence 204·3÷·36=£567·5=567l. 10s. answer.
$\cdot 36 = tr$

(8.) What principal in $3\frac{1}{4}$ years, will amount to 279*l.* 1*s.* 3*d.*, at 5 per cent. per annum?

(9.) What principal put to interest for $5\frac{1}{2}$ years, will amount to 810*l.* 16*s.* $6\frac{3}{4}d.\frac{1}{2}$, at 3 per cent. per annum?

(10.) What principal, put to interest for 240 days, at 5 per cent. per annum, will gain 23*l.* 10*s.* $7\frac{1}{2}d.\frac{8}{73}$?

(11.) What principal put to interest for 65 days, at 5 per cent. per annum, will gain 8*l.* 9*s.* $7\frac{3}{4}d.\frac{6}{7}\frac{5}{3}$ interest?

Examples to Prop. 3.

(12.) At what rate per cent. will 567*l.* 10*s.* amount to 771*l.* 16*s.* in 9 years time?

£771 16 = £771·8 = a
567 10 = 567·5 = p

204·3 = $a-p=i$, dividend.
567·5×9 = 5107·5 = pt, divisor.
Then 204·3÷5107·5 = ·04. Hence the rate is 4 per cent.

(13.) At what rate per cent. will 235*l.* amount to 279*l.* 1*s.* 3*d.* in $3\frac{1}{4}$ years?

(14.) At what rate per cent. will 715*l.* 15*s.* amount to 948*l.* 17*s.* $10\frac{3}{4}d.\frac{1}{2}$ in $7\frac{1}{4}$ years?

(15.) At what rate per cent. will 357*l.* 10*s.* gain 3*l.* 3*s.* $7\frac{1}{4}d.\frac{6}{7}\frac{5}{3}$ in 65 days?

(16.) At what rate per cent. per annum will 510*l.* amount to 679*l.* 8*s.* 4¼*d.*$\frac{7}{71}$ in 5 years and 120 days?

Examples to Prop. 4.

(17.) In what time will 567*l.* 10*s.* amount to 771*l.* 16*s.* at 4 per cent. per annum?

$$\begin{aligned}\pounds 771\ \ 16 &= \pounds 771{\cdot}8 = a\\ 567\ \ 10 &= \ \ 567{\cdot}5 = p\end{aligned}$$

$$204{\cdot}3 = a - p = i, \text{ dividend.}$$

$$567{\cdot}5 \times {\cdot}04 = 22{\cdot}700 = pr, \text{ divisor.}$$

Then $204{\cdot}3 \div 22{\cdot}7 = 9$ years, the time required.

(18.) In what time will 700*l.* 10*s.* amount to 810*l.* 16*s.* 6¼*d.*$\frac{3}{5}$, at 3 per cent. per annum?

(19.) In what time will 715*l.* 15*s.* amount to 943*l.* 17*s.* 10¾*d.*½, at 4¼ per cent. per annum?

(20.) In what time will 715*l.* 15*s.* gain 23*l.* 10*s.* 7½*d.*$\frac{15}{71}$ at 5 per cent. per annum?

(21.) In what time will 510*l.* amount to 679*l.* 8*s.* 4¼*d.*$\frac{7}{71}$ at 5 per cent. per annum?

EQUATION OF PAYMENTS AT SIMPLE INTEREST,

BY DECIMALS,

ON MALCOLM'S PRINCIPLES.

Proposition. Having two *debts, due at different times, to find the equated time for paying the whole at once, without loss either to the debtor or creditor.*

Rule 1. Divide the sum of the debts by twice the first payment, multiplied by the ratio; to the quotient add half the time between the two payments, and call the sum the first number found.

2. Multiply the second payment by the time between the two payments, and divide the product by the first payment multiplied by the ratio; call the quotient the second number found.

3. From the *square* of the first-found number subtract the second, and extract the square-root of the difference. —The first-found number, diminished by this root, will give the equated time, reckoning from the time the first payment is due.

Note. The preceding rule is the same as Mr. *Malcolm's*, though expressed in a different, and it is apprehended, more intelligible, manner. —This rule is built upon a supposition, 'That the sum of the *interests* of the debts, due *before* the equated time, from the time they become due to *that* time, ought to be equal to the sum of the *discounts* of the debts due *after* the equated time from *that* time to the time they become due.' According to this supposition, the rule given above is universally true for two payments. But, when three or more payments are to be equated for, Mr. *Malcolm's* directions for finding an equated time for the two that are first payable, then their sum and a third, &c. is not strictly true, according to the supposition on which his rule is founded; nor would it be an easy matter to give general rules or theorems for all the possible cases, on account of the variation of the debts, and the difficulty of finding between which of the payments the equated time would fall. Besides, in long and tedious operations, mistakes are frequently made; and the answer, when obtained, *admitting it to be true*, differs a mere trifle from the answer found by the old rule: hence, a rule, founded upon *simple interest* and Mr. *Malcolm's* principles, may, I think, with propriety, be considered as an useless curiosity.

Examples.

(1.) A person has now due to him 320*l.* and at the end of 5 years 96*l.* more will be due from the same debtor. Now both parties have agreed that the whole shall be paid at once, viz. at that time when the interest of the 320*l.* shall be equal to the discount of the 96*l.* both being calculated at 5 per cent. per annum. The time of payment is required.

1st. $320+96=416$l. sum of the debts.

$320\times2\times\cdot05=32$, the product of twice the first payment by the ratio.

$416\div32=13$, quotient. Then $13+\frac{5}{2}=15\cdot5$, *the first number found.*

2dly. $\overline{96\times5}\div\overline{320\times\cdot05}=30$, *the second number found.*

3dly. $\sqrt{\overline{15\cdot5}|^2-30}=\sqrt{210\cdot25}=14\cdot5$, and $15\cdot5-14\cdot5=1$ year, the time which must elapse (after the first payment is due) before the whole ought to be paid together according to the conditions of the question.

(2.) There is 100*l.* payable 1 year hence, and 105*l* payable 3 years hence; what is the equated time, allowing simple interest at 5 per cent. per annum?

(3.) At Michaelmas 1815, I lent 320*l.*, and at Michaelmas 1820, 202*l.* will be due to me from the same person. Now, on what day, and in what year, may I receive both the debts together, viz. 522*l.*, reckoning interest at 5 per cent. per annum?

ON LIFE ANNUITIES.

The value of an annuity for life depends not only on the interest of money, but also on the probability of the continuance of life, it may therefore be considered such a sum as will be sufficient to enable the person who grants the annuity to pay it without loss, allowing for the chances of mortality.

If money is supposed to bear no interest, the value of an annuity is always equal to the *expectation* of life; but as money can be improved by putting it out to interest, the value of an annuity will not be worth so many years purchase as are equal to the expectation of life, and the higher the rate of interest is, the fewer years purchase the annuity will be worth.

PROBLEM I.

To find the expectation of any single life.

RULE.

Make the number of persons living opposite the given age (in Table I.) a divisor, and the sum of the number of persons living from that age to 96 inclusive, a dividend; the quotient diminished by ·5 will be the answer.

Or, the expectation of life may be found in Table II. opposite to the given age.

Examples.

(1.) How many years may a person of 60 expect to live? Against 60 is 2038, the sum of this number, and those above it to 1 inclusive is 27953: then (27953÷2038)—·5=13·21 years. *Answer,* see Table II.

(2.) Required the expectation of a life of 45, of 75, and of 80.

PROBLEM II.

To find the probability that a person of a given age shall live a certain number of years.

RULE.

Make the number opposite to the given age (in Table I.)

the denominator of a fraction, and the number opposite to the proposed age the numerator.

In the case of joint lives, the product of the fractions found as above will shew the probabilities.

Examples.

(1.) What is the probability that a person of the age of 60 shall live 10 years?

Against 60 in Table I. stands 2038, and against 70 you will find 1232; so the probability is $\frac{1232}{2038}$. The probability of a person aged 60 being dead in 10 years, is $1-\frac{1232}{2038}=\frac{806}{2038}$.

(2.) What is the probability that each of three persons, separately, whose ages are 20, 30, and 40, shall live 15 years; and what is the probability that they shall *all* live 15 years?

PROBLEM III.

To find the probability that either the one or the other of two persons of different ages shall live a certain number of years.

RULE.

Find the probability that each of the persons shall live the proposed number of years, and the probability that they shall jointly live the said number of years: the latter result subtracted from the sum of the former will give the answer.

Examples.

(1.) Suppose there are two persons, the one aged 20, and the other 40 years, what is the probability that one of them will be alive after 30 years have elapsed?

The probability that a man of 20 shall attain the age of 50, is $\frac{2837}{5132}$, that a person aged 40 shall live to 70, is $\frac{1232}{3635}$, and that they shall jointly live 30 years, is $\frac{2837}{5132}\times\frac{1232}{3635}=\frac{3495184}{18654820}$; Hence

$\left(\frac{2837}{5132}+\frac{1232}{3635}\right)-\frac{3495184}{18654820}=\frac{13139935}{18654820}$ Answer.

(2.) What is the probability that of two persons, the one aged 50, the other 65, one of them shall be living at the expiration of 12 years?

PROBLEM IV.

To find the value of an annuity on any single life.

RULE.

Multiply the number in Table III. against the given age, by the proposed annuity, and the product will be the answer.

Examples.

(1.) What must be given for an annuity of £60 during the life of a person aged 46, reckoning interest at 4 per cent.?

Against 46, and under 4 per cent. you will find 12·089, that is, the annuity is worth 12 years purchase, hence $12·089 \times 60 = £725·34$.

(2.) What is the value of an annuity of £200 payable during the life of a person aged 25 years, reckoning interest at 5 per cent.?

(3.) What is the value of the life interest of a person aged 56 in £3000 stock in the 3 per cent. consolidated annuities. Interest at 5 per cent.?

(4.) What is the difference in value between an annuity of £80 during the life of a person aged 36, and an annuity of the same amount, certain for 20 years. Interest at 5 per cent.?

PROBLEM V.

To find what annuity any given sum will purchase during the life of a person of a given age.

RULE.

Divide the given sum by the number opposite to the given age, and under the given rate per cent. in Table III. and the quotient will shew the annuity.

Examples.

(1.) A person of 50 years of age wishes to lay out £1500 in an annuity for his life. Interest at 5 per cent. What annuity will it purchase?

Against 50 years, and under 5 per cent. you will find 10·269 hence 1500÷10·269=£146·07, the annuity required.

(2.) When the 3 per cent. consols sell for 77¾ per cent. what annuity for life should be granted to a person aged 58 for £6000 stock?

(3.) A gentleman aged 60, who receives an annuity of £200 for life, wishes to exchange it for an annuity of the same sum to continue during the life of his wife, whose age is 34, what sum ought he to give for the exchange, calculating at 4 per cent.?

(4.) A person has an annuity of £150 during the life of a gentleman aged 30, but being advanced in age, and wanting money, he is willing to exchange it for an equivalent annuity to continue during the life of a person aged 50; what annuity should be granted him? Interest at 5 per cent.

(5.) A person aged 30 is possessed of £80 a year in the government long annuities, which will terminate in January 1860; this he is willing to relinquish for an annuity during his life, to commence in January 1820; what annuity ought he to receive, reckoning interest at 5 per cent.?

PROBLEM VI.

To find the present value of a given sum to be received at the death of a person of any age: or to find what sum must be paid annually by a person of any age, that his heirs muy receive a given sum of money at his death.

RULE.

Multiply the number in Table III. against the given age, by the interest of £1 for a year, and subtract the product from an unit; divide the remainder by the amount of £1 for 1 year, the quotient multiplied by the given sum will give the value required.

To find the value in *annual* payments to the number in Table III. opposite to the given age add an unit, and divide the value found above by this result, the quotient will be the answer. *See* TABLE IV.

Examples.

(1.) What ought a person, aged 45, to pay down, that his children may receive £1000 at his death, or what sum ought he to pay annually for the same advantage, reckoning interest at 4 per cent.?

In Table III. against 45, and under 4 per cent. stands 12·283; then $1 - \frac{(12\cdot283 \times \cdot04}{1\cdot04} \times 1000 = £489\cdot115 = £489\ 2\ 3\frac{1}{2}$ the value in a *single* payment; and $\frac{489\cdot115}{1+12\cdot283} = £36\cdot82263 = £36\ 16\ 5\frac{1}{4}$ the *annual* payment; the first being paid immediately, and the remaining ones at the *beginning* of every subsequent year.

(2.) What is the present value of £1000 to be received on the death of a person aged 60, interest being reckoned at 3 per cent.*; and what ought to be paid annually to insure the same sum.

(3.) What sum must be paid annually that the heirs of a person aged 30, may receive £1000 at his decease, reckoning interest at 5 per cent.?

PROBLEM VII.

To find what sum a person ought to receive, who has insured his life to a given amount, in order that he may relinquish his claim.

RULE.

Multiply the annual payment which has been made since the insurance commenced by the value of an annuity on the life at its present age from Table III.; subtract the product from the value of the insurance of the given sum on the life at its present age, (Prob. VI.) the remainder will be the answer.

Examples.

(1.) A person whose present age is 50 has been pay-

* The rates of insurances for lives, at all the different offices established in London, are calculated from the *Northampton* Tables, at 3 per cent. interest; viz. at the *lowest* rate of interest, and the *lowest* probabilities of living. See Table IV.

ing £21·793, or £21 15s. 10½d. annually for the insurance of £1000 at his death, wishes to discontinue the payment, and relinquish the advantage which his heirs expect: what ought the office to give as a compensation for so doing, reckoning interest at 3 per cent.?

The value of an annuity on a life of 50 at 3 per cent. is 12·436, which multiplied by 21·793 produces 271·017748.

The value of £100 on a life of 50, by Table IV.* is 60·866; hence the value of £1000 is £608·66; consequently 608·66—271·017748=£337·642252=£337 12s. 10d. Answer. This solution is on a supposition that the policy is cancelled immediately *after* the annual payment becomes due, if it be cancelled immediately *before*, then 21·793 must be multiplied by 12·436+1=13·436, and the Answer will be £315·85=£315 17s.

(2.) A person aged 60 has been paying £43·588 or £43 11*s.* 9*d.* annually for the insurance of £2000, as a portion for his daughter to be received at his death; but she, unexpectedly, has died before him, and in consequence he wishes to have the policy of insurance cancelled, what ought the office to pay him, reckoning interest at 3 per cent.?

(3.) A person aged 45 insured his life for £1000 at 4 per cent., consequently he has been paying annually £36·82263, or £36 16*s.* 5¼*d.* (Prob. VI.) he is now 70 years of age, reduced in his circumstances, and has no heirs, what ought he to receive from the office for cancelling his policy?

PROBLEM VIII.

To find the value of an annuity on the longest of two lives.

RULE.

From the sum of the values of an annuity on each of the single lives, (Table III.) subtract the value of an annuity on the two joint lives; (Table V.) the remainder will be the value required?

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* If the rate be any other than 3 per cent., this value must be calculated by Prob. VI.

Examples.

(1.) What is the value of an annuity on the longest of two lives aged 20 and 40, interest at 5 per cent.?

Table III.	The value of an annuity on a life of 20	14·007
	The value of an annuity on a life of 40	11·837
	Sum	25 844
Table V.	The value of an annuity on the two joint lives..................................	9 937
	Diff.	15·907

Hence the value of an annuity on the longest of two lives, the one 20, and the other 40, would be worth nearly 16 years' purchase. That is, an annuity of £100 would be worth £1600.

(2.) What is the value of an annuity, on the longest of two lives, the one 10 and the other 15, interest at 5 per cent.?

(3.) What is the value of an annuity on the longest of two lives, the one 50 and the other 70, interest at 5 per cent.?

(4.) What is the value of an annuity on the longest of two lives, each 20, interest 5 per cent.?

PROBLEM IX.

To find the value of an annuity on three joint lives.

RULE.

Find the *value* of an annuity on the joint lives of the two elder (Table V.), and the *age* of a single life of the same value (Table III.): lastly, find the value of an annuity on the joint lives of the youngest, and that of the age just found (Table V.) the result will be the answer.

Examples.

(1.) What is the value of an annuity on three joint lives, aged 20, 30, and 40? Interest at 5 per cent.

The value on the joint lives of 30 and 40 (Table V.) is 9·576, this is nearly the value of a single age of 54 (Table III.)

The value on the joint lives of 20 and 54 (Table V.) is 8·216 nearly. Hence the value of an annuity of £100 on three joint lives of 20, 30, and 40, would be about £821 12s.

(2.) What is the value of an annuity on three joint lives, aged 10, 20, and 30?

(3.) Required the value of an annuity on three joint lives, aged 10, 30, and 60?

PROBLEM X.

To find the value of an annuity on the longest of three lives.

RULE.

From the sum of the values of an annuity on all the single lives, (Table III.) subtract the sum of the values of an annuity on each *pair of* joint lives, (Table V.) and to the difference add the value of an annuity on the three joint lives (Prob. IX.): the last sum will be the value required.

Examples.

(1.) What is the value of an annuity on the longest of three lives, aged 20, 30, and 40? Interest at 5 per cent.

Table III.	Value of a life of 20	14·007
	Value of a life of 30	13·072
	Value of a life of 40	11·837
	Sum of the values....	38·916
Table V.	Value of two lives of 20 and 30.......	10·707
	Value of two lives of 20 and 40......	9·937
	Value of two lives of 30 and 40.......	9·576
	Sum of the values.....	30·220
	This sum subtracted from the preceding sum leaves	8·696
	The value of an annuity on three joint lives, by Example 1, Problem IX. is	8·216
	The value of the longest of the three lives............	16·912

Hence the value of the longest of the three lives is about 17 years purchase.

(2.) What is the value of an annuity on the longest of three lives, each aged 20? Interest at 5 per cent.

(3.) What is the value of an annuity on the longest of three lives, aged 10, 20, and 30? Interest at 5 per cent.

PROBLEM XI.

To find the value of an anuuity granted upon THREE *lives, but to cease as soon as any* TWO *of the lives fail.*

RULE.

From the sum of the values of an annuity on each pair of joint lives (Table V.) subtract twice the value of an annuity on the *three* joint lives (Problem IX.) the remainder will be the value required.

Examples.

(1.) An annuity is purchased upon three lives aged 20, 30, and 40, on this condition, that as soon as any two of the lives fail, the annuity shall cease, required its value, interest at 5 per cent.

Table V.	Value of two lives of 20 and 30	10·707
	Value of two lives of 20 and 40	9·937
	Value of two lives of 30 and 40	9·576
	Sum of the values....	30·220

The value of an annuity on three joint lives (Prob. IX.) is 8·216. Hence 30·220—(8·216×2)=13 788, the value or number of years purchase required.

(2.) What is the value of an annuity upon three lives, each 20, to cease as soon as any two of the lives fail? Interest at 5 per cent.

(3.) What is the value of an annuity upon three lives, aged 10, 20, and 30, interest at 5 per cent.? The annuity to cease on the death of any two.

PROBLEM XII.

A person enjoys an annuity for his life, and has the right to nominate a successor at his decease, to find the value of the annuity on the succeeding life.

RULE.

Multiply the value of an annuity on the life in posses-

sion by the rate of interest divided by 100, and subtract the product from an unit; multiply the remainder by the value of an annuity on the succeeding life: the product will be the present value required *.

Examples.

(1.) A person aged 65 is in possession of an annuity, and has the power of nominating a successor; if he nominates his grandson, aged 10 years, what is the value of the annuity to the child? Interest at 5 per cent.

The value of an annuity on a life of 65 (Table III.) is 7·276, and $1-(7 \cdot 276 \times \cdot 05) = \cdot 6362$. The value of an annuity on a life of 10 (Table III.) is 15·139; hence $15 \cdot 139 \times \cdot 6362 = 9 \cdot 6314$, the number of years purchase.

(2) An annuity is held on two joint lives aged 50 and 60; on the extinction of either of them, two other joint lives, each 10 years old, are nominated as successors: the value of the annuity on the succeeding lives is required, interest at 5 per cent.

(3.) An annuity is held on the longest of two lives, aged 50 and 60, with power, on the extinction of both these lives, to nominate two other lives, who are to enjoy the annuity so long as either of them is in existence; what is the value of the annuity on these succeeding lives?

PROBLEM XIII.

To find the value of an annuity on a given life for any number of years.

RULE.

Find the value of a life as many years older than the given life, as are equal to the time for which the annuity is proposed (Table III.) Multiply this value by the present worth of £1, payable at the end of the given time,

* This rule applies equally to annuities on joint lives, or the longest of any lives, with power to nominate an equal number of similar lives to succeed.

(Table VII.) and likewise by the probability that the life will continue so long (Prob. II.) Subtract the product from the present value of the given life (Table III.), and the remainder multiplied by the annuity will give the answer.

Examples.

(1.) What is the value of an annuity of £100 for 14 years, provided a person aged 35 lives so long? Interest 5 per cent.

The value of a life of 35+14=49 (Table III.) is 10·443
The present worth of £1 due 14 years hence (Table VII.) is .... ·5050679
The value of a life of 35 (Table III.) is .... 12·502
The probability that a life of 35 will continue 14 years (by Table I. and Problem II.) is $\frac{2936}{4010}$. Then

$10{\cdot}443 \times {\cdot}5050679 \times \frac{2936}{4010} = 3{\cdot}86177$, and $12{\cdot}502 - 3{\cdot}86177 = 8{\cdot}64023$ value, or years' purchase. Hence $8{\cdot}64023 \times 100 = £864$, the value of the annuity of £100

(2.) What is the value of an annuity of £80 for 20 years, provided a person aged 45 lives so long? Interest at 5 per cent.

PROBLEM XIV.

To find the present value of an annuity certain for a given term, after the extinction of any given life or lives.

RULE.

Multiply the value of an annuity on the given life or lives by the interest of £1 for a year, subtract the product from an unit, and reserve the remainder. Find the present worth of an annuity certain for the given term, (page 252) which multiply by the reserved remainder already found, and the product will be the value required.

Examples.

(1.) A person A, or his heirs, are entitled to an annuity for 21 years, to commence at the death of a gen-

gentleman aged 70, what is the present value of A.'s interest in the annuity, interest at 5 per cent.?

The value of an annuity on a life of 70 (Table III.) is 6·023, which multiplied by ·05, and deducted from an unit leaves ·69885, the reserved remainder. The present worth of £1 annuity certain for 21 years (page 252) is 12·82115; then $12·82115 \times ·69885 = 8·96$ years' purchase, the value of A.'s interest.

(2.) A lease of an estate is held upon two lives, aged 60 and 70, and after the decease of both, for 21 years certain; what is the value of the lease, reckoning interest at 5 per cent.?

(3.) A lease of an estate is held upon three lives aged 50, 60, and 70, and after their decease, for 21 years certain; what is the value of the lease, interest at 5 per cent.?

PROBLEM XV.

To find the present value of an estate, to be entered upon at the extinction of any given life or lives.

RULE.

Find the value of an annuity of £1, to continue for ever* (by prop. 1, page 259), and the value of an annuity on the given life or lives (Table III. or V.) The difference between these two values will be the answer required.

Examples.

(1.) What is the value of a freehold estate to be entered upon at the death of a person aged 20, interest at 5 per cent.?

First $1 \div ·05 = £20$, the value of the perpetuity, and the value of a life of 20, (Table III.) is 14·007, hence $20 - 14·007 = 5·993$, the value required; so that the estate is worth about 6 years' purchase.

(2.) What is the value of a freehold estate, to be entered upon at the death of *either* of two persons, aged 40 and 45, interest at 5 per cent.?

(3.) What would be the value of a freehold estate, to

* Or for a given number of years by Theorem I, page 252.

be entered upon at the death of *both* the persons mentioned in the preceding example, interest at 5 per cent.?

(4.) A person aged 70 has the *lease* of a house for 80 years, at a ground rent of £10 per annum, which he lets for £60 a year, what must the present tenant pay down that he may hold the lease after the death of the proprietor, or what additional rent must he pay for the same advantage?

TABLE I.

Shewing the Probabilities of the Duration of Human Life, according to the Observations made at *Northampton*.

Age	Living.	Dying.	Age.	Living.	Dying.	Age.	Living.	Dying.
0	11650	3000	33	4160	75	65	1632	80
1	8650	1367	34	4085	75	66	1552	80
2	7283	502	35	4010	75	67	1472	80
3	6781	335	36	3935	75	68	1392	80
4	6446	197	37	3860	75	69	1312	80
5	6249	184	38	3785	75	70	1232	80
6	6065	140	39	3710	75	71	1152	80
7	5925	110	40	3635	76	72	1072	80
8	5815	80	41	3559	77	73	992	80
9	5735	60	42	3482	78	74	912	80
10	5675	52	43	3404	78	75	832	80
11	5623	50	44	3326	78	76	752	77
12	5573	50	45	3248	78	77	675	73
13	5523	50	46	3170	78	78	602	68
14	5473	50	47	3092	78	79	534	65
15	5423	50	48	3014	78	80	469	63
16	5373	53	49	2936	79	81	4[illegible]6	60
17	5320	58	50	2857	81	82	346	57
18	5262	63	51	2776	82	83	289	55
19	5199	67	52	2694	82	84	234	48
20	5132	72	53	2612	82	85	186	41
21	5060	75	54	2530	82	86	145	34
22	4985	75	55	2448	82	87	111	28
23	4910	75	56	2366	82	88	83	21
24	4835	75	57	2284	82	89	62	16
25	4760	75	58	2202	82	90	46	12
26	4685	75	59	2120	82	91	34	10
27	4610	75	60	2038	82	92	24	8
28	4535	75	61	1956	82	93	16	7
29	4460	75	62	1874	81	94	9	5
30	4385	75	63	1793	81	95	4	3
31	4310	75	64	1712	80	96	1	1
32	4235	75						

N. B. Of 11650 infants born, 3000 will die in the first year. Of the 8650 who live to be one year old, 1367 will die in the course of the second year, &c.

TABLE II.

Shewing the Expectations of Human Life at every Age; deduced from the Observations made at *Northampton.*

Age.	Expectation.	Age.	Expectation.	Age.	Expectation.	Age.	Expectation.
1	32·74	25	30·85	49	18·49	73	7·33
2	37·79	26	30·33	50	17·99	74	6·92
3	39·55	27	29·82	51	17·50	75	6·54
4	40·58	28	29·30	52	17·02	76	6·18
5	40·84	29	28·79	53	16·54	77	5·83
6	41·07	30	28·27	54	16·06	78	5·48
7	41·03	31	27·76	55	15·58	79	5·11
8	40·79	32	27·24	56	15·10	80	4·75
9	40·36	33	26·72	57	14·63	81	4·41
10	39·78	34	26·20	58	14·15	82	4·09
11	39·14	35	25·68	59	13·68	83	3·80
12	38·49	36	25·16	60	13·21	84	3·58
13	37·83	37	24·64	61	12·75	85	3·37
14	37·17	38	24·12	62	12·28	86	3·19
15	36·51	39	23·60	63	11·81	87	3·01
16	35·85	40	23·08	64	11·35	88	2·86
17	35·20	41	22·56	65	10·88	89	2·66
18	34·58	42	22·04	66	10·42	90	2·41
19	33·99	43	21·54	67	9·96	91	2·09
20	33·43	44	21·03	68	9·50	92	1·75
21	32·90	45	20·52	69	9·05	93	1·37
22	32·39	46	20·02	70	8·60	94	1·05
23	31·88	47	19·51	71	8·17	95	·75
24	31·36	48	19·00	72	7·74	96	·50

N. B. By expectation of life, must be understood, that out of a number of persons living, of a given age, one with another may expect to live a *certain number of years,* some of them enjoying a duration as much longer as others fall short of that period. A person of 45 years of age, may live 20·52 years; of 60 years of age, 13·21 years, &c. *Females,* in general, live longer than *males,* and *married* women live longer than *single* women.

TABLE III.

Shewing the Value of an Annuity of £1 on a *single* Life, at every Age; deduced from the Observations made at *Northampton*, reckoning Interest at 3, 4, or 5 per Cent.

Ages.	Value at 3 per cent.	Value at 4 per cent.	Value at 5 per cent	Ages.	Value at 3 per cent.	Value at 4 per cent.	Value at 5 per cent.
1	16·021	13·465	11·563	49	12·693	11·475	10·443
2	18·599	15·633	13·420	50	12·436	11·264	10·269
3	19·575	16·462	14.135	51	12·183	11·057	10·097
4	20·210	17·010	14·613	52	11·930	10·849	9·925
5	20·473	17·248	14·827	53	11·674	10·637	9·748
6	20·727	17·482	15·041	54	11·414	10·421	9·567
7	20·853	17·611	15·166	55	11·150	10·201	9·382
8	20·885	17·662	15·226	56	10·882	9·977	9·193
9	20·812	17·625	15·210	57	10·611	9·749	8·999
10	20·663	17·523	15·139	58	10·337	9·516	8.801
11	20·480	17·393	15·043	59	10·058	9·280	8·599
12	20·283	17·251	14·937	60	9·777	9·039	8·392
13	20·081	17·103	14·826	61	9·493	8·795	8.181
14	19·872	16·950	14·710	62	9·205	8·547	7·966
15	19·657	16·791	14·588	63	8·910	8·291	7·742
16	19·435	16·625	14·460	64	8.611	8·030	7·514
17	19·218	16·462	14·334	65	8.304	7·761	7·276
18	19·013	16·309	14·217	66	7·074	7·488	7·034
19	18·820	16·167	14·108	67	7·682	7·211	6·787
20	18·638	16·033	14·007	68	7·367	6·930	6·536
21	18·470	15·912	13·917	69	7·051	6·647	6·281
22	18·311	15·797	13·833	70	6·734	6·361	6·023
23	18·148	15·680	13·746	71	6·418	6·075	5·764
24	17·983	15·560	13·658	72	6·103	5·790	5·504
25	17·814	15·438	13·567	73	5·794	5·507	5·245
26	17·642	15·312	13·473	74	5·491	5·230	4·990
27	17·467	15·184	13·377	75	5·199	4·962	4·744
28	17·289	15·053	13·278	76	4·925	4·710	4·511
29	17·107	14·918	13·177	77	4·652	4·457	4·277
30	16·922	14·781	13·072	78	4·372	4·197	4·035
31	16·732	14·639	12·965	79	4·077	3·921	3·776
32	16·540	14·495	12·854	80	3·781	3·643	3·515
33	16·343	14·347	12·740	81	3·499	3·377	3·263
34	16·142	14·195	12·623	82	3.229	3·122	3·020
35	15·938	14·039	12·502	83	2·989	2·887	2·797
36	15·729	13·880	12·377	84	2·793	2·708	2·627
37	15·515	13·716	12·249	85	2·620	2·543	2·471
38	15·298	13·548	12·116	86	2·462	2·393	2·328
39	15·075	13·375	11·979	87	2·312	2·251	2·193
40	14·848	13·197	11·837	88	2·185	2·131	2·080
41	14·620	13·018	11·695	89	2·013	1.967	1·924
42	14·391	12·838	11·551	90	1·794	1·758	1·723
43	14·162	12·657	11·407	91	1·501	1·474	1·447
44	13·929	12·472	11·258	92	1.190	1·171	1·153
45	13·692	12·283	11·105	93	·839	.827	·816
46	13·450	12·089	10·947	94	·536	.530	·524
47	13·203	11·890	10·784	95	·242	·240	·238
48	12·951	11·685	10·616	96	·000	·000	·000

TABLE IV.

Shewing the Value of an Insurance of £100 on a *single* Life, payable in one payment, or in annual Payments, Interest at 3 per Cent. deduced from the Observations made at *Northampton*.—N. B. This Table is adopted by all the Insurance Offices in London.

Age.	Whole Premium.	Annual Premium.	Age.	Whole Premium.	Annual Premium.
8 to 14	....	1·879	41	54·505	3·487
15	39·834	1·929	42	55·172	3·583
16	40·481	1·983	43	55·839	3·683
17	41·113	2·033	44	56·517	3·787
18	41·710	2·083	45	57·208	3·896
19	42·272	2·133	46	57·913	4·008
20	42·802	2·179	47	58·632	4·129
21	43·291	2·225	48	59 366	4·254
22	43·756	2·267	49	60·117	4·392
23	44·229	2·312	50	60·866	4·533
24	44·710	2·354	51	61·603	4·675
25	45·202	2·403	52	62·340	4·821
26	45·703	2·450	53	63·086	4·979
27	46·213	2·504	54	63·784	5·142
28	46·732	2·554	55	64·612	5·317
29	47·261	2·612	56	65 392	5·504
30	47·800	2·671	57	66·182	5·700
31	48·353	2·725	58	66·980	5·908
32	48·913	2·787	59	67·792	6·133
33	49·486	2·854	60	68·611	6·367
34	50·072	2·921	61	69·438	6·617
35	50·666	2·992	62	70·277	6·887
36	51·275	3·067	63	71·136	7·179
37	51·898	3·142	64	72·007	7·492
38	52·530	3·225	65	72·901	7·837
39	53·180	3·308	66	73·804	8·204
40	53·841	3·396	67	74·713	8·604

TABLE V.

Shewing the Value of an Annuity during the joint Continuance of *two* Lives, deduced from the Observations made at *Northampton*, reckoning Interest at 5 per Cent.

Ages.	Value.	Ages.	Value.	Ages.	Value.	Ages.	Value.
5—5	11·984	15—55	8·403	30—70	5·442	55—65	5·671
5—10	12·315	15—60	7·622	30—75	4·365	55—70	4·893
5—15	11·954	15—65	6·705	30—80	3·290	55—75	4·006
5—20	11·561	15—70	5·631	35—35	9·680	55—80	3·076
5—25	11·281	15—75	4·495	35—40	9·331	60—60	5·888
5—30	10·959	15—80	3·372	35—45	8·921	60—65	5·372
5—35	10·572	20—20	11·232	35—50	8·415	60—70	4·680
5—40	10·102	20—25	10·989	35—55	7·849	60—75	3·866
5—45	9·571	20—30	10·707	35—60	7·174	60—80	2·992
5—50	8·941	20—35	10·363	35—65	6·360	65—65	4·960
5—55	8·256	20—40	9·937	35—70	5·882	65—70	4·373
5—60	7·466	20—45	9·448	35—75	4·327	65—75	3·685
5—65	6·546	20—50	8·861	35—80	3·268	65—80	2·873
5—70	5·472	20—55	8·216	40—40	9·016	70—70	3·930
5—75	4·362	20—60	7·463	40—45	8·643	70—75	3·347
5—80	3·238	20—65	6·576	40—50	8·171	70—80	2·675
10—10	12·665	20—70	5·532	40—55	7·654	75—75	2·917
10—15	12·302	20—75	4·424	40—60	7·015	75—80	2·381
10—20	11·906	20—80	3·325	40—65	6·240	80—80	2·018
10—25	11·627	25—25	10·764	40—70	5·298	85—85	1·256
10—30	11·304	25—30	10·499	40—75	4·272	90—90	·909
10—35	10·916	25—35	10·175	40—80	3·236	*Difference of Ages*	
10—40	10·442	25—40	9·771	45—45	8·312	*10 Years.*	
10—45	9·900	25—45	9·301	45—50	7·891	16—26	11·193
10—50	9·260	25—50	8·739	45—55	7·411	18—28	10·939
10—55	8·560	25—55	8·116	45—60	6·822	22—32	10·498
10—60	7·750	25—60	7·383	45—65	6·094	24—34	10·285
10—65	6·803	25—65	6·515	45—70	5·195	26—36	10·062
10—70	5·700	25—70	5·489	45—75	4·206	28—38	9·826
10—75	4·522	25—75	4·396	45—80	3·197	32—42	9·320
10—80	3·395	25—80	3·308	50—50	7·522	34—44	9·058
15—15	11·960	30—30	10·255	50—55	7·098	36—34	8·781
15—20	11·535	30—35	9·954	50—60	6·568	38—48	4·487
15—25	11·324	30—40	9·576	50—65	5·897	42—52	7·875
15—30	11·021	30—45	9·135	50—70	5·054	44—54	7·569
15—35	10·655	30—50	8·596	50—75	4·112	46—56	7·249
15—40	10·205	30—55	7·999	50—80	3·140	48—58	6·915
15—45	9·690	30—60	7·292	55—55	6·785	52—62	6·222
15—50	9·076	30—65	6·447	55—60	6·272	54—64	5·860

TABLE VI.

Shewing the Value of an Insurance of £100 on *two* joint Lives, payable in one Payment, or in annual Payments, Interest at 3 per Cent. deduced from the Observations made at *Northampton.*—N. B. This Table is adopted by all the Insurance Offices in London.

Ages.	Whole Premium.	Annual Premium.	Ages.	Whole Premium.	Annual Premium.
10—10	49·498	2·855	25—55	69·461	6·625
10—15	51·177	3·053	25—60	72·343	7·619
10—20	52·958	3·279	25—65	75·621	9·035
10—25	54 319	3·463	30—30	60·418	4·446
10—30	55·873	3·688	30—35	61·754	4·703
10—35	57·693	3·972	30—40	63 392	5·044
10—40	59·832	4·339	30—45	65·271	5·474
10—45	62·206	4·794	30—50	67·495	6·048
10—50	64·919	5·390	30—55	69·915	6·769
10—55	67·801	6·133	30—60	72·685	7·751
10—60	71·012	7·135	30—65	75·866	9·156
10—65	74·606	8·557	35—35	62·944	4·947
15—15	52·731	3·249	35—40	64·428	5·275
15—20	54·388	3·473	35—45	66·149	5·692
15—25	55·641	3·653	35—50	68·217	6·252
15—30	57·083	3·874	35—55	70·492	6·958
15—35	58·783	4·154	35—60	73·125	7·925
15—40	60·799	4·517	35—65	76·181	9·316
15—45	63·047	4·969	40—40	65·736	5·588
15—50	65·634	5·563	40—45	67·274	5·988
15—55	68·395	6·303	40—50	69·154	6·530
15—60	71·485	7·302	40—55	71·250	7·218
15—65	74 960	8·719	40—60	73·713	8·168
20—20	55·923	3·695	40—65	76·612	9·511
20—25	57·065	3·871	45—45	68.611	6·367
20—30	58·390	4·087	45—50	70·278	6·887
20—35	59·968	4·363	45—55	72·164	7·551
20—40	61·856	4·723	45—60	74·424	8·476
20—45	63·979	5·173	45—65	77·134	9·825
20—50	66·438	5·766	50—50	71·705	7·381
20—55	69·077	6·506	50—55	73·344	8·014
20—60	72·049	7·508	50—60	75·357	8·907
20—65	75·406	8·930	50—65	77·831	10·226
25—25	58·106	4·040	55—55	74 713	8 606
25—30	59·322	4·248	55—60	76·443	9·451
25—35	60·786	4·515	55—65	78·637	10·721
25—40	62·559	4·867	60—60	77·846	10·235
25—45	64·571	5·308	60—65	79·699	11·434
25—50	66 923	5·893	65—65	81·152	12·541

TABLE VII.

Shewing the present Value of £1, to be received at the End of any Number of Years not exceeding 40.

Years.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.
1	.9708738	.9061836	.9615385	.9569378	.9523809
2	.9425959	.9335107	.9245562	.9157299	.9070295
3	.9151417	.9019427	.8889964	.8762966	.8638376
4	.8884870	.8714422	.8548042	.8385613	.8227025
5	.8626088	.8419732	.8219271	.8024511	.7835262
6	.8374843	.8135006	.7903145	.7678957	.7462154
7	.8130915	.7859910	.7599178	.7348285	.7106813
8	.7894092	.7594116	.7306902	.7031851	.6768394
9	.7664167	.7337310	.7025867	.6729044	.6446089
10	.7440939	.7089188	.6755642	.6439277	.6139133
11	.7224213	.6849457	.6495809	.6161988	.5846793
12	.7013799	.6617833	.6245971	.5896639	.5568374
13	.6809513	.6391041	.6005741	.5642716	.5303214
14	.6611178	.6177818	.5774751	.5399729	.5050679
15	.6418619	.5968906	.5552645	.5167204	.4810171
16	.6231669	.5767059	.5339082	.4944693	.4581115
17	.6050164	.5572038	.5133733	.4731764	.4362967
18	.5873946	.5383611	.4936281	.4528004	.4155207
19	.5702860	.5201557	.4746424	.4333018	.3957340
20	.5536758	.5025659	.4566870	.4146429	.3768895
21	.5375493	.5855709	.4388336	.3967874	.3589424
22	.5218925	.4691506	.4219554	.3797009	.3418499
23	.5066917	.4532856	.4057263	.3633501	.3255713
24	.4919337	.4379571	.3901215	.3477035	.3100679
25	.4770056	.4231470	.3751168	.3327306	.2953028
26	.4636947	.4088378	.3606892	.3184025	.2812407
27	.4501891	.3950123	.3468166	.3046914	.2678483
28	.4370768	.3816543	.3334775	.2915707	.2550936
29	.4243464	.3687482	.3206514	.2790150	.2429463
30	.4119868	.3562784	.3083187	.2670000	.2313775
31	.3999871	.3442304	.2964603	.2555024	.2203595
32	.3883370	.3325897	.2850579	.2444999	.2098662
33	.3770263	.3213427	.2740942	.2339712	.1998726
34	.3660449	.3104761	.2635521	.2238959	.1903548
35	.3553834	.2999765	.2534155	.2142544	.1812903
36	.4350324	.2898327	.2436687	.2050280	.1726574
37	.3349829	.2800316	.2342969	.1961992	.1644356
38	3 252262	.2705619	.2252854	.1877504	.1566054
39	.3157536	.2614125	.2166206	.1796655	.1491479
40	.3065568	.2525725	.2082890	.1719287	.1420457

Note. Those who wish for farther information on Life Annuities, may consult the works of Mr. *De Moivre*, Mr. *Simpson*, Mr. *Dodson*, Dr. *Price*, Mr. *Emerson*, Mr. *Morgan*, Baron *Maseres*; or the Doctrine of Life Annuities and Assurances, by Mr. *Baily*, in two volumes octavo.

ON RATIOS.

1. RATIO is the relation which one quantity bears to another of the same kind, with respect to magnitude; and the comparison is made by considering how often the one is contained in the other, or how often the one contains the other.

Thus the ratio of A to B is expressed by $\frac{A}{B}$, and the ratio of B to A by $\frac{B}{A}$, the former of these quantities, or the numerator, is called the antecedent; and the latter, or the denominator, is called the consequent of the ratio.

2. When the antecedent is equal to the consequent, viz. if $\frac{A}{B}=1$, it is called a ratio of equality if $\frac{A}{B}$ be greater than 1, we call it a ratio of greater inequality; and if $\frac{A}{B}$ be less than 1, it is called a ratio of less inequality.

3. The antecedent and consequent are called the terms of the ratio, and the quotient of the two terms is called the index, or exponent of the ratio.

Thus, if $\frac{A}{B}=m$, then m is called the exponent of the ratio of A to B.

4. Compound ratio is made up of two or more ratios, by multiplying their terms and exponents together.

If $\frac{A}{B}=m$, and $\frac{C}{D}=n$, then $\frac{A}{B}\times\frac{C}{D}=mn=\frac{AC}{BD}$, so that the ratio of AC to BD, is said to be compounded of the ratios of A to B, and C to D.

5. If a ratio be compounded of two equal ratios, it is called a duplicate ratio; if of three equal ratios, it is called a triplicate ratio, &c.

Thus, if $\frac{A}{B}=m$, $\frac{C}{D}=m$, then, $\frac{AC}{BD}=m^2$, hence the ratio of AC to BD is duplicate of the ratio of A to B, or of C to D. And if $\frac{A}{B}=m$, $\frac{C}{D}=m$, $\frac{E}{F}=m$, then, $\frac{ACE}{BDF}=m^3$, hence the

ratio of ACE to BDF, is triplicate to the ratio of A to B, C to D, or E to F.

6. *If the terms of a ratio be prime to each other, no other quantities can be found in the same ratio, but what shall be some multiple* thereof.*

Let $\frac{A}{B} = m$, and $\frac{C}{D} = m$, where A and B are prime to each other, I say C shall be a multiple of A, and D a multiple of B. For $\frac{A}{B} = \frac{C}{D}$, multiply by D, then $C = \frac{DA}{B}$; now it is evident, if B measures DA, it must measure D alone, because A is prime to B: consequently D is some multiple of B, therefore C is some multiple of A.

7. Cor. 1. *The like multiples, or the like parts of the terms of a ratio, have the same ratio as the terms themselves.*

8. Cor. 2. *Numbers that are prime to each other, are the least of all numbers in the same ratio.*

9. *Having the terms of a ratio given in large numbers that are prime to each other, to find a ratio, nearly equivalent, whose terms are expressed by smaller numbers.*

This is performed by reducing the terms of the given ratio into a series, of what are called *continued fractions.*

Thus, let the given ratio be expressed by $\frac{b}{a}$; and let a be contained in b, c times, with a remainder d; again let d be contained in a, e times, with a remainder f, and so on, we shall have

$$\begin{array}{l} a)b(c \\ \overline{d)a(e} \\ \overline{f)d(g} \\ \overline{h)f(i} \\ \overline{k)h(l} \\ \overline{m}, \&c. \end{array}$$

$$\begin{aligned} b &= ac + d \\ a &= de + f \\ d &= fg + h \\ f &= hi + k, \&c. \end{aligned}$$

* One number is said to be a multiple of another, when the former contains the latter some exact number of times. Thus, m B is a multiple of B, and $\frac{1}{n}$ A is a multiple of A.

Hence $\frac{b}{a}=\frac{ac+d}{a}=c+\frac{d}{a}$, but $a=de+f$.

Therefore $\frac{b}{a}=c+\frac{d}{de+f},=c+\cfrac{1}{e+\cfrac{f}{d}}$, but $d=fg+h$.

Therefore $\frac{b}{a}=c+\cfrac{1}{e+\cfrac{f}{fg+h}}=c+\cfrac{1}{e+\cfrac{1}{g+\cfrac{h}{f}}}$, but $f=hi+k$,

Therefore $\frac{b}{a}=c+\cfrac{1}{e+\cfrac{1}{g+\cfrac{h}{hi+k}}}=c+\cfrac{1}{e+\cfrac{1}{g+\cfrac{1}{i+\cfrac{k}{h}}}}$ &c.

Viz. $\frac{b}{a}=c+\frac{1}{e}+\frac{1}{g}+\frac{1}{i}+$ &c. Then collecting the terms of this series one after another, beginning at c, we continually approximate towards the ratio of $\frac{b}{a}$; and this approximation is alternately less, and greater than the true ratio.

The first value is c, or $\frac{c}{1}$, the second $c+\frac{1}{e}=\frac{ce+1}{e}$,

The third $c+\frac{1}{e}+\frac{1}{g}=c+\cfrac{1}{\cfrac{ge+1}{g}}=c+\frac{g}{ge+1}=\frac{cge+c+g}{ge+1}=$

$\frac{g\times\overline{ce+1}}{ge},\ \frac{+c}{+1}$. The fourth $c+\frac{1}{e}+\frac{1}{g}+\frac{1}{i}=c+\frac{1}{e}+\cfrac{1}{\cfrac{ig+1}{i}}$

$=c+\frac{1}{e}+\frac{i}{ig+1}=c+\cfrac{1}{\cfrac{eig+e+i}{ig+1}}=c+\frac{ig+1}{eig+e+i}=\frac{ceig+ce+ei+ig+1}{eig+e+i}$

$=\left(\frac{g\times\overline{ce+1}}{ge};\ \frac{+c}{+1}\right)\times\frac{i}{i};\ \frac{+ce+1}{+e}.$

Hence we deduce the following general rule.

1, Divide the greater term by the less, and that di-

visor by the remainder, &c. as in Prop. 1, page 65, Vulgar Fractions. Then, if the antecedent be greater than the consequent, the first quotient divided by 1, gives the first ratio; if less, an unit divided by the first quotient, will express the first ratio.

2. Multiply the terms of the first ratio by the second quotient, and add an unit to the numerator, or denominator, according as the antecedent of the original terms is greater or less than its consequent, and you will have the second ratio.

3. Then, in general, multiply the terms of the ratio last found by the next succeeding quotient, and to the two products add the corresponding terms of the preceding ratio, and you will have the next succeeding ratio, &c.

Example 1. Let it be required to find a series of ratios in less numbers, constantly approaching to the ratio of 314159 to 100000, which is nearly the ratio of the circumference of a circle to its diameter.

100000)314159(3=c

d=14159.)100000(7=e

f=887)14159(15=g

h=854)887(1=i

k=33, &c.

$3=\frac{3}{1}$ the first ratio.

$\frac{3\times7+1}{1\times7}=\frac{22}{7}$ the second ratio, being the approximation of *Archimedes*.

$\frac{22\times15}{7\times15};\ \frac{+3}{+1}=\frac{333}{106}$ the third ratio.

$\frac{333\times1}{106\times1};\ \frac{+22}{+7}=\frac{355}{113}$ the fourth ratio, the approximation of *Metius*.

Hence $\frac{314159}{100000}=3+\frac{1}{7}+\frac{1}{15}+\frac{1}{1}$ &c. in a continued fraction.

Example 2. Let it be required to find a series of ratios in less numbers, constantly approaching to the

ratio of 7853981633 to 10000000000, which is nearly the ratio of the area of a circle to the square of its diameter.

7853981633)10000000000(1

2146018367)7853981633(3

1415926532)2146018367(1

730091835

730091835)1415926532(1

685834697)730091835(1

44257138)685834697(15

20977627)44257138(2

2301884, &c.

$1 = \frac{1}{1}$ first ratio.

$\frac{1\times 3}{1\times 3+1} = \frac{3}{4}$ second ratio.

$\frac{3\times 1}{4\times 1}; \frac{+1}{+1} = \frac{4}{5}$ third ratio.

$\frac{4\times 1}{5\times 1}; \frac{+3}{+4} = \frac{7}{9}$ fourth ratio.

$\frac{7\times 1}{9\times 1}; \frac{+4}{+5} = \frac{11}{14}$ *fifth ratio.*

$\frac{11\times 15}{14\times 15}; \frac{+7}{+9} = \frac{172}{279}$ sixth ratio.

$\frac{172\times 2}{119\times 2}; \frac{+11}{+14} = \frac{355}{452}$ *seventh ratio.*

ON PROPORTION.

10. Proportion is the equality of ratios.

Thus, if $\frac{A}{B} = m$, and $\frac{C}{D} = n$; then, if m be equal to n, the ratios are equal; that is, A has the same ratio to B, which C has to D, and the four quantities are said to be proportional; viz. A : B :: C : D, or $\frac{A}{B} = \frac{C}{D}$.

If m be greater than n, then A has to B a greater ratio than C has to D, and the four quantities are not proportional.

If m be less than n, then A has to B a less ratio than C has to D, and the four quantities are not proportional.

If m and n are each equal to an unit, then the ratios of A to B, and C to D, are ratios of equality.

11. *If four quantities be proportional, the rectangle, or product of the extremes, will be equal to the rectangle, or product of the means.*

For, if A : B :: C : D, then $\frac{A}{B}=\frac{C}{D}$ by the definition.

The fractions $\frac{A}{B}$ and $\frac{C}{D}$ reduced to a common denominator will be $\frac{AD}{BD}$ and $\frac{BC}{BD}$, but when two *equal* fractions have the same denominator, their numerators are equal, therefore $AD=BC$; A and D being the extremes, and B and C the means.

12. *If the product of two quantities be equal to the product of two others, the four quantities may be turned into a proportion, by making the terms of one product the means, and the terms of the other the extremes.*

Thus, if $AD=BC$, divide each by BD, then $\frac{AD}{BD}=\frac{BC}{BD}$ viz. $\frac{A}{B}=\frac{C}{D}$, or A : B :: C : D.

13. *If four quantities be proportional, they shall also be proportional when taken inversely,* viz. if A : B :: C : D, then B : A :: D : C. For $AD=BC$, mult. by $\frac{1}{AC}$, then $\frac{AD}{AC}=\frac{BC}{AC}$, or $\frac{B}{A}=\frac{D}{C}$, hence, INVERTENDO, B : A :: D : C.

14. *If four quantities be proportional, they shall also be proportional when taken alternately,* viz. if A : B :: C : D, then A : C :: B : D. For, $\frac{A}{B}=\frac{C}{D}$, mult. by $\frac{B}{C}$, then $\frac{BA}{BC}=\frac{BC}{DC}$, or $\frac{A}{C}=\frac{B}{D}$, hence, ALTERNANDO, A : C :: B : D.

15. *When four quantities are proportional, the first together with the second, is to the second; as the third together with the fourth, is to the fourth.*

Thus, if $A : B :: C : D$, then

COMPONENDO, $A+B : B :: C+D : D$.

For $AD=BC$ (article 11) add DB to each.

Then $AD+DB=BC+DB$; or $\overline{A+B}\times D=\overline{C+D}\times B$, there fore (art. 12.) $A+B : B :: C+D : D$.

16. *If four quantities be proportional, the difference between the first and second, is to the second; as the difference between the third and fourth, is to the fourth.*

Thus, if $A : B :: C : D$, then

DIVIDENDO, $A-B : B :: C-D : D$.

For $AD=BC$ (art 11.) take DB from each,

then $AD-DB=BC-DB$; or $\overline{A-B}\times D=\overline{C-D}\times B$, therefore (art. 12.) $A-B : B :: C-D : D$.

17. *If four quantities be proportional; the first, is to the difference between the first and second, as the third, is to the difference between the third and fourth.*

Thus, if $A : B :: C : D$, then

CONVERTENDO, $A : A-B :: C : C-D$.

For $\frac{A-B}{B}=\frac{C-D}{D}$ art. 16th, and $\frac{B}{A}=\frac{D}{C}$, art. 13th,

Hence, $\frac{A-B}{B}\times\frac{B}{A}=\frac{C-D}{D}\times\frac{D}{C}$, or $\frac{A-B}{A}=\frac{C-D}{C}$,

that is, $A-B : A :: C-D : C$, and INVERTENDO,

$A : A-B :: C : C-D$.

18. *If several quantities be proportional, as one of the antecedents is to its consequent; so is the sum of all the antecedents, to the sum of all the consequents.*

Thus, if $A : B :: C : D :: E : F :: G : H$, &c.

Then $A : B :: A+C+E+G : B+D+F+H$.

For, $\frac{A}{B}=\frac{A}{B}=\frac{C}{D}=\frac{E}{F}=\frac{G}{H}$, hence $AB=BA$, $AD=BC$, $AF=BE$, $AH=BG$, therefore $AB+AD+AF+AH=BA+BC+BE+BG$, or $A\times\overline{B+D+F+H}=B\times\overline{A+C+E+G}$; by art. 12, $A : B :: A+C+E+G : B+D+F+H$.

19. *Cor. As any antecedent is to its consequent, so is any other antecedent to its consequent.*

20. *If four quantities be proportional, and if any antecedent and its consequent, or the two antecedents and their consequents, be both multiplied or both divided by the same quantity, the four quantities will still be proportional.*

$$\text{Thus, if } A : B :: C : D$$
$$\text{Then } mA : mB :: C : D$$
$$A : B :: \frac{C}{n} : \frac{D}{n}$$
$$mA : \frac{B}{n} :: mC : \frac{D}{n}$$

For in each case, $\frac{A}{B} = \frac{C}{D}$.

21. *If there be four proportional quantities in one rank, and four more in another; or if there be several such ranks; the products of the correspondent terms will be proportional.*

$$\begin{array}{llllllll} \text{Thus, if} & A & : & B & :: & C & : & D \\ & E & : & F & :: & G & : & H \\ & I & : & K & :: & L & : & M, \&c. \\ \text{Then} & AE & : & FB & :: & CG & : & DH \\ \text{Or,} & AEI & : & BFK & :: & CGL & : & DHM, \&c. \end{array}$$

For $\frac{A}{B} = \frac{C}{D}$, $\frac{E}{F} = \frac{G}{H}$, $\frac{I}{K} = \frac{L}{M}$, &c. Hence $\frac{AE}{BF} = \frac{CG}{DH}$ and $\frac{AEI}{BFK} = \frac{CGL}{DHM}$.

22. Cor. *The like powers, or the like roots, of proportional quantities, are proportional.*

Thus, if $A : B :: C : D$, then

$$A^m : B^m :: C^m : D^m; \text{ or, } A^{\frac{1}{m}} : B^{\frac{1}{m}} :: C^{\frac{1}{m}} : D^{\frac{1}{m}}$$

This is obvious, by supposing A, E, and I, equal to each other; also B, F, K.

23. *If there be any number of quantities in one rank, and an equal number of quantities in another rank; so constituted that the first is to the second in the first rank, as the first is to the second in the second rank; or the*

second is to the third in the first rank, as the second is to the third in the second rank, &c.; then shall the first be to the last in the first rank, as the first is to the last in the second rank. And any four of these quantities, in the form of a square, or parallelogram, shall be proportional. The same is general, let the number of ranks be ever so many.

Thus, if A : B : C : D : E ::
F : G : H : I : K ::
L : M : N : O : P ::
Q : R : S : T : V :: &c.

Then, EX ÆQUO ORDINATA, A : E :: F : K, or A : E :: Q : V. For,

A : B :: F : G
B : C :: G : H
C : D :: H : I
D : E :: I : K. Hence, art. 21;

ABCD : BCDE :: FGHI : GHIK. Conseq.

$\frac{ABCD}{EBCD} = \frac{FGHI}{KGHI}$, or $\frac{A}{E} = \frac{F}{K}$, or, A : E :: F : K, and so on for any other two ranks.

From this demonstration it follows, that the ratio of A to E, is compounded of the ratios of A to B, B to C, C to D, and D to E.

24. *If there be any number of quantities in one rank, and an equal number of quantities in another rank; so constituted that the first is to the second in the first rank, as the last but one in the second rank is to the last; and the second of the first rank is to the third, as the last but two in the second rank is to the last but one, &c.; then shall the first be to the last in the first rank, as the first is to the last in the second rank.*

Thus, if A · B · C · D · E be the first rank,
And F · G · H · I · K the second rank,
Then EX ÆQUO PERTURBATA, A : E :: F : K.

For, A : B :: I : K
B : C :: H : I
C : D :: G : H
D : E :: F : G by the proposition.

Hence, by compounding the terms as in art. 23.

A : E :: F : K.

ON NUMBERS, ODD AND EVEN.

25. *If any number of even numbers be added together, the sum will be an even number.*

For, let $2A$, $2B$, $2C$, &c. be even numbers. Then will $2A+2B+2C$, &c. be their sum. But this sum can be divided by 2, therefore it is an even number, Defin. 7, p. 2.

26. *If any even number of odd numbers be added together, the sum will be an even number.*

For, let $2A+1$, $2B+1$, $2C+1$, $2D+1$, &c. represent the odd numbers, then $2A+2B+2C+2D$ is an even number, and $1+1+1+1$ is also an even number, that is, an even number of units is an even number; it is therefore obvious that the whole is even.

27. *If an odd number of odd numbers be added together, the sum will be an odd number.*

This is evident from above, for $2A+1;+2B+1;+2C+1=2A+2B+2C+3$, an odd number.

28. *If an even number be taken from an even number, the remainder will be even.*

For, since $2A$ and $2B$ are even numbers, if A be greater than B; $2A-2B$, beingdivisible by 2, is an even number.

29. *If an odd number be taken from an odd number, the remainder will be even.*

Let $2A+1$ and $2B+1$ be odd numbers, where A is greater than B; $\overline{2A+1}-\overline{2B+1}=2A-2B$ an even number.

30. *If an even number be taken from an odd number, or an odd number from an even one, the remainder will be odd.*

Let $2A$, $2B$, be two even numbers, and $2C+1$, $2D+1$, two odd numbers, where C is greater than A, and B greater than D. Then $\overline{2C+1}-2A$ and $2B-2D+1$ are evidently odd numbers.

31. *If an odd number be multiplied by an odd number, the product will be odd.*

Let $2A+1$ and $2B+1$ be any two odd numbers, their product $4AB+2B+2A+1$ is evidently an odd number.

Cor. *The quotient of an odd number by an odd number, is an odd number.*

32. *If an even number be multiplied by any number whatever, the product will be even.*

Let $2A$ and $2B$ be any even numbers, and $2C+1$ an odd number, $2A\times 2B=4AB$ an even number, also $2A\times\overline{2C+1}$ and $2B\times\overline{2C+1}$ are even numbers.

Cor. *If an even number contain an odd number a certain number of times, the quotient will be an even number. Hence also an even number cannot be contained an exact number of times in an odd number.*

Other particular properties of numbers are given at page 6, 10, 15, 66, 70, 105, 106, 200, 202, &c.

ON SQUARE AND CUBE NUMBERS, &c.

33. *The sum of any number of terms of the series of odd numbers,* 1. 3. 5. 7. 9. 11. *&c. is equal to the square of that number.*

1 · 2 · 3 · 4 · 5 · 6 number of terms.
1 · 3 · 5 · 7 · 9 · 11, &c. series of odd numbers.

Then $1+3=2^2$; $1+3+5=3^2$; $1+3+5+7=4^2$; and so on as far as you please.

34. *If to the sum of any number of terms of the series of squares* 1. 4. 9. 16. 25. 36. 49. *&c. you add the square of half the sum of the same number of terms, and increase that sum by an unit, the last sum will always be a square number.*

Thus $1+4+\left(\frac{1+4}{2}\right)^2+1=1+4+6{\cdot}25+1=12{\cdot}25.$

$$1+4+9+\left(\frac{1+4+9}{2}\right)^2+1=1+4+9+49+1=64,$$

$$1+4+9+16+\left(\frac{1+4+9+16}{2}\right)^2+1=1+4+9+16+225+1=256$$

Hence may be found as many square whole numbers as you please, whose sum shall universally be a square number.

35. *In a series of squares proceeding from an unit, the second differences will be equal to each other; in cubes the third differences; in biquadrates the fourth differences, &c.*

Thus, 1 · 4 · 9 · 16 · 25 · 36, &c. *series of squares.*
3 · 5 · 7 · 9 · 11, &c. 1*st* order of differences.
2 · 2 · 2 · 2, &c. 2*d* order of differences.
And, 1 · 8 · 27 · 64 · 125 · 216, &c. *series of cubes.*
7 · 19 · 37 · 61 · 91, &c. 1*st* order of diff.
12 · 18 · 24 · 30, &c. 2*d* order of diff.
6 · 6 · 6, &c. 3*d* order of diff.

In the same manner the fourth order of differences in the series of biquadrates, 1 . 16 . 81 . 256 . 625 . 1296, &c. will be 24. These orders of differences are obtained, by subtracting the first term from the second, the second from the third, the third from the fourth, &c. in the series; and in each of the orders of differences.

36. *If a be the first term of any series, d' the first term of the first order of differences; d'' the first term of the second order of differences; d''' the first term of the third order of differences; d*iv *the first term of the fourth order of differences, &c. The last or nth term will be*

$$a+\frac{n-1}{1}d'+\frac{n-1}{1}\times\frac{n-2}{2}d''+\frac{n-1}{1}\times\frac{n-2}{2}\times\frac{n-3}{3}d'''+\frac{n-1}{1}\times\frac{n-2}{2}\times\frac{n-3}{3}\times\frac{n-4}{4}d^{iv}\ \&c.$$

And the sum of n terms will be $na+n\times\frac{n-1}{2}d'+n\times\frac{n-1}{2}\times\frac{n-2}{3}d''+n\times\frac{n-1}{2}\times\frac{n-2}{3}\times\frac{n-3}{4}d'''+n\times\frac{n-1}{1}\times\frac{n-2}{3}\times\frac{n-3}{4}\times\frac{n-4}{5}d^{iv}$ &c. See Mr. *Emerson's* differential method page 12th and 13th. Any term of a given series, or the sum of any number of its terms, may be accurately determined, when any of the orders of differences become at last equal to each other.

Examples.

1. Required the 20th term of the series 1 · 4 · 8 · 13 · 19 · 26 · 34, &c.

1 · 4 · 8 · 13 · 19 · 26 · 34 &c. series.
3 · 4 · 5 · 6 · 7 · 8 &c. 1st order of diff.
1 · 1 · 1 · 1 · 1 &c. 2d order of diff.

Here $d'=3$, $d''=1$, $a=1$, and $n=20$.

Then, $a+\frac{n-1}{1}d'+\frac{n-1}{1}\times\frac{n-2}{2}d''=$

$$1+\left(\frac{20-1}{1}\times 3\right)+\left(\frac{20-1}{1}\times\frac{20-2}{2}\times 1\right)=$$

$1+(19\times 3)+(19\times 9\times 1)=$
$1+57+171=229$ *answer.*

2. Required the sum of a thousand terms of the series of squares $1^2 . 2^2 . 3^2 . 4^2 . 5^2$ &c.

1 . 4 . 9 . 16 . 25 . 36 &c. series.
3 . 5 . 7 . 9 . 11 &c. *first* order of diff.
2 . 2 . 2 . 2 &c. *second* order of diff.

Here $d'=3$, $d''=2$, $a=1$, and $n=1000$.

Then, $na+(n\times\frac{n-1}{2}d')+(n\times\frac{n-1}{2}\times\frac{n-2}{3}d'')=$

$$(2000\times 1)+(1000\times\frac{1000-1}{2}\times 3)+(1000\times\frac{1000-1}{2}\times\frac{1000-2}{3}\times 2)=1000+\left(\frac{1000}{1}\times\frac{999}{1}\times\frac{3}{2}\right)+\left(\frac{1000}{1}\times\frac{998}{1}\times\frac{3}{3}\right)=$$

$1000+1498500+332334000=$
333833500 *answer.*

3. Required the sum of twenty terms of the series of cubes $1^3 . 2^3 . 3^3 . 4^3 . 5^3 . 6^3$ &c.

1 . 8 . 27 . 64 . 125 . 216 &c. series.
7 . 19 . 37 . 61 . 91 &c. *first* order of diff.
12 . 18 . 24 . 30 &c. *second* order of diff.
6 . 6 . 6 &c. *third* order of diff.

Here $d'=7$, $d''=12$, $d'''=6$, $a=1$, and $n=20$.

Then, $na+(n\times\frac{n-1}{2}d')+(n\times\frac{n-1}{2}\times\frac{n-2}{3}d'')+(n\times\frac{n-1}{2}\times\frac{n-2}{3}\times\frac{n-3}{4}d''')=$

$$(20\times 1)+(20\times\frac{19}{2}\times 7)+(20\times\frac{19}{2}\times\frac{18}{3}\times 12)+(20\times\frac{19}{2}\times\frac{18}{3}\times\frac{17}{4}\times 6)$$

$=20+1330+13680+29070=$
44100 *answer.*

37. ***The sum of any two square numbers whatever; their difference; and twice the product of their roots; will express the three sides of a right angled triangle in rational numbers.***

Let 4 and 9 be the two squares, then $4+9=13$ their sum, $9-4=5$ their difference, and $\sqrt{4}\times\sqrt{9}\times 2=12$, twice the product of their roots; hence the three sides of the triangle will be 13, 5, and 12; for $12^2+5^2=13^2$.

38. *The cube of any number divided by 6, will leave the same remainder as the number itself, when divided by 6. Or, the difference between any number and its cube will divide even by 6.*

Let 47 be proposed, the cube of which is 103823; each of these numbers divided by 6 will leave 5 for a remainder.

Or, $103823-47$ will divide by 6 without a remainder.

39. *The sum of any number of the cubes of the natural series 1. 2. 3. 4. 5. &c. taken from the beginning always makes a square number; the roots of these squares are 1. 3. 6. 10. 15. 21. &c. whose differences are, 2. 3. 4. 5. 6. &c.*

Let 1. 8. 27. 64. 125. 216. 343. &c. be a series of cubes; the sum of the two first is 9, the sum of three 36, of four 100, &c. whose roots are 3. 6. 10. &c.

40. *An even square number will divide by 4, and leave no remainder, but an odd square number divided by 4 will leave a remainder of 1.*

Since the square of an odd number must be an odd number, let $2R+1$ express the root; the square of which is $4RR+4R+1$, this divided by 4 leaves 1 for a remainder; and the first term of it, viz. the square of $2R$, is divisible by 4.

Other particular properties of numbers may be seen in INVOLUTION, EVOLUTION, PROGRESSION, &c.

END OF THE SECOND PART.

THE

COMPLETE PRACTICAL ARITHMETICIAN.

PART III.

AN USEFUL COLLECTION OF

BILLS OF PARCELS, &c. &c.

CLASS I.

Exercising the Rules in COMPOUND MULTIPLICATION.

(1.) James Lamb, esq.

Bought of John Simpson,

Jan. 1, 1822. £. s. d.

$7\frac{1}{2}$ lb. of green tea, at 10*s.* 4*d.* per lb. - -
$14\frac{1}{2}$ do. finest bloom, at 14*s.* 8*d.* - - -
$10\frac{1}{4}$ do. fine green, at 16*s.* 5*d.* - - -
21 do. hyson, at 10*s.* $10\frac{1}{4}$*d.* - - - -
19 do. good hyson, at 13*s.* $9\frac{1}{2}$*d.* - - -
$8\frac{1}{4}$ do. bohea, at 6*s.* 9*d.* - - - - -

£

(2.) Sir John Guchim,
Hull, 1822. To S. Jefferson, Dr.

Jan. 11. For $37\frac{1}{4}$ yds. of sheeting, at 1*s.* $4\frac{1}{4}$*d.* per yard - - - - £. s. d.
Feb. 3. For $43\frac{1}{8}$ yds. of lace, at 4*s.* $0\frac{3}{4}$*d.* per yard - - - - -
—— 16. For $75\frac{3}{4}$ ells of Irish, at 2*s.* 3*d.* per ell
May 12. For 209 do. dowlas, at $9\frac{1}{4}$*d.* - -
—— 15. For 730 do. muslin, at 7*s.* $0\frac{1}{2}$*d.* -

£

Received the contents,
S. Jefferson.

(3.) Andrew Wines, esq.

To W. Johnson and Co. Dr.

London, 1822.		£.	s.	d.
July 11.	$473\frac{1}{8}$ gallons of British spirits, at 4*s.* $7\frac{1}{2}$*d.* per gallon - - -			
———	$308\frac{1}{2}$ gallons of fine old rum, at 9*s.* 10*d.* per gallon - - -			
———	$610\frac{5}{8}$ gallons of Holland's gin, at 5*s.* 2*d.* per gallon - - -			
Aug. 5.	$207\frac{3}{4}$ gallons of rum, at 8*s.* $9\frac{3}{4}$*d.* per gallon - - - - -			
———	$119\frac{1}{2}$ galls. of cognac brandy, at 10*s.* $0\frac{1}{2}$*d.* per gallon - - -			
Sept. 22.	$401\frac{1}{6}$ gallons of Maidstone gin, at 4*s.* 6*d.* per gallon - - -			
		£		

Received, Dec. 24, 1822, the contents for self and Co.

W. Johnson.

(4.) George Veres, esq.

Bought of Charles West,

London, Dec. 3, 1822.	£.	s.	d.
A loin of lamb, weight $7\frac{1}{4}$lb. at $10\frac{3}{4}$*d.* per lb.			
A fillet of veal, weight $16\frac{1}{4}$lb. at $6\frac{1}{2}$*d.* - -			
A buttock of beef, weight $37\frac{1}{2}$lb. at $4\frac{1}{2}$*d.* -			
A pig, weight $12\frac{3}{4}$lb. at $7\frac{1}{4}$*d.* - - -			
A leg of pork, weight $16\frac{1}{4}$lb. at $5\frac{1}{2}$*d.* - -			
A leg of mutton, weight $13\frac{3}{4}$lb. at $4\frac{3}{4}$*d.* - -			
	£		

(5.) Hugh Abbot,

Bought of C. Hartley,

London, Aug. 19, 1822.	£.	s.	d.
174$\frac{5}{8}$lb. of Quinquina, at 3*l.* 14*s.* 8*d.* per lb.			
821$\frac{3}{5}$lb. of gum lac, at 5*s.* 9*d.* - - -			
607$\frac{7}{8}$lb. of rhubarb, at 12*s.* 4*d.* - - -			
720$\frac{1}{2}$lb. of mastic, at 1*s.* 0$\frac{3}{4}$*d.* - - -			
509$\frac{1}{3}$lb. of sassafras, at 6$\frac{1}{2}$*d.* - - -			
	£		

Received, at the same time, the contents,

C. Hartley.

(6.) Miss Evitt,

Bought of William Wilson,

London, Sept. 22, 1822.	£.	s.	d.
19$\frac{3}{4}$ yards of Flander's lace, at 9*s.* 8*d.* per yard			
27$\frac{5}{8}$ do. Dresden lace, at 15*s.* 5$\frac{1}{2}$*d.* - -			
113$\frac{1}{2}$ do. gauze, at 2*s.* 2$\frac{3}{4}$*d.* - - -			
215$\frac{1}{4}$ do. muslin, at 7*s.* 5$\frac{3}{4}$*d.* - - -			
25$\frac{5}{6}$ dozen of napkins, at 27*s.* 6*d.* per doz. -			
118 pair of kid gloves, at 1*s.* 8*d.* per pair -			
	£		

(7.) Mr. Crowther,

Bought of Mary Griffiths,

London, October 5, 1822.	£.	s.	d.
114 yards of muslin, at 8*s.* 4$\frac{1}{2}$*d.* per yard -			
17$\frac{3}{5}$ do. Holland, at 4*s.* 6*d.* - - -			
715$\frac{1}{2}$ do. cambric, at 10*s.* 7*d.* - -			
126$\frac{3}{4}$ ells of dowlas, at 1*s.* 2$\frac{1}{2}$*d.* per ell - -			
221$\frac{1}{4}$ do. Irish, at 2*s.* 9$\frac{1}{2}$*d.* - - -			
419$\frac{3}{4}$ do. chints, at 5*s.* 10*d.* - - -			
	£		

Received, at the same time, the contents,

Mary Griffiths.

(8.) Mrs. Mertown,

Bought of John Linsdall,

Winestead, July 9, 1822.	£.	s.	d.
Tares, 104 bushels, at 2*s*. 7*d*. per bush. -			
Peas, 127 bush. 3 pecks, at 1*s*. 10½*d*. per bush.			
Malt, 46¼ quarters, at 1*l*. 14*s*. 4*d*. per quarter			
Oats, 204⅜ do. at 18*s*. 6*d*. per qr. - -			
Beans, 17⅞ do. at 1*l*. 17*s*. 3*d*. per qr. -			
	£		

(9.) Mr. Ochterlony,

To Hudson and Co., Dr.

London, 1822.		£.	s.	d.
Sept. 19.	170 pieces Norwich crapes, at 2*l*. 7*s*. 8*d*. per piece - -			
———	204 pieces of Kendal cottons, at 1*l*. 16*s*. 3*d*. per piece - -			
Oct. 5.	175⅓ yards of Lancashire sheeting, at 1*s*. 2¾*d*. per yard - -			
———	698⅔ yards of Manchester velvet, at 7*s*. 8*d*. per yard - -			
Nov. 7.	537⅛ ells of Yorkshire drab, at 3*s*. 4*d*. per ell - - -			
———	1001 ells of ditto forester, at 6*s*. 8*d*. per ell - - - -			
		£		

(10.) Sir George Lovell,

Bought of Simpson and Co.

London, Dec. 5, 1822.			£.	s.	d.
2	pieces of	fustian, each 27 yds, at 1*s*. 4*d*. per yard - - - - -			
5	do.	Irish, each 25 yds, at 2*s*. 4*d*. -			
7	do.	check, each 31 yds, at 10*d*. -			
8	do.	dowlas. each 29 yds, at 7¼*d*. -			
12	do.	plaid, each 37 yds, at 1*s*. 10*d*. -			
11	do.	dimity, each 18 yds, at 2*s*. 1½*d*. -			
			£		

CLASS II.

Exercising the RULE *of* THREE, *or* PRACTICE.

(11.) Mr. Measurewell,

Bought of Edward Kent,

Dublin, May 5, 1822. *Six parcels of muslin, viz.*

	E. Flem.	qr.	n.			£.	s.	d.
No. 1	24	2	1,	at 6*s*. 9*d*.	per yard			
2	27	1	3,	at 7*s*. 2*d*.	- -			
3	21	0	2,	at 8*s*. 4*d*.	- -			
4	34	1	1,	at 5*s*. 3*d*.	- -			
5	19	2	2,	at 7*s*. 9*d*.	- -			
6	27	1	3,	at 7*s*. 2½*d*.	- -			
						£		

(12.) Mr. Cudworth,

Bought of Hilton Morrison,

London, July 4, 1822.

Cwt.	qr.	lb.		£.	s.	d.
14	1	19	of tobacco, at 4*l*. 17*s*. 2*d*. per cwt.			
17	2	17	of snuff, at 5*l*. 19*s*. 4*d*. - -			
18	3	16	of tobacco in leaf, at 3*l*. 10*s*. 8*d*.			
9	0	15	of sugar, at 2*l*. 12*s*. 6*d*. - -			
	3	10	of soap, at 2*l*. 17*s*. 4*d*. - -			
49	1	9	of molasses, at 1*l*. 16*s*. 4*d*. - -			
				£		

(13.) Anthony How, esq.

	Dr.	£.	s.	d.
To 4 puncheons of Jamaica rum, each 48 gall, at 12*s*. 9*d*. per gallon - - - -				
To 7 pipes of mountain, at 6*s*. 5*d*. per gallon				
To 5 hhds of malaga, at 8*s*. 6*d*. - - -				
		£		

	Cr. £. s. d.
By our bill on Geo. Giles, esq. exchange at 4s. 2¼d. per crown for 450 crowns - -	
By ditto on Monsieur Arbre, exchange at 4s. 5d. per crown, for 840 crowns - - -	
By ditto on Mr. Cordigon, exchange at 4s. 9d. per crown, for 1090$\frac{4}{5}\frac{5}{7}$ crowns - - -	
	£

Errors excepted. £

London, Geo. Keith and Co.
Sept. 22, 1822.

(14.) George Germaine, esq.

To Fairbank, Elmer, and Co., Dr.

London, 1822.		£. s. d.
May 15.	4½ pieces of muslin, each 37¼ yds. at 10s. 7½d. per ell English - -	
——	7¼ pieces of chintz, each 47½ yds. at 4s. 6¼d. per ell English - -	
June 11.	4⅜ pieces of Holland, each 25½ ells Fl. at 3s. 10d. per yard - - -	
——	10⅜ pieces of serge de Nismes, each 19¾ ells Fr., at 2s. 9½d. per yard	
July 1.	1749⅝ yds of Kendal cottons, at 9⅛d. per ell Flemish - - - -	
——	947¼ yds of Manchester stuff, at 10¾d. per ell Flemish - - -	
		£

(15.) William West, esq.

London, Bought of Daniel Jones,

May, 19, 1822.	oz.	dwt.	gr.		£. s. d.
A punch bowl, weight	24	10	4	at 6s. 5d. per oz.	
A tankard, ——	15	7	6	at 5s. 10d. —	
A tea pot and lamp,	35	8	9	at 5s. 7½d. —	
12 plates, ——	126	2	7	at 5s. 8¼d. —	
18 spoons, ——	41	0	16	at 6s. 1d. —	
A waiter, ——	16	8	0	at 6s. 2½d. —	
					£

(16.) Theodore King, esq.

Bought of James Bird,

Hull, June 15, 1822.	£.	s.	d.
Shalloon, 174 ells Eng. at 14¼*d*. per yard -			
Muslin, 24¾ ells Flem. at 4*s*. 9*d*. per yard -			
Russia tick, 17 pieces, each 71½ ells French, at 1*s*. 1⅛*d*. per yard - - - - - -			
Callico, 20 pieces, each 34¼ yards, at 4*s*. 9¾*d*. per ell English - - - - - -			
Gamblet, 174 yards, at 1*s*. 0½*d*, per ell English			
Yorkshire drab, 1000 yards at 3⅙*s*. - -			
	£		

Received, at the same time, the contents,

James Bird.

(17.) Mr. Torin,

To Norris and Co., Dr.

Beverley, 1822.		£.	s.	d.
Feb. 5.	Palmsack, 12½ doz. at 2*l*. 7*s*. 8*d*. per doz.			
May 11.	Port, red, 1¼ hhd. at 6*s*. 9*d*. per gall.			
———	Claret, ½ hhd. at 10*s*. 11*d*. - -			
June 5.	Lisbon, white, 31¼ gal. at 1*s*. 10*d*. per qt.			
———	Rhenish, do., 17¾ gal. at 2*s*. 7½*d*. per qt.			
July 4.	Sherry, do. 25½ gal. at 6*s*. 4*d*. per gal.			
		£		

Received Sept. 22, 1822, the contents in full,

J. Norris.

(18.) Valentine Fawkes, esq.

Bought of William Vickerman,

Hull, August 9, 1822.	£.	s.	d.
194 yds. 1qr. 2n. of muslin, at 5*s*. 9*d*. per yard			
1761 1 3 of linen, at 2*s*. 3*d*. - -			
47 ells Eng. 1qr. 1n. of velvet, at 10*s*. 7½*d*. -			
10 pieces of chintz, ea. 27¼ yds. at 3*s*. 8*d*. per ell E.			
7 pieces of cotton, each 31½ yds. at 1*s*. 10*d*. -			
9 pieces of do. each 34 ells E., at 1*s*. 9*d*½*d*. per yd.			
	£		

(19.) Mr. Carpenter,

To George Minot, Dr.

London, 1822.		£.	s.	d.
Jan. 1.	For 17cwt. 1qr. 15lb. of Virginia tobacco, at 10*l*. 10*s*. per cwt. -			
—— 7.	For 14cwt. 2qr. 4lb. of Jamaica sugar, at 4*l*. 11*s*. 2*d*. per cwt. -			
March 9.	For 15cwt. 11lb. of Barbadoes ditto, at 4*l*. 4*s*. per cwt. - - -			
————	For 179¾lb. of Jamaica pepper, at 7¼*d*. per lb. - - - -			
May 15.	For 4 puncheons of rum, each 84 gal. at 5*s*. 10*d*. per gallon -			
	£			

Received, Aug. 5, 1822, the contents,

George Minot.

(20.) Mr. Willet,

Bought of William Winch,

London, Nov. 4, 1822.	£.	s.	d.
47 pieces of shalloon, ea. 47⅛ yds. at 1*s*. 9½*d*. per ell - - - - - -			
16 pieces of camblet, ea. 39¼ yds, at 1*s*.6¾*d*. —			
27½ —— of drugget, each 27¾ yds, at 2*s*. 2¼*d*, per E. F. - - - -			
49 pieces of calimanco, each 34¼ yds, at 1*s*.8½*d*. per E. F. - - - -			
19⅛ pieces of calico, each 42 ells E. at 2*s*. 11½*d*. per yard - - - -			
17 pieces of chintz, each 41½ ditto at 4*s*, 10¼*d*. per yard - - - -			
£			

CLASS III.

Exercising the RULE of THREE, or PRACTICE, *and* TARE and TRET.

(21.) Mr. Cole,

Bought of George Mitchell,

London, May 1, 1822.

cwt.	qr.	lb.		£.	s.	d.
16	1	19	gross of sugar, tare 121lb., at 3*l*.10*s*. per cwt. neat - -			
21	2	17	—— of ditto, tare 137lb. at 4*l*. 4*s*. per cwt. neat - - -			
19	1	21	—— of raisins, tare 96lb., at 2*l*. 7*s*. per cwt. neat - -			
11	3	14	—— of currants, tare 85lb., at 2*l*.10*s*. 4*d*. per cwt. neat - -			
5	1	17	—— of Pimento, tare 47lb. at 5*l*. 5*s*. per cwt. neat - -			
7	2	19	—— of ginger, tare 74lb., at 5*l*.6*s*.6*d*.			
				£		

Received, at the same time, the contents,

George Mitchell,

(22.) Mr. George Lane,

Bought of James Khuff, 5 bags of cotton, viz.

London, June 5, 1822.

No.	Cwt.	qr.	lb.			qr.	lb.	
No. 1.	5	1	4	gross,	tare	1	4	£. s. d. at 4 18 11 per cwt. neat.
2.	7	2	11	——	—	2	$5\frac{1}{2}$	
3.	4	3	9	——	—		$21\frac{1}{4}$	
4.	5	0	14	——	—	1	$19\frac{1}{4}$	
5.	6	2	17	——	—	2	$14\frac{1}{2}$	
								£

(23.) Messrs. Langton and Co.

To Stephen Memprize, Drs.

Hull, 1822.		£. s. d.
April 8.	To 17cwt. 2qr. 24lb. gross of lump sugar, tare 14lb. per cwt., at 4*l.* 17*s.* 6*d.* per cwt. neat - - -	
———	To 27cwt. 1qr. 19lb. gross of double refined sugar, tare 16lb. per cwt., at 5*l.* 5*s.* per cwt. neat - -	
May 10.	To 19cwt. 3qr. 16lb. gross of rice, tare 8lb. per cwt., at 1*l.* 10*s.* 4*d.* per cwt. neat - - - -	
—— 17.	To 10cwt. 8lb. gross of Malaga raisins, tare 14lb. per cwt., at 3*l.* 1*s.* 5*d.* per cwt. neat - - -	
June 6.	To 8cwt. 3qr. 7lb. gross of currants, tare 7lb. per cwt., at 2*l.* 17*s.* 8*d.* per cwt. neat - - - -	
———	To 1cwt. 1qr. 21lb. of pepper, tare 12lb. per cwt., at 6*l.* 8*s.* 2*d.* per cwt. neat - - - - -	
		£

Received, July 17, 1822, 50*l.* 10*s.* 6*d.* in part of this bill

Stephen Memprize.

(24.) Mr. Henry Chapman,

Bought of George Evitt, 5 barrels of indigo,

London, May 1, 1822.

	Cwt. qr. lb.			
No. 1. qt.	10 2 14	gross, tare 7lb. per cwt.		at 2*s.* 4½*d.* per lb. neat.
2 —	11 3 12	——— 7 ———		
3 —	12 1 17	——— 8 ———		
4 —	9 2 14	——— 8 ———		
5 —	10 1 14	——— 7 ———		
			£	

(25.) Mr. Amutie,

Bought of William Wilson,

London, March 5, 1822. £. s. d.

7 hhds of sugar, each 10cwt. 1qr. 12lb. gross, tare 17lb. per hhd, at 2*l.* 8*s.* 10*d.* per cwt. neat - - - - - - - -

3 hhds of pimento, each 4cwt. 7lb. gross, tare 21lb. per hhd, at 5*l.* 1*s.* 6*d.* per cwt. neat

5 hhds of ginger, each 7cwt. 3qr. gross, tare 13lb. per hhd at 6*l.* 7*s.* 4*d.* per cwt. neat

6 hhds of pepper, each 3 cwt. 2qr. 9lb. gross, tare 19lb. per hhd, at 5*l.* 7*s.* 3*d.* per cwt. neat

3 hhds of tobacco, each 12cwt. 1qr. 24lb. gross, tare 29lb. per hhd, at 6*l.* 6*s.* 8*d.* per cwt. neat

£

(26.) Francis Clarke, esq.

Bought of George Jenkins,

London, April 9, 1822.

Five butts of currants, *viz.*

No. 1. 4cwt. 1qr. 12lb. gross, tare 19lb. per cwt. tret 4lb. per 104lb.
2. 9cwt. 2qr. 17lb. gross, tare 21lb. per cwt. tret 4lb. per 104
3. 8cwt. 3qr. gross, tare 9lb. per cwt. tret 4lb. per 104 -
4. 7cwt. 11lb. gross, tare 47lb. in the whole, tret 4lb. per 104
5. 9cwt. 1qr. 9lb. gross, tare 7lb. per cwt. tret 4lb. per 104 -

at £$2\frac{1}{11}$ per cwt. neat.

£

(27.) Granville King, esq.

Bought of John Russel,

London, May 10, 1822.	£.	s.	d.
Tobacco in leaf, 19cwt. 1qr. 27lb. gross, tare 149lb. at 5*l*. 0*s*. 4*d*. per cwt. neat - -			
Ditto in rolls, 12cwt. 3qr. 19lb. gross, tare 48½lb. at 5*l*. 17*s*. 8*d*. per cwt. neat - -			
Pimento, 4cwt. 2qr. 25lb. gross, tare 17¾lb. at 7*l*. 13*s*. 5*d*. per cwt. neat - - -			
Cotton, 16cwt. 0qr. 17lb. gross, tare 125lb. at 4*l*. 15*s*. 4*d*. per cwt. neat - - -			
Sugar, 21cwt. 1 qr. 2lb. gross, tare 158¼lb. at 2*l*. 1*s*. 7*d*. per cwt. neat - - -			
Nutmegs, 3cwt. 0qr. 6lb. gross, tare 12¼lb. at 15*l*. 8*s*. 9*d*. per cwt. neat - - -			
	£		

Received, at the same time, the contents,

John Russel.

(28.) Mr. John Grant, Dr.

To J. H. Wicks, for 5 bags of Pimento,

London, Oct. 8, 1822. *viz.*

	Cwt.	qr.	lb.	lb.	
No. 21. Wt. gross	0	3	19, tare	7½	£. s. d.
22. ———	1	0	4, ——	9¼	at 4 19 2
36. ———	0	3	2, ——	8¾	per cwt.
37. ———	1	1	5, ——	10	neat.
41. ———	1	0	9, ——	9½	

£

(29.) Wilmer Willet, esq.

Bought of Francis Duke, 6 butts of madder,

London, Nov. 4, 1822. £. *s.* *d.*

No. 1. Wt. gross 11cwt. 2qr. tare 14lb. per cwt. tret 4lb. per 104lb. and cloff 2lb. for every 3cwt. at 3*l.* 5*s.* per cwt. neat -

2. Wt. gross 10cwt. 1qr. 14lb. tare 7lb. per cwt. tret 4lb. per 104lb. and cloff 2lb. for every 3cwt. at ditto - - -

3. Wt. gross 9cwt. 3qr. tare 16lb. tret 4lb. per 104lb. and cloff 2lb. for every 3cwt. at ditto - - - -

4. Wt. gross 12cwt. 16lb. tare 8lb. per cwt. tret 4lb. per 104lb. and cloff 2lb. for every 3 cwt. at ditto - - -

5. Wt. gross 9cwt. 1qr. 14lb. tare 12lb. per cwt. tret 4lb. per 104lb. and cloff 2lb. for every 3cwt. at ditto - -

6. Wt. gross 10cwt. tare 10lb. per cwt. tret 4lb. per 104lb. and cloff 2lb. for every 3cwt. at ditto - - - -

£

CLASS IV. *Exercising* DUODECIMALS, &c.

(30.) Sir Leonard Hurt,

Sproatley, 1822. To John Simpson, Dr. £. *s.* *d.*

To flooring three rooms, each 17ft. 9in. 4pts. by 14ft. 9in., the fire-place in each being 4ft. 4in. by 3ft. 7in., at 7*l.* 4*s.* 6*d.* per square -

To wainscoting 4 rooms, each 47ft. 9in. round, and 9ft. 7in. high, the 4 window-shutters each 6ft. by 3ft. 7in., and the doors 7ft. by 4ft., are reckoned work and half, at 6*s.* 9½*d.* per square yard - - - - - - - -

To 74½ days work, at 2*s.* 9*d.* per day - -

Received, December 5, the contents,

£

John Simpson.

(31.) Mr. Gresham,

To William Milner, Dr.

Sproatley, Jan. 1, 1822.	£.	s.	d.
To raising a brick wall, 174ft. 6in. long, 6ft. 11in. high, and $3\frac{1}{2}$ bricks thick, at 5*l.* 17*s.* per rod			
To the brick-work of a house, viz. 56ft. round, and 28ft. 7in. high, 15ft. 4in. of which is $4\frac{1}{2}$ bricks thick, and 13ft. 3in. $3\frac{1}{2}$ bricks thick, at 6*l.* 11*s.* per rod - - - - - -			
To paving a court-yard, 61ft. 6in. by 54ft. 9in. at $3\frac{1}{4}$*d.* per foot - - - - - -			
To tiling 15 squares, 75ft. by 54ft. 9in. at 2*l.* 2*s.* per square - - - - - -			
£			

(32.) John Carttar, esq.

Bought of Thomas Adamson,

Hull, June 5, 1822.	£.	s.	d.
15 planks of beech, each 10ft. 11in. long, and 15 inches broad, at $2\frac{1}{4}$*d.* per square foot -			
27 fir ditto, each 12ft. 8in. long, and 14in. broad, at $1\frac{1}{2}$*d.* per square foot - - - - -			
$7\frac{1}{2}$ loads of oaken timber, at 2*l.* 14*s.* per ton -			
$10\frac{3}{4}$ ton of elm ditto, at 2*l.* 12*s.* per load -			
1437 deal boards, at 5*l.* 8*s.* 4*d.* per hundred -			
£			

Received, at the same time, the contents,

Thomas Adamson.

(33.) Messrs. Mount and Son,

To Henry Edmons, Dr.

London, April 4, 1822.	£.	s.	d.
To paving a court-yard, 50ft. 9in. by 40ft. 7in. 8pts. at 2*s*. 8½*d*. per yard - - - - -			
To paving a stable with clinkers, 17ft. 10in. 5pts. by 10ft. 4in. 11pts. at 4*s*. 2¼*d*. per yard -			
To a brick wall 294ft. by 8ft. 10in. 2pts. and 2½ bricks thick, at 5*l*. 9*s*. per rod - - -			
To ceiling 5 rooms, each 12ft. 4in. 10pts. by 8ft. 0in. 8pts. at 8¾*d*. per yard - - - - -			
To slating a house, 29ft. by 18ft. 10in. the roof of a true pitch, and the eaves-boards projecting 15 inches, at 6*l*. 9*s*. per square - - -			
	£		

(34.) Mr. Constant,

To Benjamin Lancaster, Dr.

York, May 5, 1822.	£.	s.	d.
To flooring 3 rooms, each 17ft. 6in. by 13ft. 4in. at 6*l*. 17*s*. per square - - - - -			
To wainscoting ditto, being 61ft. 8in. round, and 10ft. high, (including the cornice and moulding) at 6*s*. 2*d*. per square yard - - -			
To 174 oaken planks, at 7*l*. 12*s*. 6*d*. per hundred			
To 215 deal planks, at 4*l*. 17*s*. 2*d*. per hundred			
To flooring an out-room, being 19ft. 7in. 4pts. by 11ft. 2in. at 4*l*. 12*s*. 1*d*. per square -			
	£		

(35.) Mr. Craven, To Francis Oldfield, Dr.

Helmsley, June 4, 1822. £. s. d.

To slating a barn, length 27ft. breadth 15ft. the eaves boards projecting 1ft. 4in. at 6*l.* 7*s.* 4*d.* per square - - - - - - -

To plastering a room 19ft. 10in. by 9ft. 6in. at 1*s.* 3½*d.* per yard - - - .

To white washing 3 rooms, each 9ft. high, 27ft. long, and 18ft. wide, the 3 doors each 6ft. 6in. by 3ft. 9in. and 9 windows each 6ft. by 4ft. 9in. at 2½*d.* per yard - - -

£

CLASS V. INVOICES, ACCOUNTS OF SALES, &c.

(36.) *Invoice of* 547 *firkins of butter and* 70 *barrels of pork, laden by me, James Donegall, on board the Cork, Patrick Fitzgerald, master, for the proper account and risk of Thomas Saunders, merchant in London, under the mark, per margin, contents, costs, and charges, viz.*

Dublin, Aug. 7, 1822

					£.	s.	d.
	547 firkins of butter bought of James O'Brien, weight 56lb. each neat, at 1*l.* 4*s.* 7*d.* per cwt. - - -						
	70 barrels of pork, bought of Patrick O'-Neille, at 19*s.* 4*d.* per barrel -						
	CHARGES.	£.	s.	d.			
	To custom of the butter -	2	11	0			
V	Ditto of the pork -	1	14	0			
* *	For 547 firkins - -	15	19	11			
A	Cooperage, hoops, heading, &c.	8	4	0			
T.S.	For 70 barrels, cooperage, &c. of ditto - -	4	5	0			
	Lighterage and Wharfage -	0	17	9			
	Cartage and Portage -	0	19	7			
	To my commission at 2½ per cent.						
	Errors excepted, £						
	James Donegall.						

Quest. What sterling money does this invoice amount to, exchange at 12 per cent.?

(37.) *Invoice of* 14*hhds of tobacco, laden on board the Speedwell, George Panton, master, consigned to James Porter, merchant in London, for his proper account and risk, marked as per margin, contents, costs, and charges, viz.*

Kingston, Jamaica, June 5, 1822.

					£.	s.	d.
	To 14hhds of the best tobacco, weight 84cwt. 1qr. 14lb. tare 2qr. 4lb. per hhd. tret 4lb. per 104lb. at 9¼*d.* per lb.						
	CHARGES.	£.	s.	d.			
	For 14 empty hhds - -	1	8	0			
No8.	Cooperage, hooping, and heading, &c. - - - -	1	1	0			
to22. J.P.	Warehouse-room - - -	0	17	9			
	Boatage and stowage - -	0	14	5			
	Charges at shipping - -	0	19	4			
	For my commission at 3½ per cent.						
	£						

Errors excepted,
Per George Minot.

Quest. What sterling money does the above invoice amount to, exchange at 25 per cent.?

(38.) *Invoice of* 12 *pieces of Holland,* 11 *pieces of cambric, and* 10 *pieces of Ghentish cloth, shipped by me, Abraham Van Schooten, on board the Nancy, Robert Cooke, master, for the proper account and risk of John Harrison, merchant at Hull, marked as per margin, contents, costs, and charges, viz.*

Amsterdam, Jan. 11, 1822.

			G.	S.	F.
	To 12 pieces of Holland, qt. 479⅓ ells Fl. at 1 guild. 4½ stiv. per ell - -				
	To 11 pieces of cambric, qt. 347¼ ells Fl. at 1 guild. 3 stiv. 12 p. per ell -				
	To 10 pieces of Ghentish cloth, qt. 117½ ells Flem. at 18½ stivers per ell -				
	CHARGES.	G. S.			
No.1 to 33. J.H.	To customs and brokerage of the Holland, at 3 guild. per piece	36 0			
	To charges in buying - -	4 17			
	To custom of cambric and Ghentish cloth - - - - -	10 12			
	To canvass, folding and tacking -	4 11			
	To warehouse-room - -	3 14			
	To boatage aboard - - -	1 11			
	To my commission at 2½ per cent.				

Errors excepted,
Abraham Van Schooten.

Quest. What sterling money does the above invoice amount to, exchange at 34*s.* 6*d.* Flemish per £ sterling?

(39.) *Invoice of half a tun of wine and 25 puncheons of prunes, shipped on board the Friendship, John Sampson, master, for the account and risk of Charles Hood Chicheley Plowden, merchant in London, marked as in the margin, contents, costs, and charges, viz.*

Bourdeaux, Nov. 4, 1822.

	Liv.	Sol.	Den.
To 2 hhds of claret, at 50 crowns per tun			
To 25 puncheons of prunes, viz.			

C.H.C.P.

No.	lb.	No.	lb.
1, wt	1000;	14, wt	955
2, —	1120;	15, —	960
3, —	750;	16, —	1710
4, —	594;	17, —	1410
5, —	740;	18, —	940
6, —	1140;	19, —	310
7, —	943;	20, —	412
8, —	1110;	21, —	1101
9, —	541;	22, —	941
10, —	742;	23, —	375
11, —	494;	24, —	948
12, —	175;	25, —	549
13, —	419;		

Tare 79½ lb. per cask, at 2 liv. 17s. 7d. per quintal neat.

CHARGES.

	Liv.	s.	den.
To custom and brokerage of the wine, at 20 liv. per tun			
Ditto of prunes, 4 liv. 15s. per puncheon - - -			
To sledage and boatage of the wine - - -	0	15	0
To ditto for the prunes, at 9 sols per puncheon - -			
To the ship-broker for the prunes, 11 sols per tun -			
To average poor's box, 27 sols per tun - - -			

To my commission at 2½ per cent.

Errors excepted,
Jean Jacques d'Anville.

Quest. What sterling money does the above invoice amount to, exchange at 54¼*d.* per ecu?

(40.) *Invoice of sundry goods shipped on board the Faithful, Hilton Morrison, master, for Barbadoes, by order and for account of Jones and Co., and to them consigned.*

London, July 2, 1822.

		£.	s.	d.	£.	s.	d.
1 to 25	25 Boxes, containing 90 doz. lb. of mould candles, at 10*s*. 4*d*. per dozen -						
26 to 40	15 boxes 40 doz. dipped ditto, at 9*s*. per dozen -						
	40 boxes, at 2*s*. 6*d*. each						
41 to 54	14 boxes 7 cwt. soap, at 90*s*. per cwt. - -						
	14 boxes at 2*s*. each - -						
	Bond to recover drawback	0	17	0			
	Drawback - -	6	6	0			
55 to 56	2 puncheons of refined sugar, weight neat 12 cwt. at 1*s*. per pound - -						
57 to 60	3 chests of tea, at £6 per chest - - -						
	CHARGES.						
	Cartage, lighterage, and wharfage - - -	3	18	6			
	Entry bond, shipping charges, and bills of lading	4	18	9			
	Commission 2½ per cent.						
	Premium of insurance on 200*l*. at 3 per cent. -						
	Stamp duty - -	0	5	0			
	Commission ½ per cent.						

Errors excepted,
John Croft.

(41.) Dr. *Account of Sales of* 15 *pipes of Port Wine, received*

	CHARGES.	£.	s.	d.	£.	s.	d.
1822. Jan. 20.	To duty on 2000 gallons at 2*s*. per gallon - -						
	Excise at 14*l*. 5*s*. per tun of 252 gallons - -						
	Freight, primage, &c. -	11	8	6			
	Cooperage, 3*s*. 6*d*. per pipe						
	Cartage, wharfage, &c. -						
	Vault rent, insurance from fire, and taking stock -	3	5	0			
	Brokerage, 6*s*. per pipe -						
	Landwaiter's fees - -	0	10	0			
	Postage of letters - -	0	10	6			
	Interest on duty and excise advanced to this day at 5 per cent. being 95 days						
	Commission $2\frac{1}{2}$ per cent						
Feb. 24.	To Croft and Co. their account current for net proceeds - - -						

Errors and bad debts excepted,

John Pennell.

by the Hope from Oporto, on account of Croft and Co. Cr.

		£.	s.	d.
1822. Feb. 5.	By Raikes, Newbery, and Co. sold them payable at 2 months. 9 *Pipes.* No. Gallons. 1 - - - - - - 146 2 - - - - - - 140 3 - - - - - - 138 4 - - - - - - 141 5 - - - - - - 139 6 - - - - - - 140 7 - - - - - - 141 8 - - - - - - 141 9 - - - - - - 139 1265 deducting 3 gallons allowed for ullage, viz. 1262 gallons, at £72 per pipe of 139 gallons.			
Feb. 24.	By John Plasket, sold him payable in 3 months. 6 *Pipes.* No. Gallons. 10 - - - - - - 121 11 - - - - - - 122 12 - - - - - - 123 13 - - - - - - 120 14 - - - - - - 119 15 - - - - - - 130 735 deducting 2 gallons for ullage, at £73 a pipe of 136 gallons - -			

CLASS VI.

BILLS OF EXCHANGE, PROMISSORY NOTES, RECEIPTS, &c.

1. Inland Bills of Exchange.

(42.) *Hull*, June 5, 1822.

Sir,

Pay Mr. Thomas Strange, or bearer, one hundred and fifty pounds, and place it to my account.

To Mr. Saunders, C. Hartley.
merchant, *London*.

London, Feb. 10, 1822.

(43.) Messrs. Jones and Co.

Pay William Simpson, or bearer, ninety pounds on account.

Joseph James.

Bristol, Feb. 11, 1822.

(44.) At sight, pay Mr. John Russel the sum of fifty pounds, the value received of Mr. John Hill, and place it to account, as per advice from

To Mr. Stephen Munn, James Trueman.
grocer, *Strand*, *London*.

£300. *Newcastle*, April 5, 1822.

(45.) At fifteen days sight, pay Mr. Richard Thorpe, or order, the sum of three hundred pounds for value received of Sir James Jukes, and place it to account, as per advice from

To Mr. John Harrison, Rd. Hutton.
merchant at *Hull*.

£500. *Glasgow*, May 4, 1822.

(46.) Two months after sight, pay to Sir Christopher Sykes, or order, five hundred pounds, value received of the Right Hon. the Lady Dundas, and place it to account, as per advice from

To Sir James Allpay, *Accepted, May 8th, J. Allpay.* Collin M'Donald.
merchant in *London*.

2. Foreign Bills of Exchange *.

(47.) For £571 18*s.* sterling, at 34*s.* 4*d.* Flemish per £. sterling at usance.

London, Sept. 22, 1822.

At usance, pay this my first bill of exchange to Jacob Vanderberghausen, or order, five hundred seventy-one pounds eighteen shillings sterling, at thirty-four shillings and four-pence Flemish per £. sterling, value received of Samuel James, esq., and place it to account, as per advice from

Your humble servant,
James Willis.

To Mr. Van Schwellingberg,
merchant, *Amsterdam.*

Quest. What is the value of this bill in Flemish money?

(48.) For 7494 guild. 14 stiv. at 35*s.* 4*d.* per £. sterling, at usance.

Amsterdam, June 2, 1822.

At usance, pay this my second bill of exchange, my first not paid, to Charles Johnson, or order, seven thousand four hundred and ninety-four guilders fourteen stivers, at thirty-five shillings and four-pence Flemish per £. sterling, value received of Herman Vanbeck, and place it to account, as per advice from

To James Hall, esq.
merchant in *London.*

Your humble servant,
Simon Van Busching.

Quest. What is the value of this bill in sterling money?

(49.) For 5000 crowns, at 4*s.* 3*d.*

Paris, Sept. 17, 1822.

At one month after sight, pay this my first bill of exchange to James Philips, or order, the sum of five thou-

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* *See the Definitions of Exchange, page* 169, 170, *&c.*

Foreign bills of exchange drawn in sets, according to the custom of merchants, every bill of each set, where the sum shall not exceed £100, is charged with a *stamp duty of one shilling* and *sixpence;* exceeding £100 to £200, *three shillings;* exceeding £200 to £500, *four shillings;* exceeding £500 to £1000, *five shillings;* exceeding £1000 to £2000, *seven shillings* and *sixpence;* exceeding £2000 to £3000, *ten shillings;* and exceeding £3000, *fifteen shillings.*

sand crowns, at four shillings and three-pence each, value received, and place it to account, as per advice of

To Mr. Wm. King, Your humble servant,
merchant in *London*. Jean Du Faur.

Accepted, October 16th,
W. King.

Quest. What is the value of this bill in sterling money?

(50.) For 1576 pieces of eight of Mexico, at $54\frac{1}{2}d.$ each, at three months.

Leghorn, Feb. 14, 1822.

Three months after date, pay this my first bill of exchange to Mr. John La Motte, or order, one thousand five hundred and seventy-six pieces of eight of Mexico, for the value received of himself, at $54\frac{1}{2}d.$ sterling per piece, and place it to account, as per advice from

Your humble servant,
To Mr. Wm. Hintz, James Morini.
merchant in *London*.

Quest. What is the value of this bill in sterling money?

(51.) For 1749*l.* 18*s.* sterling, at $54\frac{1}{4}d.$ per ducat bank, at usance.

London, Jan. 5, 1822.

At usance, pay this my third bill of exchange, my first and second not paid, to Mr. Joshua Sommers, or order, one thousand seven hundred and forty-nine pounds eighteen shillings sterling, in ducats, at fifty-four pence farthing each, and place it to the account of

Your humble servant,
To Mr. Michael Tassoni, James Lamb.
merchant at *Venice*.

Quest. What is the value of this bill in ducats bank?